Lecture 3 - Network Measures

Wednesday, January 21, 2026 6:17 PM

Plan for today:

- Discuss paper presentations
  - 15-20 minutes
  - What to include?
    - Intro: What problem is the paper trying to address/solve?
    - Background: What other relevant work has considering this problem or other similar problems.
    - Methods: How exactly are the authors addressing/solving the problem under considerations?
    - Results: What experiments have the authors performed to validate their approach or assumptions or theoretical results?
    - Discussion: What are the broad takeaways from the methods and results with respect to prior work and the problem as a whole?
    - Future Work: What are the next steps this work could take?
  - For examples see: gmslota.com -> Publications -> 'slides' OR
  - gmslota.com -> Presentations -> 'slides'
- Go over lec02.py code example
- k-edge-connectivity and network flow
- Network measures and distributions
- Code example????????

Review of last class

k-connectivity

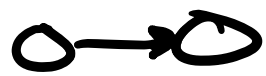
→ network resilience

→ how many vertices to remove to disconnect the graph

directed graphs

directed graphs

- edge have direction



- strong/weak connectivity

=> Connectivity in general gives us insight into some global or underlying structure of some graph

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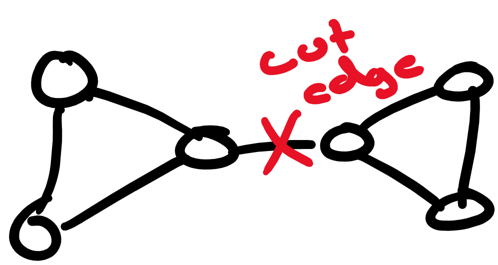
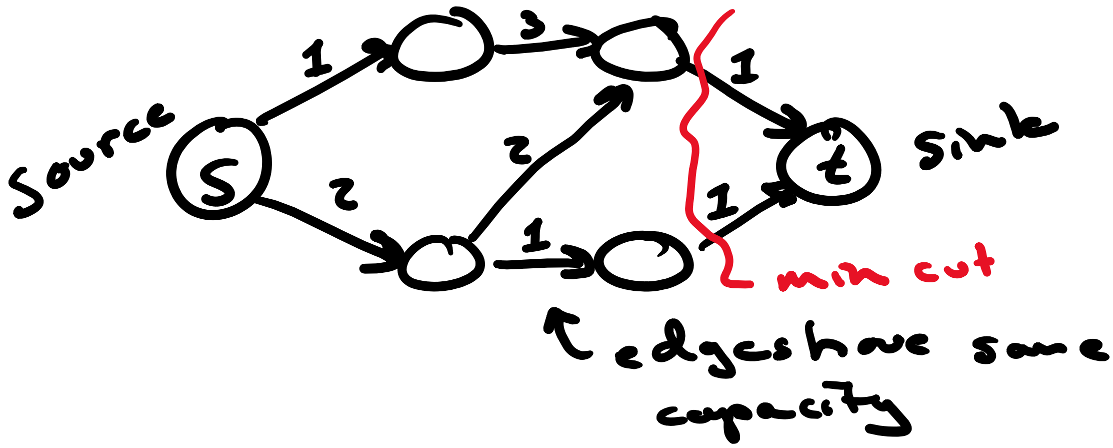
k-edge-connectivity

↳ how many edges to remove to disconnect some G



Algorithmic approaches

Network Flow



Goal: determine max flow

max flow = min cut
(weighted cut)

To determine a min cut G:

consider all possible u, v pairs
as source, sink

Solve for max flow

Min cut?

$\Rightarrow O(n^4)$

Faster algorithms (using variants of flow)

$\Rightarrow O(n^2)$

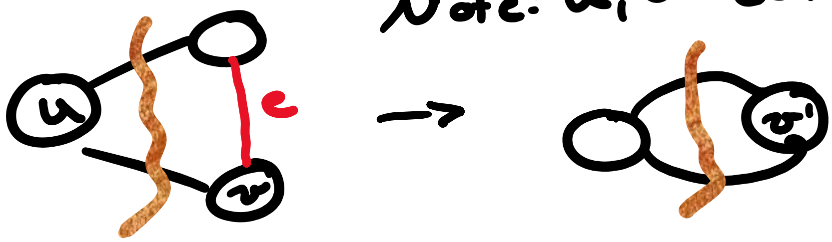
of flow)

$\Rightarrow O(n^2)$

Other approaches

→ consider edge contraction

Note: u, v cut is the same



To contract edge e

→ delete e

→ we combine the endpoints of e, retaining all other adjacencies

auxiliary graph construction

↳ transformation of G into some G' using edge contraction or similar techniques

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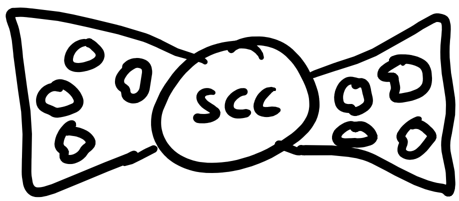
A little more on strong connectivity

context: strong connectivity decompositions



where are all of the strong components?

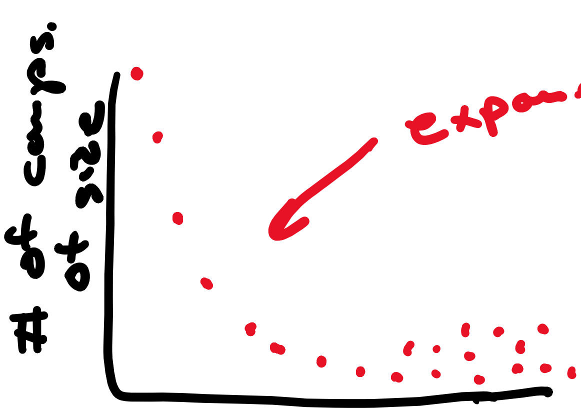
Bring it to "Network measures"

- largest SCC size, proportionally
- total # of SCCs
- <u>size distributions</u> of SCCs
- How many size-1 SCCs?

  How many size 2 SCCs?

  ...

  max size SCCs?



linear fit

fat tail, aka more large SCCs than expected (outliers)

Note: many properties of our real-world fit the above "shape"

→ power-law-ish fit

→ fat tail

E.g. component size distributions

degree distributions

"cluster" size distributions (dense subgraph)

Degree Distributions

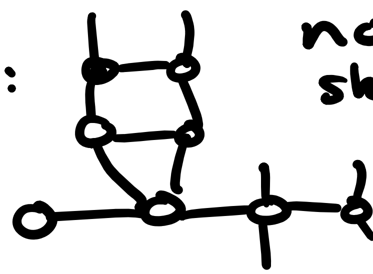
of vertices with a given degree, over all degrees in G

Why do we care?

→ This simple measure that captures and drives a lot of the network measure and topology

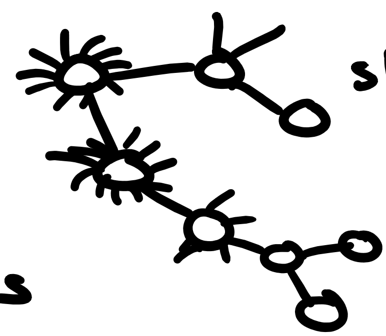
Eg. "skew" → existence of power-law
→ gini coefficient

Road networks:



not skewed

Social networks:



skewed

→ our other measures usually follow the notion skewedness that arises from degree distribution

Side note: Random graph generation and hypothesis testing

→ If we generate some G' with

↳ If we generate some G' with an equivalent D.D. as G, but with randomly wired edges,

→ we can measure the difference in properties of G and G' to explicitly determine the relative "importance" of those properties

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More detail on power-law degree distributions

$$\boxed{P(k) \sim k^{-\gamma}}$$ ← power-law exponent

probability of degree k

proportional

All together: frequency of a degree decreases exponentially with the magnitude of the degree

To actually measure degree dists. and compare them and compare them

(recall: measurements are useless in a vacuum)
(sp?)

1. compute Gini coefficient or similar given a distribution

2. Determine γ power-law exponent

↳ less straightforward

$$\gamma = 1 + n\left[\sum_{v \in V} \log\left(\frac{d(v)}{d_{min}}\right)\right]^{-1}$$

γ = power-law exponent

$d(v)$ = degree of v

d_{min} = min degree (probably 1)

n = number of verts in our distribution

Most real-world graphs

$1 \le \gamma \le 3$ for their degree dist.

more skewed

less skewed

As we've noted:

This relation generally holds for more than just degrees

↳ connectivity distributions
cluster sizes
shortest path lengths

All together, this gives us

=> <u>scale-free</u> graphs

Generally: Fat-tailed distributions with power-law fits define many measurable properties of real networks

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How to graph mine?

Can compare degree distributions
connectivity distributions
cluster distributions

cluster distributions

→ pairwise against other networks

OR

against a null model

Main takeaways:

1. Measurements should not exist in a vaccuum

2. Real graphs are usually skewed
   → arising from power-law degree distribution driving other properties