Pop Quiz 6

CSCI 4030 / CSCI 6220: Randomized Algorithms

Fall 2026

Rensselaer Polytechnic Institute

[10 points] Problem 1 (Empty bins and dependence).

Suppose that n balls are thrown independently and uniformly at random into m bins. Let X be the number of empty bins.

(a) (4 points) Compute $\mathbb{E}[X]$. Justify your answer using indicator random variables and linearity of expectation.

(b) (6 points) Assume $n = \Theta(m)$ as $m \to \infty$. Explain why the standard multiplicative Chernoff bound for a sum of independent indicator variables cannot be applied directly to X. Then identify and briefly explain at least one tool, or combination of tools, from class that could still show that, for every fixed $\varepsilon > 0$,

$$\Pr(|X - \mathbb{E}[X]| \geq \varepsilon \mathbb{E}[X]) \longrightarrow 0.$$

A complete concentration proof is not required.