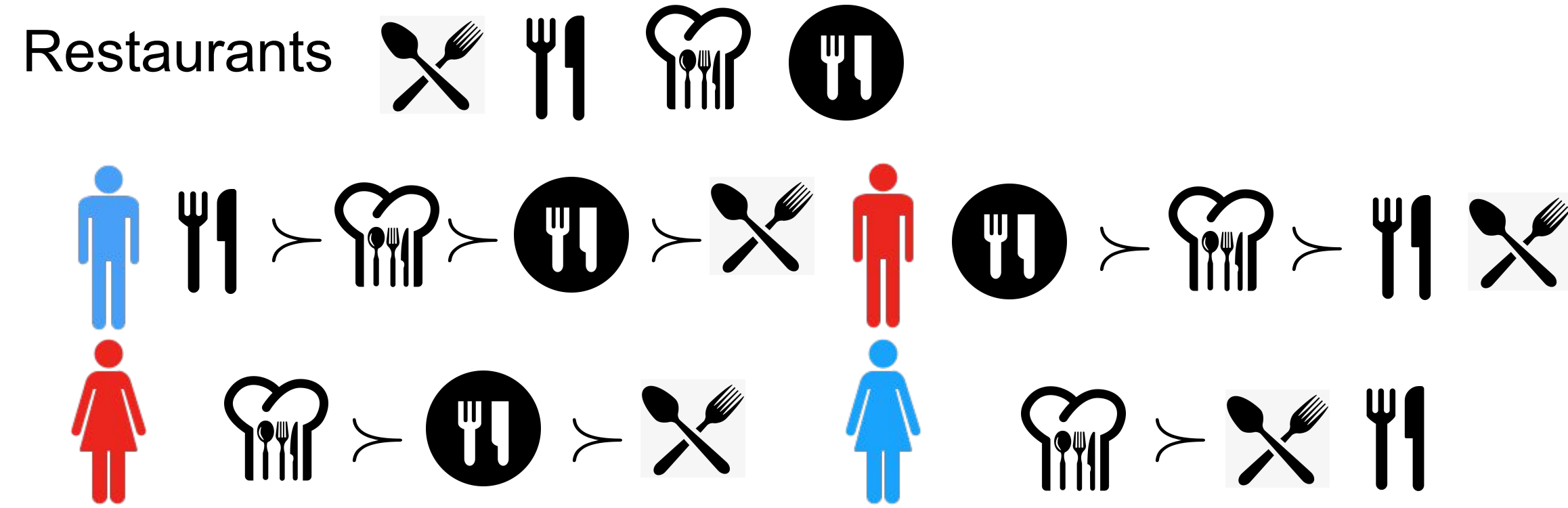


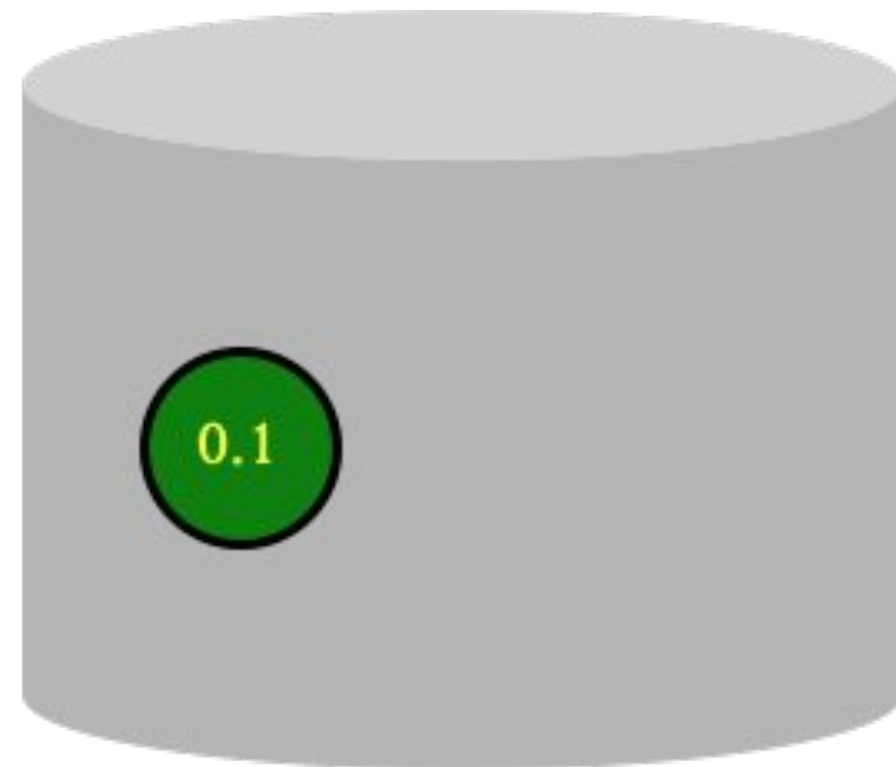
Introduction:

Background



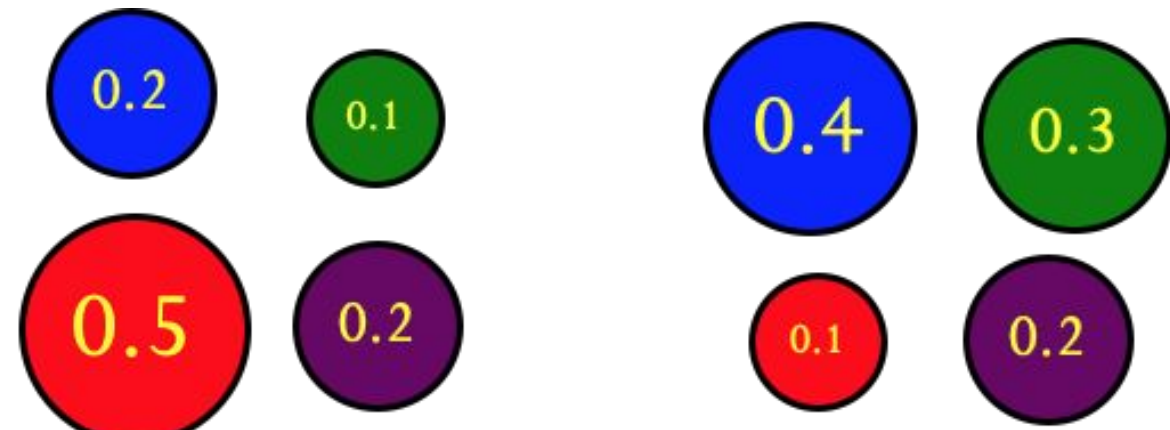
Plackett-Luce Model

$$\frac{0.2}{1} \times \frac{0.5}{0.5 + 0.2 + 0.1} \times \frac{0.2}{0.1 + 0.2}$$



Mixtures of Plackett-Luce Models

$$\text{Pr}_{\text{PL}}(R|\vec{\theta}) = \prod_{i=1}^{m-1} \frac{\theta_i}{\sum_{p=i}^m \theta_p}$$



$$\text{Pr}_{k\text{-PL}}(R|\vec{\theta}) = \sum_{r=1}^k \alpha_r \text{Pr}_{\text{PL}}(R|\vec{\theta}_r) \quad \alpha = 0.2 \quad 1 - \alpha = 0.8$$

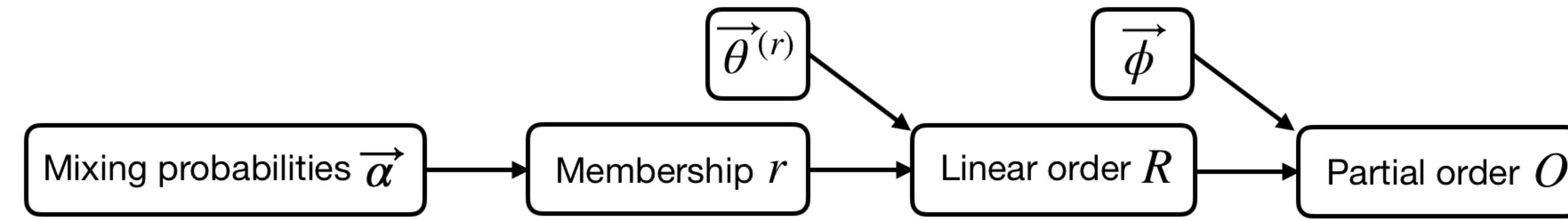
Identifiability

- Full rankings (Zhao et al. 16)
- Structured partial orders?



Contributions:

Modeling



Theoretical Contribution

- **Nonidentifiability:** given top- l_1 and l_2 -way orders, k-PL is not identifiable if $k \leq (l_1 + l_2 - 1)/2$.
- **Identifiability:** 2-PL over 4 or more alternatives are identifiable given: (i) top-3, (ii) top-2 and pairwise, (iii) 4-way, or (iv) choice over 2, 3, 4 alternatives.

Algorithmic Contribution: Generalized-Method-of-Moments

- First step:

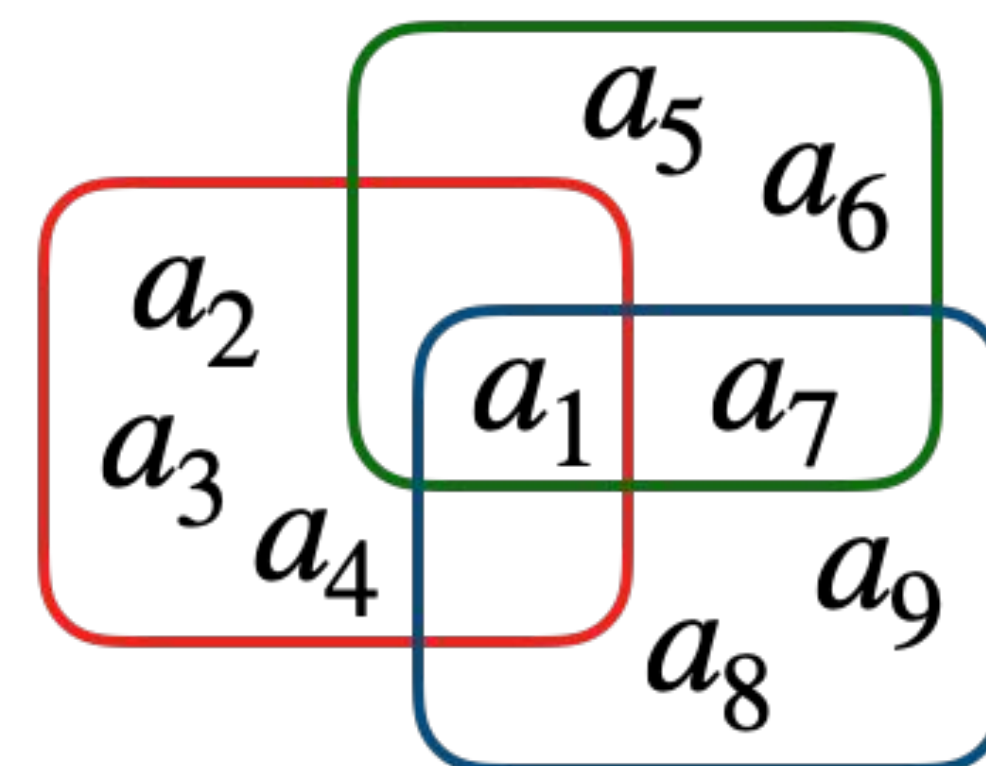
$$\phi_{\mathcal{A}_s}^s = \frac{\# \text{ of orders with structure } (s, \mathcal{A}_s)}{n}$$

- Second step:

$$\vec{\theta}' = \arg \min \sum_{t=1}^q \left(\frac{\text{Pr}_{k\text{-PL}-\Phi}(\mathcal{E}_t|\vec{\theta})}{\phi_{\mathcal{A}_t}^{s_t}} - \frac{\# \text{ of } \mathcal{E}_t}{n\phi_{\mathcal{A}_t}^{s_t}} \right)^2$$

- Selection of events:

- top-2 and 2-way: all
- choice-2, 3, 4: groups of 4 alternatives with at least one overlapping alternative.



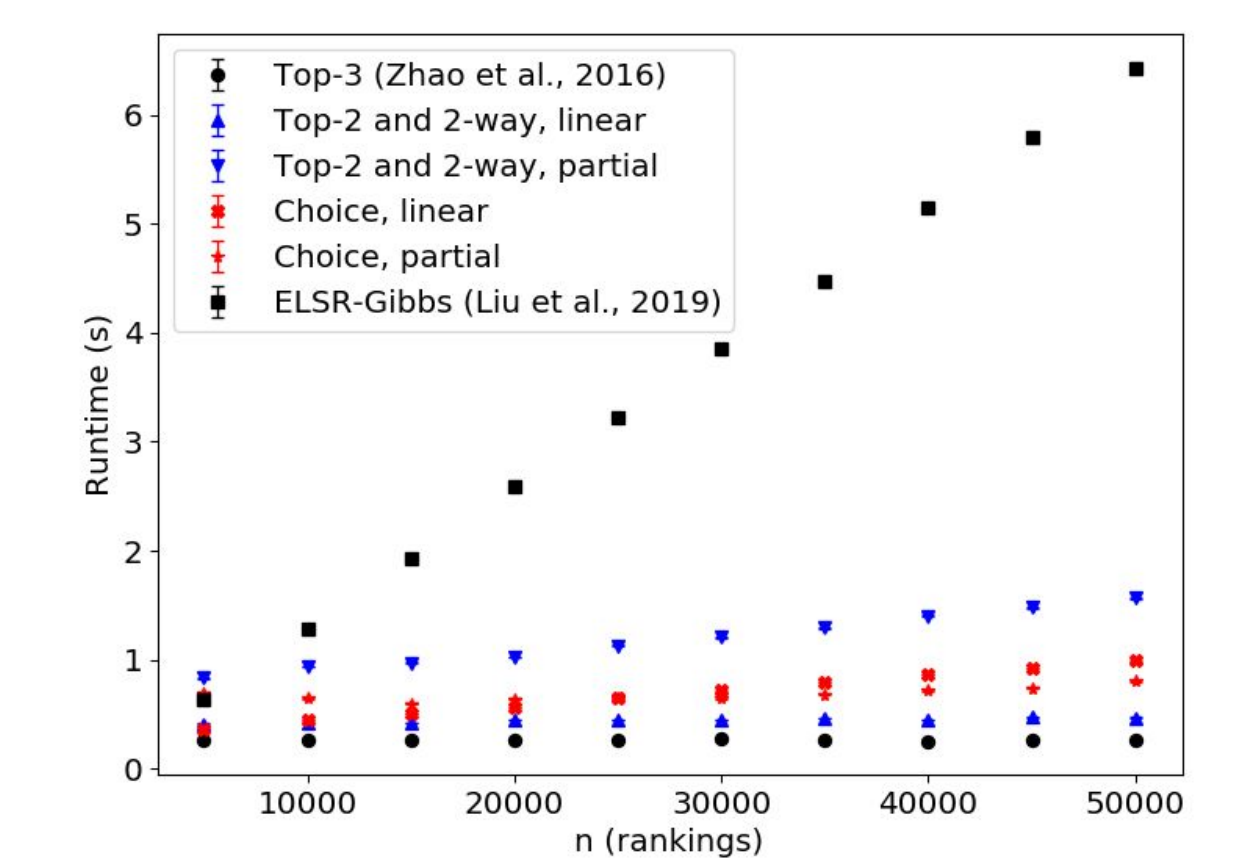
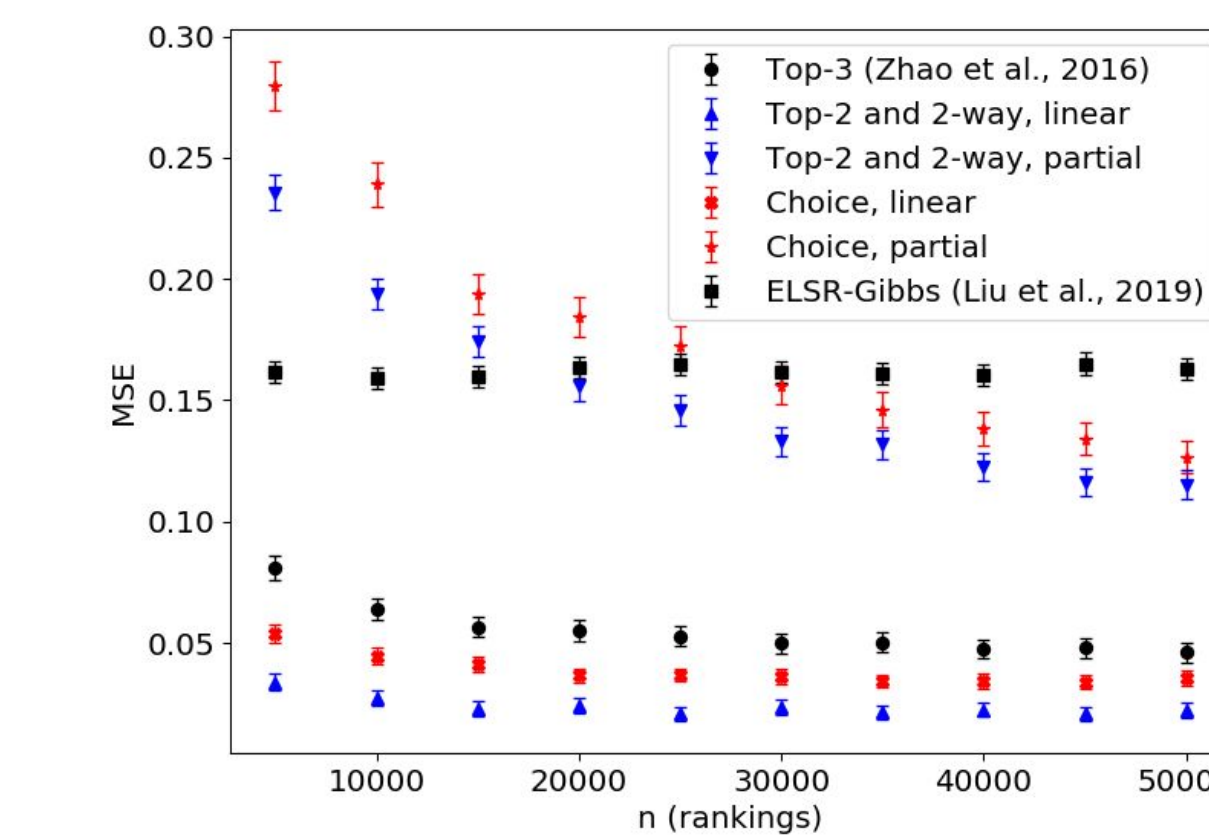
- Each group has four alternatives
- Each group has at least one alternative which is also in another group

Results:

Synthetic data with the following settings:

- Full rankings (linear orders) available
 - Only structured partial orders available
- ### Algorithms
- Top-3: GMM with events of top-3 orders (Zhao et al. 2016)
 - Top-2 and 2-way: GMM with events of ranked top-2 orders and pairwise comparisons (proposed)
 - Choice: GMM with events of choice-2, 3, 4 orders

Results



Conclusions and Future Work:

- We propose a class of models (k-PL-partial) that generate partial orders from k-PL; we prove theorems on (non-)identifiability of k-PL-partial under certain combinations of structures; we propose efficient GMM based algorithms to learn 2-PL-partial.

Interesting future directions include:

- Efficient algorithms for more general partial orders
- Theoretical and algorithmic results for 3 or more components
- Exploring mixtures of more general RUMs beyond PL

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Code available at <https://github.com/zhaozb08/MixPL-SPO>