Pop Quiz 7

CSCI 4030 / CSCI 6220: Randomized Algorithms

Let $G = (V, E)$ be a connected undirected multigraph with $n \geq 2$ vertices. Recall Karger's contraction algorithm:

Algorithm 1: Karger's contraction algorithm

Input: Connected undirected multigraph G, $n \geq 2$
Output: A cut of the original graph, and its number of crossing edges
$H \leftarrow G$; initially each current vertex represents one original vertex
while *H has more than two supervertices* **do**
    Sample an edge uniformly among all current edge *instances*
    Contract its endpoints (remove loops; keep parallel edges)
end
$S \leftarrow$ original vertices represented by either remaining supervertex
return $(S, |\delta_G(S)|)$

(a) (2 points) How many distinct cuts does G have? Briefly justify your answer.

(b) (8 points) Last lecture we proved that for any fixed minimum cut C of G, $\mathbb{P}(\text{Karger's algorithm returns } C) \geq 1/\binom{n}{2}$. Also, tautologically, $\mathbb{P}(\text{Karger's algorithm returns a minimum cut}) \leq 1$. Use these two facts to find an upper bound, as a function of n, on the number of distinct minimum cuts of G. Justify each step of your argument.