Homework 6

IMPORTANT NOTE:
Use the PDF problem set posted to the course website; its problem numbering differs from the textbook.

Recitation problems (ungraded)

Attempt these before recitation. We will use recitation to work through selected solutions and the ideas behind them; they are chosen to prepare you for the graded problems below.

(1) DMC Problem 11.8
(2) DMC Problem 11.26. [Similar techniques may be helpful for Homework Problem 11.11!]
(3) DMC Problem 12.2
(4) DMC Problem 12.5
(5) DMC Problem 12.8. [A good warm-up for Homework Problem 12.7!]
(6) DMC Problem 12.20 (a)
(7) DMC Problem 12.28 (j). [A good warm-up for Homework Problem 12.27!]
(8) DMC Problem 12.30
(9) DMC Problem 12.32 (a), (b). Also give a vertex order for which greedy coloring uses two colors.
(10) DMC Problem 12.35 (a)

Problems to submit (100 points)

Show your work and explain your reasoning. A correct answer without adequate justification may receive little or no credit. Write in complete, grammatically sound sentences and submit legible work. The number in square brackets is the number of points each problem is worth.

(1) **[8]** DMC Problem 11.11
(2) **[10]** DMC Problem 11.22 (b), (c). Note that (c) asks you to prove that

$$\sum_{i=1}^{n} \frac{1}{\delta_i} \sum_{j \in N(i)} (\delta_j - \delta_i) = \sum_{(i,j) \in E} (\delta_i - \delta_j) \left(\frac{1}{\delta_j} - \frac{1}{\delta_i} \right).$$

(3) **[10]** DMC Problem 11.22 (d). You may assume the graph is connected.
Hint: What is $(\delta_i - \delta_j)\,(1/\delta_j - 1/\delta_i)$ when $\delta_i = \delta_j$? When $\delta_i \neq \delta_j$?
(4) **[8]** DMC Problem 12.7
(5) **[12]** DMC Problem 12.9 (b). Assume the common vertex degree is at least 1.
(6) **[8]** DMC Problem 12.12 (a)
(7) **[10]** DMC Problem 12.10 (d)
(8) **[6]** DMC Problem 12.20 (b), including the regret calculations.
(9) **[8]** DMC Problem 12.21
(10) **[8]** DMC Problem 12.27
(11) **[12]** DMC Problem 12.32 (c)

Bonus challenge: Tricky Graph Problems (up to 10 bonus points)

This problem is completely optional and is meant to be substantially more challenging than the required homework. Each of these two bonus problems are graded all-or-nothing: a complete solution earns the listed bonus points, while an incomplete proof earns no points. Bonus points are added to your homework score, up to a maximum score of **110/100**.

Bonus Problems:

(1) **[5 bonus points]** DMC Problem 11.70

(2) **[5 bonus points]** DMC Problem 12.39