

Graph Theory Homework 3

Due: 20 Feb 2026 at midnight EST as a PDF on Submittity

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1. Bipartite graph $G_{X,Y}$ has matching M . $\exists v \in X$ such that there is no M -augmenting path from v . Prove that v will not be matched in a maximum match M' .
Hint: consider using induction on the value of $\alpha'(G) - |M|$, where $\alpha'(G)$ is the size of a maximum match on G . (4 pts)
2. We have matchings M and M' on G , where $|M| < |M'|$. Prove or disprove whether $M \Delta M'$ must contain an alternating path as a component. (4 pts)
3. Consider connected graph G where $\forall v \in V(G) : d(v) = 3$ and $\forall e \in E(G) : G - e$ is connected. Place a tight upper and lower bound on the size of possible maximum matchings for G . (4 pts)
4. Draw a connected graph G with $|V(G)| = 6$ and $|M| = 2$, where M is a maximum match. Use Tutte's condition to prove that M is a maximum match. (4 pts)
5. Prove that the Gale-Shapley algorithm will always output a stable solution.
Hint: A straightforward way to do this is by showing that no man is ever rejected by all women. (4 pts)
6. Consider the below graph.
 - (a) Provide a minimum vertex cover C and minimum dominating set D for the below graph. (2 pts)
 - (b) Provide and justify tight bounds on the cardinality of maximum match M' relative to $|C|$. (2 pts)
 - (c) Provide the complement of C , and show that it is an independent set. (2 pts)

