Pop Quiz 8

CSCI 4030 / CSCI 6220: Randomized Algorithms

Fall 2026, RPI

Let $[n] = \{1, 2, \ldots, n\}$. Suppose we are given M subsets $S_1, \ldots, S_M \subseteq [n]$, each of size at most k. For a two-coloring of the items, assign each $i \in [n]$ a sign $\chi_i \in \{-1, +1\}$ independently and uniformly at random. For each set S_j, define its discrepancy

$$Z_j = \sum_{i \in S_j} \chi_i.$$

You may use Hoeffding's inequality in any standard form from class.

(a) (4 points) Fix a particular set S_j. Use Hoeffding's inequality to bound $\Pr(|Z_j| \geq t)$ as a function of t and k.

(b) (4 points) Use the union bound to bound the probability that at least one of the M sets has discrepancy at least t. Then choose an appropriate value $t^\star$ and conclude that there exists a two-coloring of $[n]$ for which every set S_j has discrepancy strictly less than $t^\star$.

(c) (2 points) Given the following Las Vegas algorithm: repeatedly sample an independent uniform random two-coloring of $[n]$ until every set S_j has discrepancy strictly less than $t^\star$. Using your bound from part (b), give a simple upper bound on the expected number of trials until success.