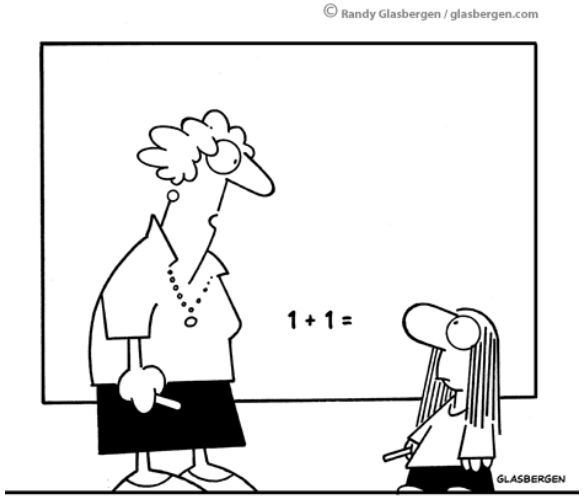
Foundations of Computer Science
Lecture 1

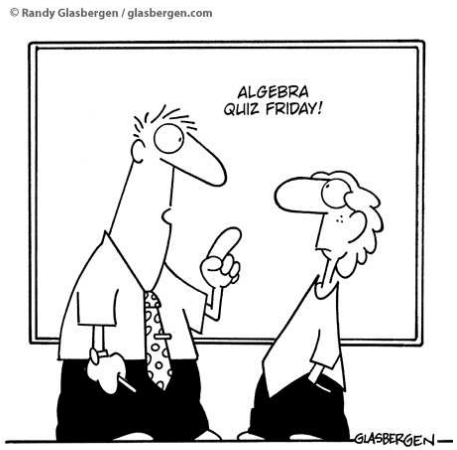
Warmup: A Taste for Discrete Math and Computing

Background

Disease spread, speed-dating, friendship networks



"Yes, this will be useful to you later in life."



"It's important to learn math because someday you might accidentally buy a phone without a calculator."

"In 1953 you were my math teacher. You promised that algebra would come in handy someday. How much longer do I have to wait?"

(Today) Warmup: A Taste for Discrete Math and Computing

1. Resources and Rules
2. Storyline
3. Background
4. A Taste of Discrete Math
   - Two-Contact Ebola on a Grid
   - Scheduling Speed Dates
   - Friendship Networks and Ads
   - Modeling Computers
5. Getting Good at Discrete Math
   - Computing is Mathematics
   - Polya's Mouse

Resources and Rules

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   MATH 1010 (Calc I) MATH 1020 (Calc II STRONGLY recommended)

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The Storyline

1. Discrete objects.

- concepts/concrete
- proof/theory/abstract
- theory of computation

our language will be mathematics . . .
. . . it will be everywhere

The Storyline

1. Discrete objects.
2. Reasoning about discrete objects

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The Storyline

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3. Counting discrete objects

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3. Counting discrete objects
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5. What can we compute?
6. What can we compute efficiently?

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Background

Programming, numbers, geometry, algebra, calculus, . . .

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- Does this set of *positive* numbers have a minimum element:

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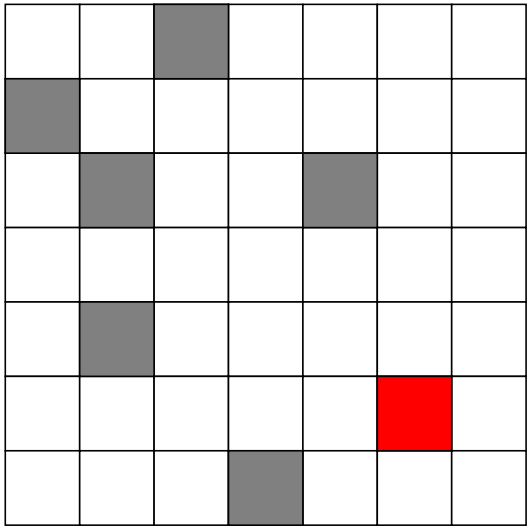
- Does this set of *positive* numbers have a minimum element:

$$\{25, 97, 107, 100, 18, 33, 99, 27, 2014, 2200, 23, \ldots\}$$

Any (non-empty) set containing only **positive integers** has a minimum element.

Two-Contact Ebola on a Grid

A square gets infected if two or more neighbors (N,S,E,W) are infected.



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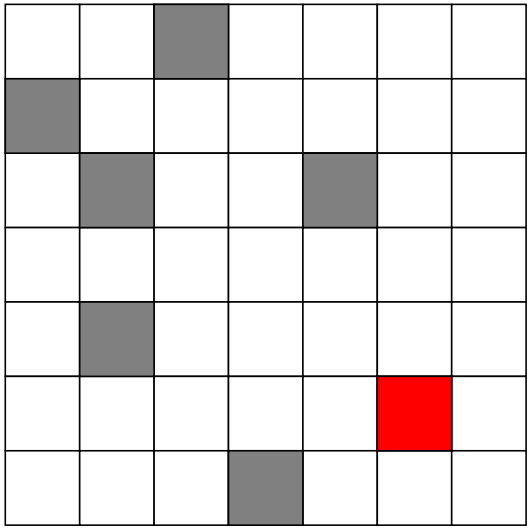

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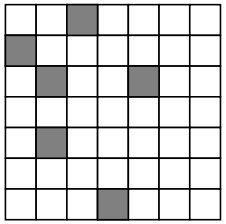

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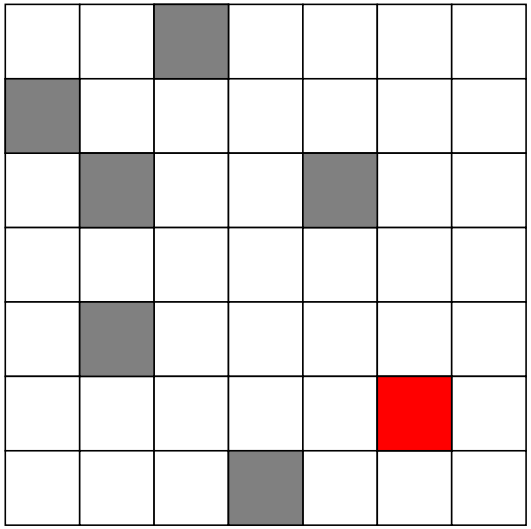

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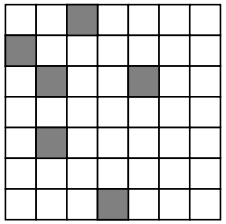

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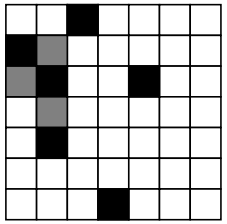

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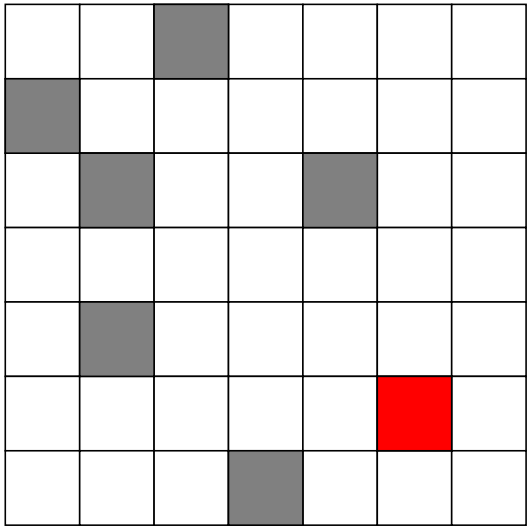

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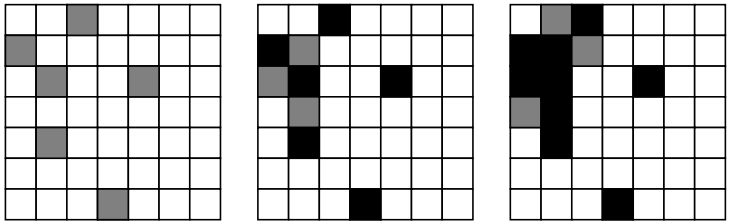

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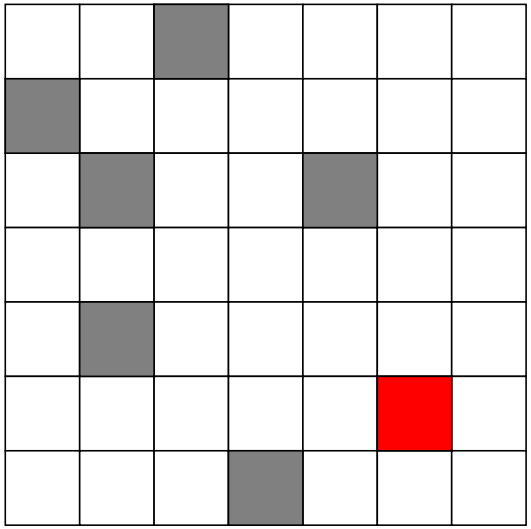

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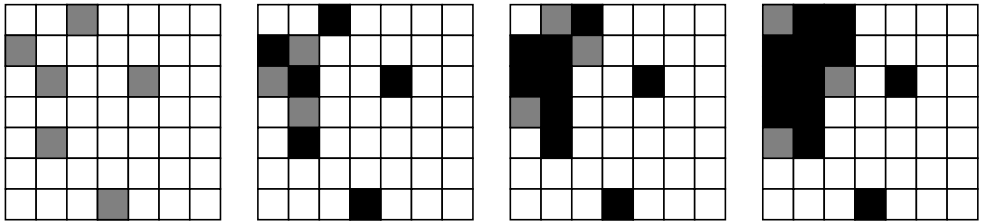

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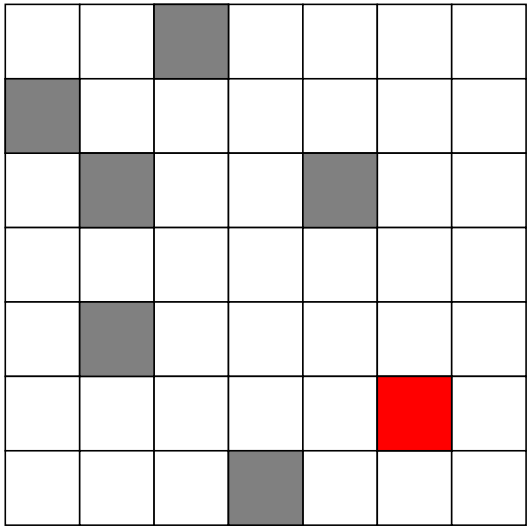

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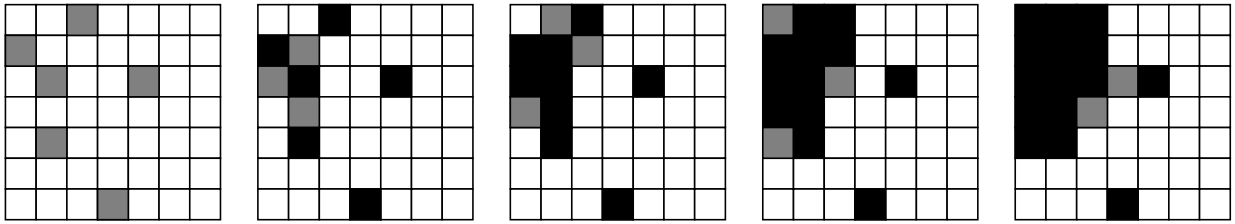

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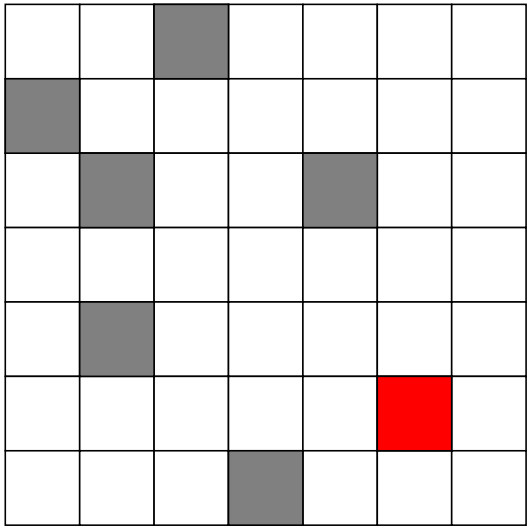

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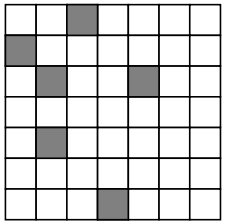
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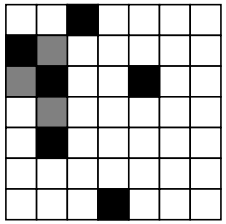
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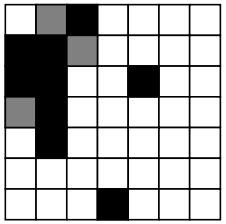
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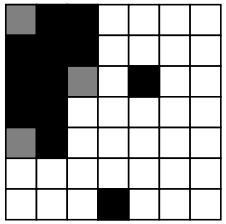
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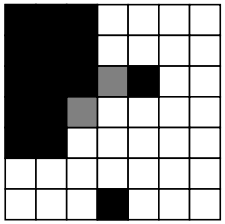
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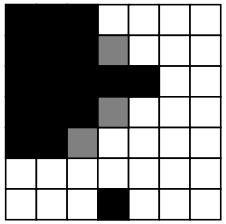
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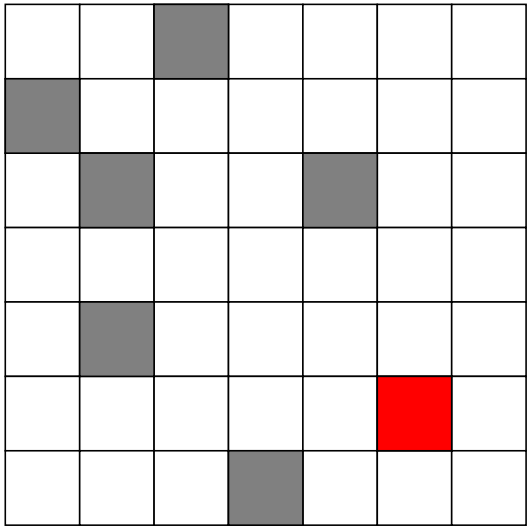

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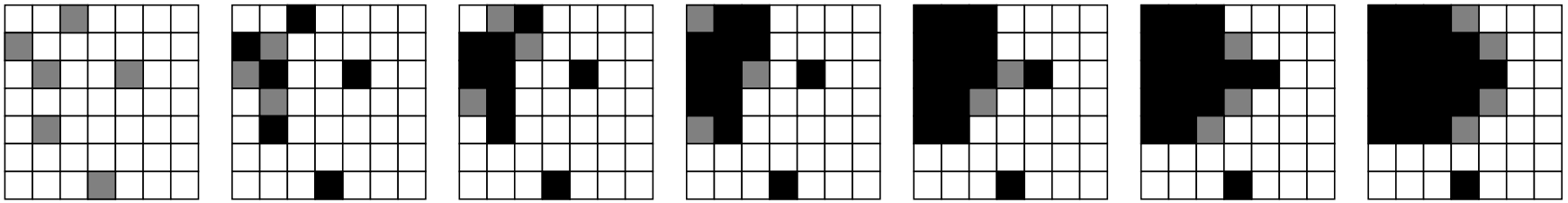

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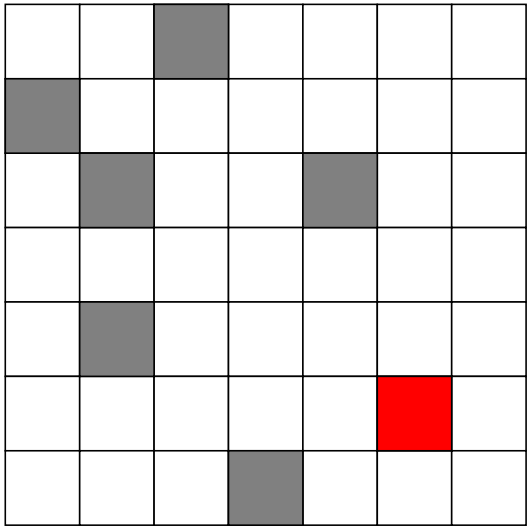

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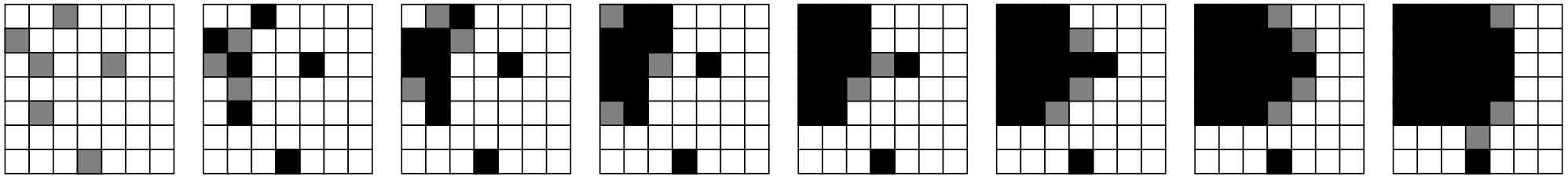

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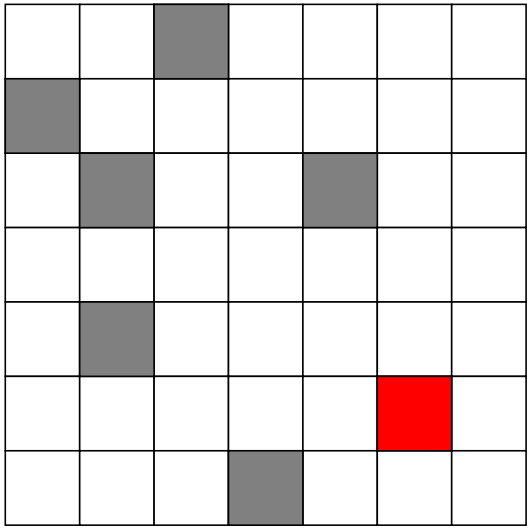
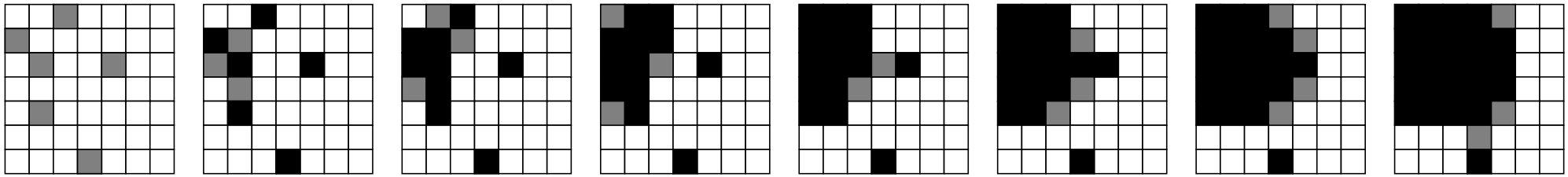


- Given initial gray infections, who ultimately gets infected?
- Minimum infections to infect everyone?



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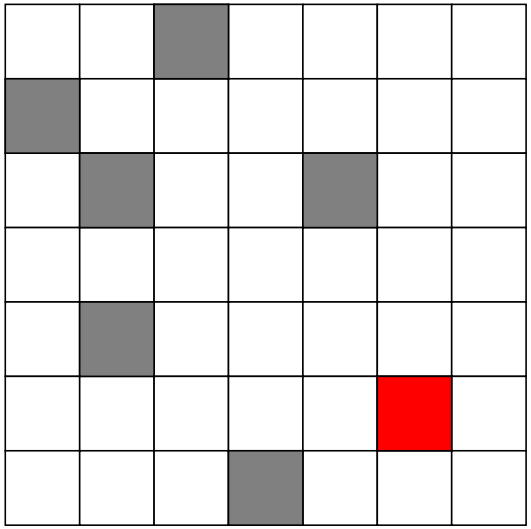
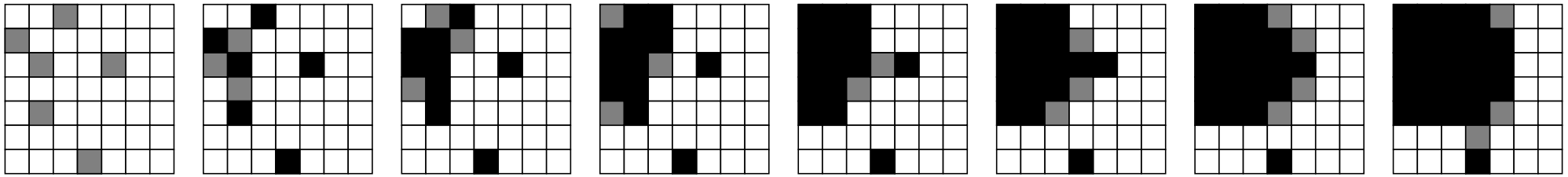


- Given initial gray infections, who ultimately gets infected?
- Minimum infections to infect everyone?
- Given few vaccines, who to immunize?



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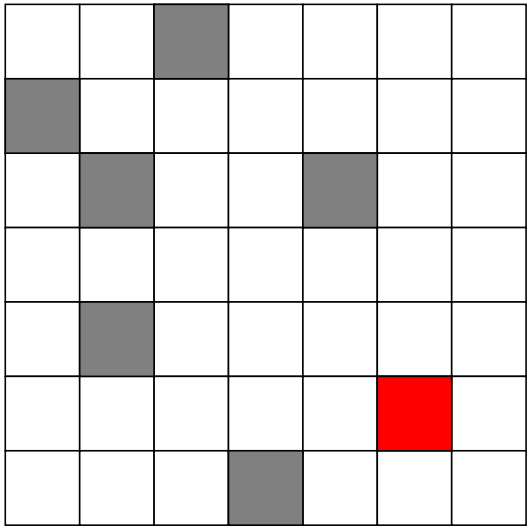

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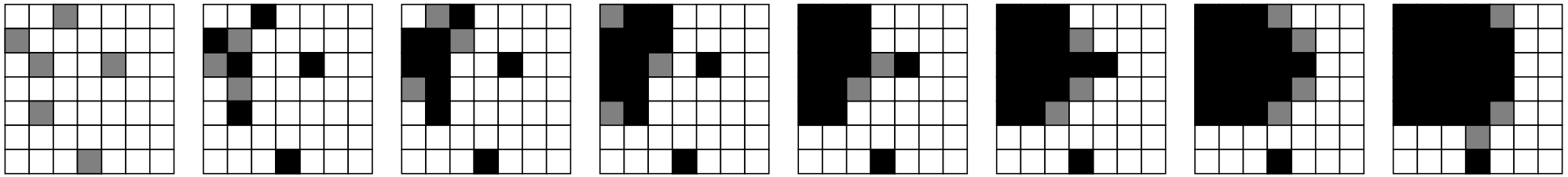
- What were the "entry points"?



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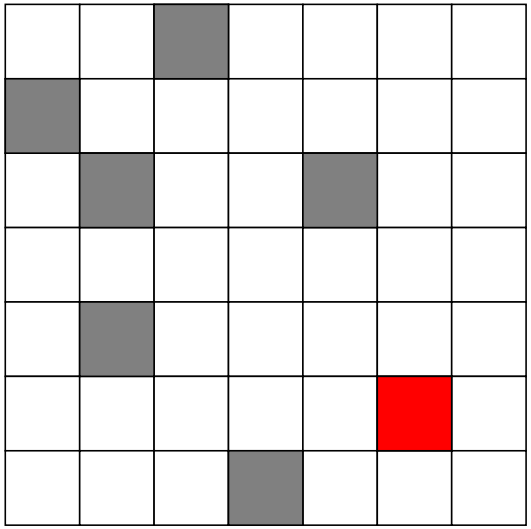

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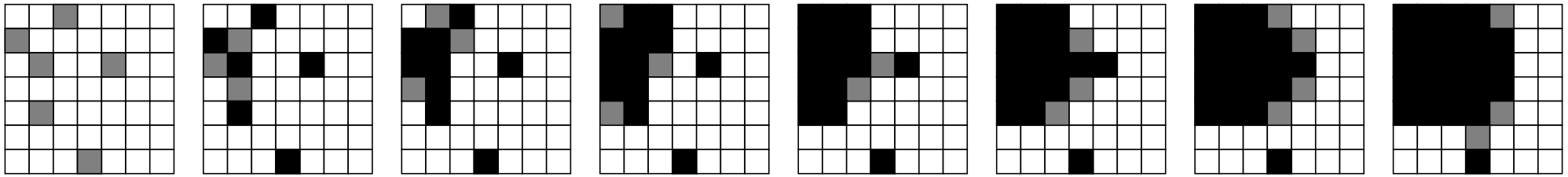
Answers involve discrete math.



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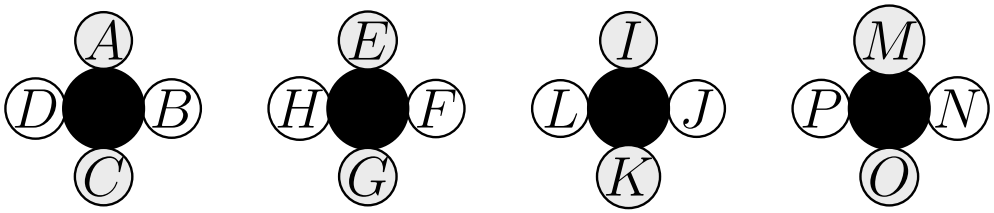
Scheduling Speed Dates

In each round 4 people "group"-speed-date around a table. (4 rounds in all)



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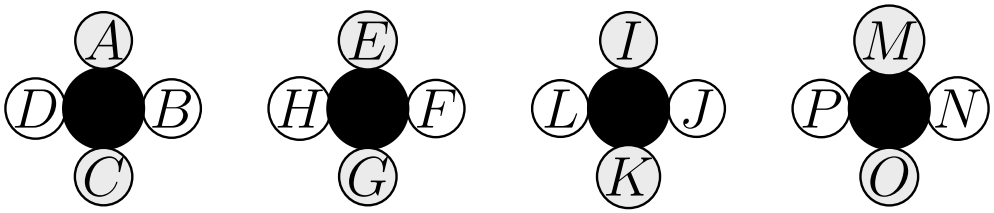

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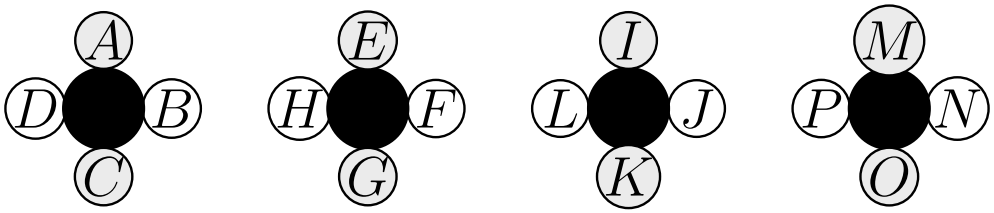

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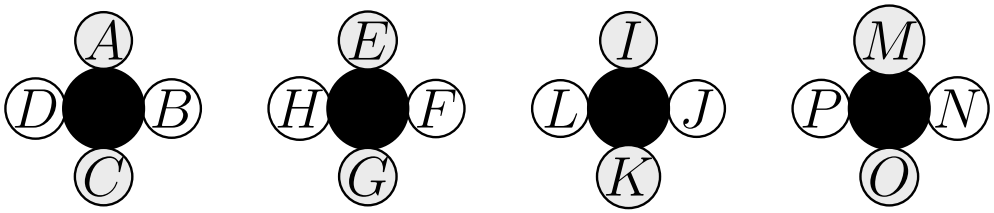

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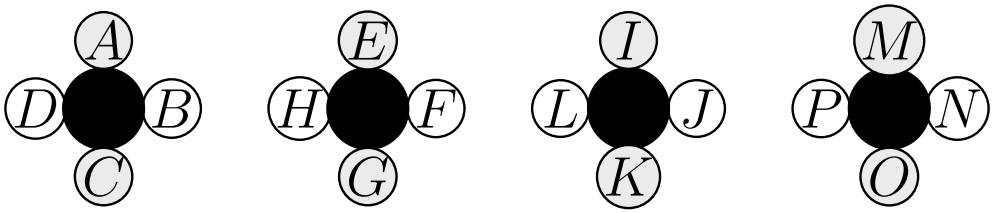
In each round 4 people "group"-speed-date around a table. (4 rounds in all)



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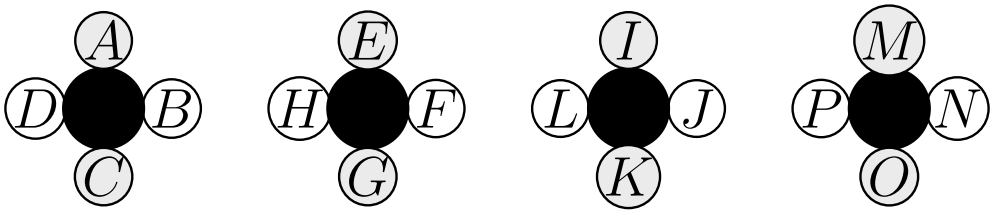

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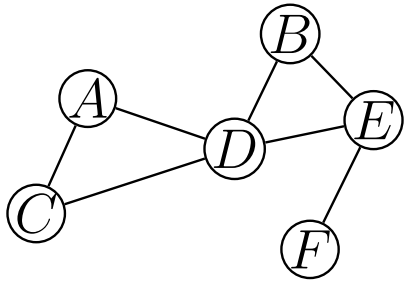

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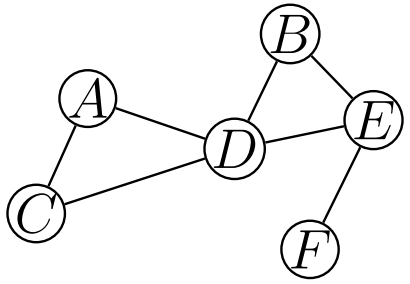

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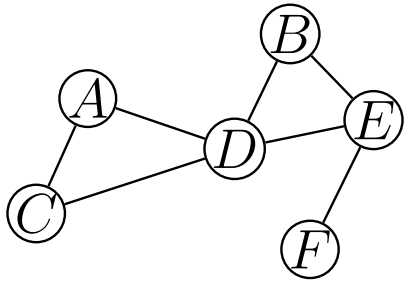

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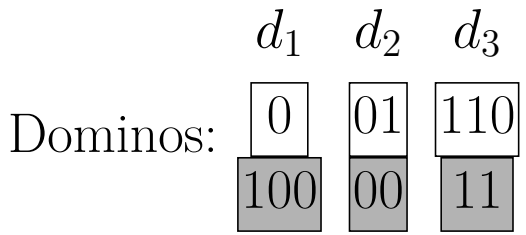
Modeling Computers

Desktop, smartphone, fitbit, ...

What is computing?



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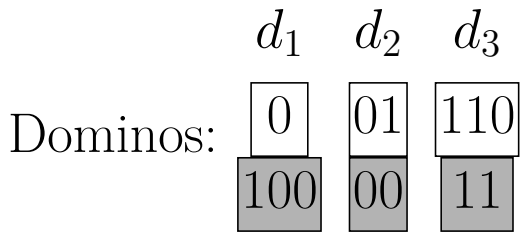

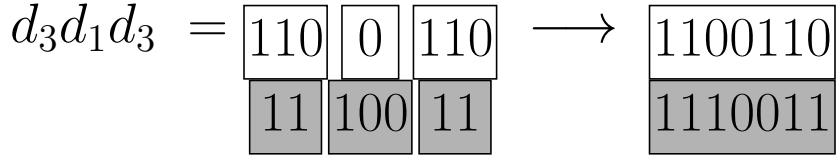

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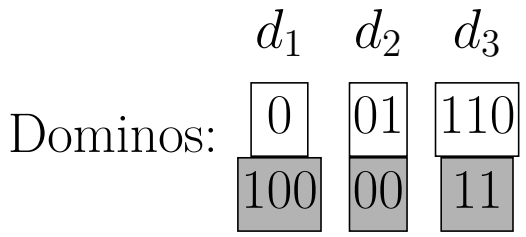

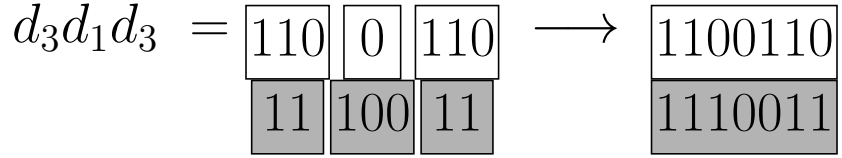

Domino puzzle: Want same top and bottom.

Domino program:

Input: dominos

Output: sequence that works

or

say it can't be done

Answers involve discrete math.

Modeling Computers

Desktop, smartphone, fitbit, ...

We have deep questions:

1. What can we compute?
2. What *can't* we compute?
3. How fast?

Answers involve discrete math.

Computing is Mathematics

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"The required techniques of effective reasoning are pretty formal, but as long as programming is done by people who don't master them, the software crisis will remain with us and will be considered an incurable disease. And you know what incurable diseases do: they invite the quacks and charlatans in, who in this case take the form of Software Engineering Gurus."

– Edsger Dijkstra

Polya's Mouse

"A mouse tries to escape from an old fashioned cage. After many futile attempts bouncing back-and-forth, thumping his body against the cage bars, he finally finds one place where the bars are *slightly* wider apart. The mouse, bruised and battered escapes through this small opening, and to his elation, finds freedom." – Polya

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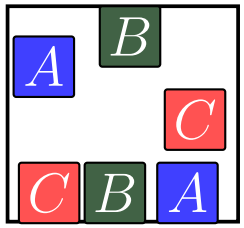

Connect tiles of the same letter with wires. Wires cannot cross, enter tiles, or leave the box. How can it be done? If it can't be done, why not?

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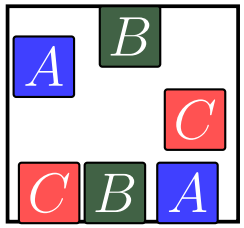

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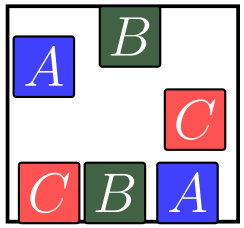

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To solve such problems, "*You need brains and good luck. But, you must also sit tight and wait till you get a bright idea.*" – Polya.

Getting Good at Discrete Math

The *professional*'s workflow in addressing a discrete math problem:

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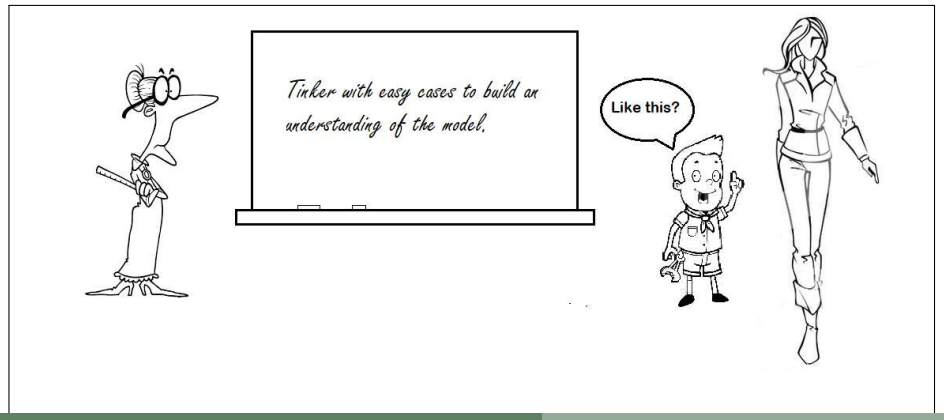
4: Prove the conjecture and make it a theorem. You now *know* something new.

Tinker, Tinker, Tinker,



Tinker!