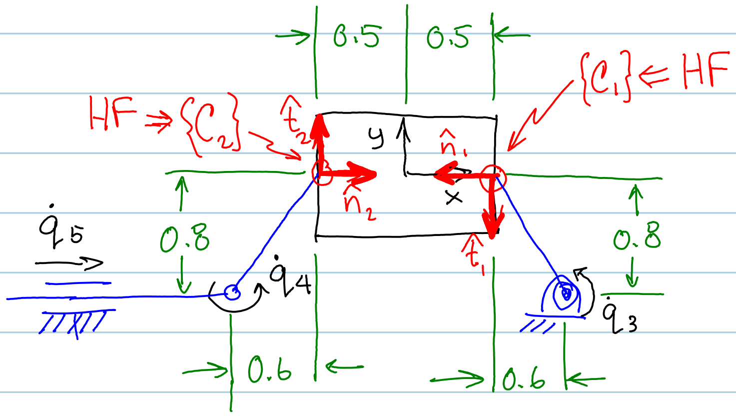
2009_mdtrm_soln

Saturday, February 28, 2009

12:57 PM

0. $\boxed{\text{2D Problem!}}$

21 pts



a. Construct G & J assuming HF contacts.

$$G = \begin{bmatrix} -1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & -\frac{1}{2} & 0 & -\frac{1}{2} \end{bmatrix} \qquad J = \begin{bmatrix} 0.8 & 0 & 0 \\ 0.6 & 0 & 0 \\ 0 & -0.8 & 1 \\ 0 & 0.6 & 0 \end{bmatrix}$$

b.) With the correct G & J, Matlab gives:

$$\text{Rank}(G) = 3 \qquad \text{Rank}(GJ) = 3$$

$$\text{null}(G) = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \qquad \text{null}(J^T) = \begin{bmatrix} 0.6 \\ -0.8 \\ 0 \\ 0 \end{bmatrix}$$

b.i.) Do the contacts provide enough constraint to move the object with a full 3 degrees of freedom? Why or why not?

Yes. $\text{Rank}(G) = 3 = n_\nu$

b.ii.) Can the fingers control all 3 degrees of freedom of the object? Why or why not?

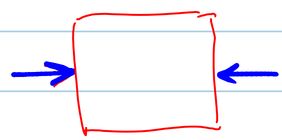
Yes. $\text{Rank}(GJ) = 3 = n_\nu$

b.iii.) Can the fingers control all internal forces? Why or why not?

Yes. Because $\mathcal{N}(G) \cap \mathcal{N}(J^T) = 0$ as seen by the fact that both $\mathcal{N}(G)$ & $\mathcal{N}(J^T) = \mathbb{R}^1$, but the basis vectors are not the same.

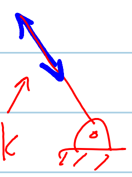
b.iv.) null(G) & null(J^T) represent unachievable contact twists on the hand and object respectively. Give a physical interpretation of these unachievable twists.

$\mathcal{N}(G) \Rightarrow$



can't compress or strech object

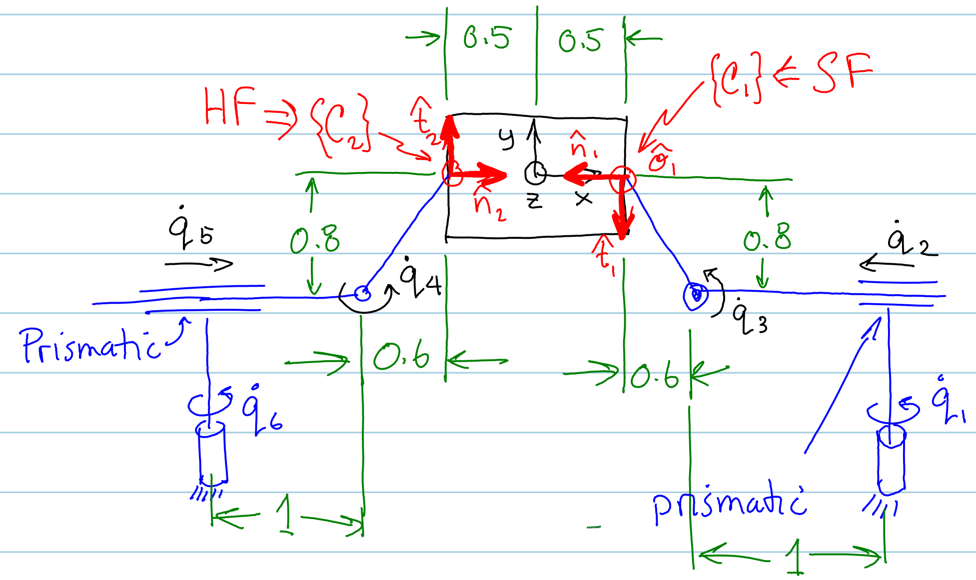
$\mathcal{N}(J^T) \Rightarrow$



can't stretch or compress link

1. $\boxed{\text{3D Problem!}}$

25 pts



a.) Construct G & J.

$$G_{(6\times7)} = \underbrace{\left[\begin{array}{cccc|ccc} -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -\frac{1}{2} & 0 & 0 & 0 & \frac{1}{2} \\ 0 & -\frac{1}{2} & 0 & 0 & 0 & -\frac{1}{2} & 0 \end{array}\right]}_{\text{SF} \qquad\qquad \text{HF}}_{(6\times7)}$$

$$J_{7\times6} = \left[\begin{array}{ccc|ccc} 0 & 1 & 0.8 & & & \\ 0 & 0 & 0.6 & & 0 & \\ 1.6 & 0 & 0 & & & \\ 0 & 0 & 0 & & & \\ \hline & & & -0.8 & 1 & 0 \\ & 0 & & 0.6 & 0 & 0 \\ & & & 0 & 0 & -1.6 \end{array}\right]$$
$\qquad$ finger 1 $\qquad$ finger 2

b.) With the correct G & J, Matlab gives:

$\text{Rank}(G) = 6 \qquad\qquad \text{Rank}(GJ) = 5$

$$\text{null}(G) = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} \left.\vphantom{\begin{matrix}1\\0\\0\\0\end{matrix}}\right\} \text{SF} \\ \left.\vphantom{\begin{matrix}1\\0\\0\end{matrix}}\right\} \text{HF} \end{matrix} \qquad\qquad \text{null}(J^T) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} \left.\vphantom{\begin{matrix}1\\0\\0\\0\end{matrix}}\right\} \text{SF} \\ \left.\vphantom{\begin{matrix}1\\0\\0\end{matrix}}\right\} \text{HF} \end{matrix}$$

b.i.) Do the contacts provide enough constraint to move the object with a full 6 degrees of freedom? Why or why not?

Yes. $\text{rank}(G) = 6 = n_\nu$

b.ii.) Can the fingers control all 6 degrees of freedom of the object? Why or why not?

No. $\text{rank}(GJ) = 5 \neq n_\nu$

(SEE ADDITIONAL EXPLANATION AFTER SOLN b.iv.)

b.iii.) Can the fingers control all internal forces? Why or why not?

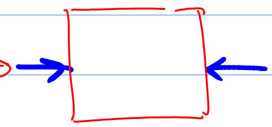
Yes! $\mathcal{N}(G) \cap \mathcal{N}(J^T) = 0$, since both are $\mathbb{R}^1$, but basis vectors don't align.

b.iv.) null(G) and null(J^T) represent unachievable contact twists on the object and hand respectively. Give a physical interpretation of these unachievable twists.

$\mathcal{N}(J^T) \longrightarrow$ $f_{?}$

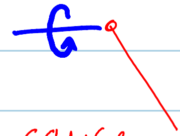
twists.

$\eta(G) \rightarrow$



can't compress object

$\eta(J^T) \rightarrow$



joints can't cause rotation of contact point about $\hat{n}$.

Also note that $J\dot{q} = \nu_{cc}$ & $G^T\nu = \nu_{cc}$. Control means that we can generate any ν by choice of $\dot{q}$

$$J\dot{q} = G^T\nu \Rightarrow \nu = (G^T)^+ J\dot{q} + N(G)\alpha$$

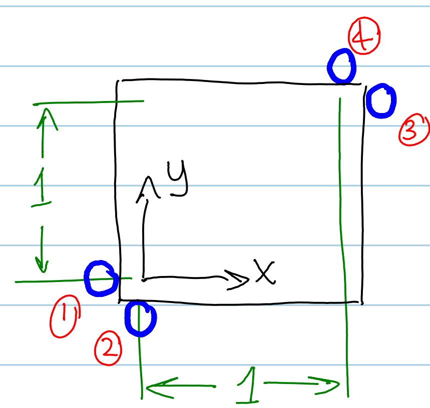
$\uparrow$ arbitrary

Control is thru the mapping $(G^T)^+ J$, which has a row of zeros corresponding to ω_x. This means that no choice of $\dot{q}$ can affect/control ν's ω_x component!

2. $\boxed{\text{2D Problem!}}$

25 pts

Determine analytically if the grasp has form closure.



$$G_n^T \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} \geq 0 \overset{?}{\Rightarrow} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} = 0$$

form closure.

$$G_n = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & -1 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{\theta} \end{bmatrix} \geq 0$$

$\dot{x} \geq 0$

$\dot{y} \geq 0 \Longrightarrow -\dot{y} \leq 0$

$-\dot{x} + \dot{\theta} \geq 0 \Longrightarrow \dot{\theta} \geq \dot{x} \geq 0 \Rightarrow \dot{\theta} \geq 0$

$-\dot{y} - \dot{\theta} \geq 0 \qquad \dot{\theta} \leq -\dot{y} \leq 0 \Rightarrow \dot{\theta} \leq 0$

$\Rightarrow \Rightarrow \boxed{\dot{\theta} = 0}$

Substitute $\dot{\theta} = 0$ back in to get

$\dot{x} \geq 0, \; -\dot{x} \geq 0 \Longrightarrow \dot{x} = 0$

$\dot{y} \geq 0, \; -\dot{y} \geq 0 \Longrightarrow \dot{y} = 0$

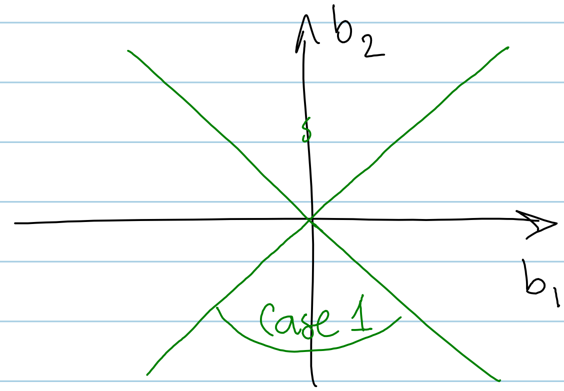
q.e.d.

3. Given the following LCP:

(25 pts)

$$0 \leq \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} \perp \begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \geq 0$$

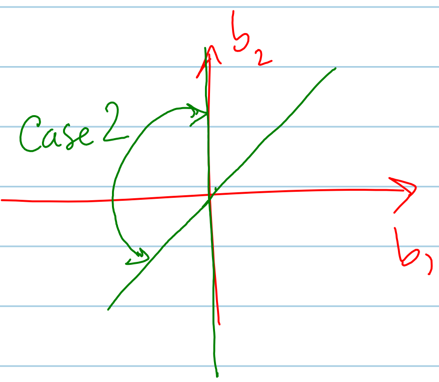
a.) Determine the set of $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ consistent with each of the 4 LCP solution cases $\left[\begin{array}{cc|cc|cc|cc} + & 0 & + & 0 & 0 & + & 0 & + \\ + & 0 & 0 & + & + & 0 & 0 & + \end{array}\right]$ and sketch the sets on the axes below and sketch the sets on the axes below



Case 1:

$$\begin{bmatrix}1 & -1\\ 1 & 1\end{bmatrix}\begin{bmatrix}p_1\\ p_2\end{bmatrix} + \begin{bmatrix}b_1\\ b_2\end{bmatrix} = \begin{bmatrix}0\\ 0\end{bmatrix}$$

$$\begin{bmatrix}p_1\\ p_2\end{bmatrix} = \frac{\begin{bmatrix}-1 & -1\\ +1 & -1\end{bmatrix}}{2}\begin{bmatrix}+b_1\\ +b_2\end{bmatrix} \geq \begin{bmatrix}0\\ 0\end{bmatrix} \Rightarrow \begin{array}{l} -b_1 - b_2 \geq 0 \Rightarrow \underline{b_2 \leq -b_1} \\ b_1 - b_2 \geq 0 \Rightarrow \underline{b_2 \leq b_1} \end{array}$$



Case 2:

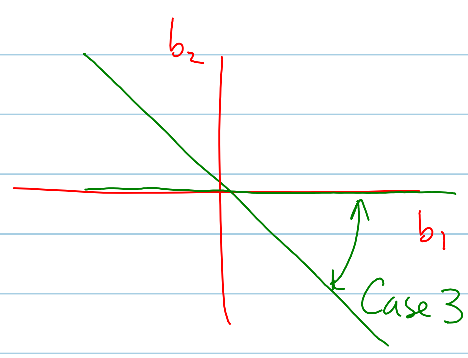
$p_1 - \cancel{p_2} + b_1 = 0 \Rightarrow p_1 = -b_1$

$p_2 = 0$

$p_1 \geq 0 \Rightarrow -b_1 \geq 0 \Rightarrow \underline{b_1 \leq 0}$

$p_1 + \cancel{p_2} + b_2 \geq 0 \Rightarrow -b_1 + b_2 \geq 0 \Rightarrow \underline{b_2 \geq b_1}$

0



Case 3:

$p_1 = 0$

$\cancel{p_1} + p_2 + b_2 = 0 \Rightarrow p_2 = -b_2$

0

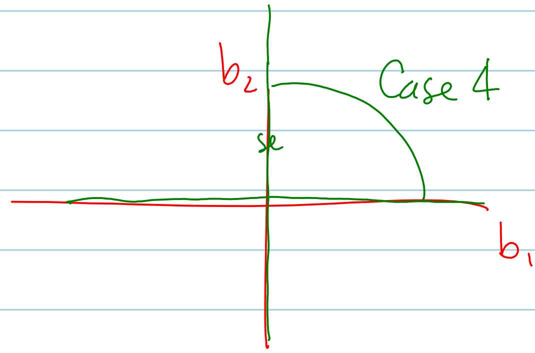
$p_2 \geq 0 \Rightarrow b_2 \leq 0$

$\cancel{p_1} - p_2 + b_1 \geq 0 \Rightarrow b_2 + b_1 \geq 0 \Rightarrow b_2 \geq -b_1$

0

Case 4:

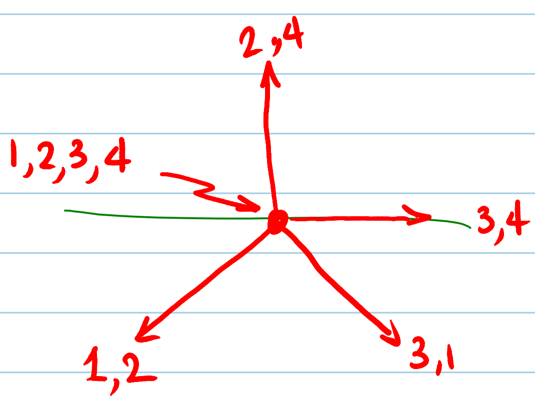
$$\left.\begin{array}{l} p_1 = 0 \\ p_2 = 0 \end{array}\right\} \Rightarrow \begin{array}{l} b_1 \geq 0 \\ b_2 \geq 0 \end{array}$$

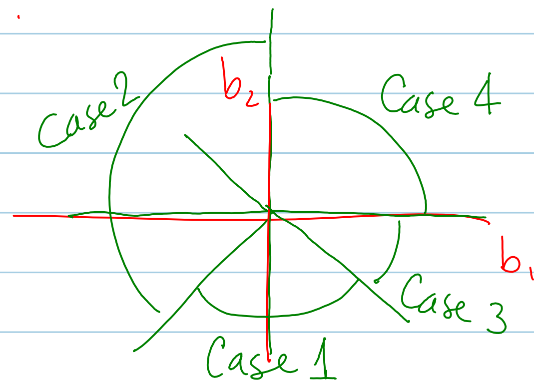


b. Is there at least one consistent case for each $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \in \mathbb{R}^2$?

Yes.

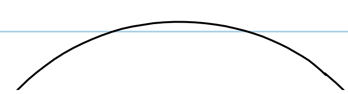
c. Show all $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ on the sketch that are consistent with more than one case.



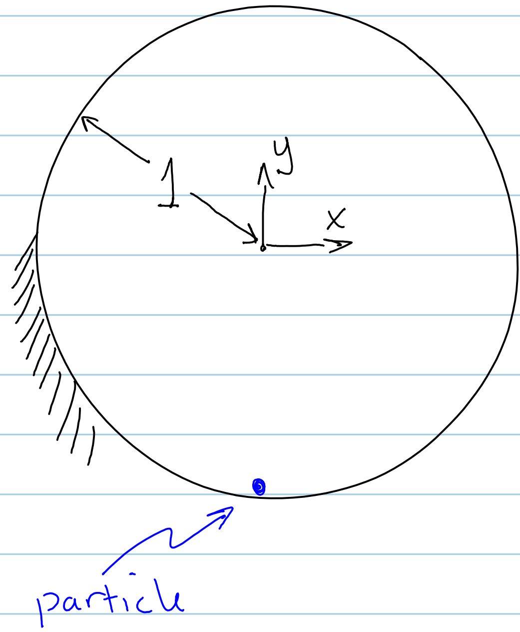


4. 2D Problem!

25 pts



(25 pts) A particle is moving inside a circular cavity in the plane.



Find the position of the particle at $t=2$.

Using LCP time stepping, determine the positions of the particle at times $t=1$ and $t=2$.

Assume: $h = m = 1$, $\mu = 0$

$$\nu^0 = \begin{bmatrix} 1 \\ -1 \end{bmatrix} \qquad u^0 = \begin{bmatrix} 0 \\ -1 \end{bmatrix} \qquad p_{ext} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\boxed{\text{If particle isn't touching surface of circle use } \hat{n} \text{ at point on circle closest to particle}}$

$$\underbrace{\cancel{M}}_{I} \nu^{l+1} = G_n p_n^{l+1} + \underbrace{\cancel{M}}_{I} \nu^l + \overset{0}{\cancel{p_{ext}}} \qquad G_n = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$0 \le p_n^{l+1} \perp G_n^T \nu^{l+1} + \psi_n^l / \underbrace{\cancel{h}}_{1} + \overset{0}{\cancel{\frac{\partial \psi_n}{\partial t}}} \ge 0$$

$$0 \le p_n^{\ell+1} \perp G_n^T G_n p_n^{\ell+1} + G_n^T \nu^\ell + \psi_n^\ell \ge 0$$

$$0 \le p_n^{\ell+1} \perp p_n^{\ell+1} + \dot{y}^\ell + \psi_n^\ell \ge 0 \qquad \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = 1$$

$$\psi_n^\ell = 1 - \sqrt{x^2 + y^2} \qquad \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix} = \dot{y}$$

At time $t=1$, we get:

$$0 \le p_n^{\ell+1} \perp p_n^{\ell+1} - 1 \ge 0 \implies p_n^{\ell+1} = \underline{1}$$

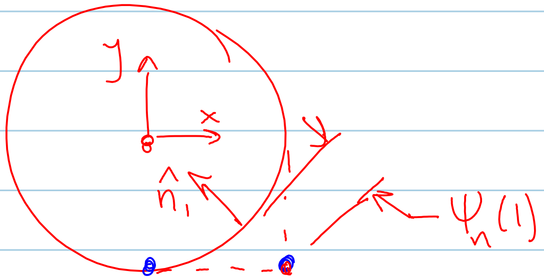
substituting back into $\nu^{\ell+1}$, we get:

$$\nu^{\ell+1} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \underline{1} + \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \nu^{\ell+1} = \nu(1)$$

Update position

$$u(1) = u(0) + \cancel{h}^{1} \nu(1) = \begin{bmatrix} 0 \\ -1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Next time step.



$$G_n = \begin{bmatrix} -\sqrt{2}/2 \\ \sqrt{2}/2 \end{bmatrix}$$

$$\psi_n(1) = 1.414 - 1 = 0.414$$

Substitute into:

$$0 \le p_n^{\ell+1} \perp G_n^T G_n p_n^{\ell+1} + G_n^T \nu^\ell + \psi_n^\ell \ge 0$$

$= 0.414$

$$0 \le p_n^{l+1} \perp G_n^T G_n p_n^{l+1} + G_n^T \nu^l + \psi_n^l \ge 0$$

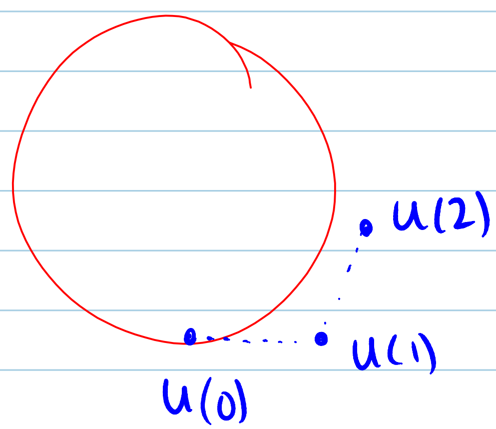
$$0 \le p_n^{l+1} \perp p_n^{l+1} + \begin{bmatrix} -\sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} - 0.414 \ge 0$$

$$\therefore\ p_n = 0.707 + 0.414 \simeq \boxed{1.121 = p_n}$$

$$\nu^2 = G_n p^2 + \nu^1 = \begin{bmatrix} -\sqrt{2}/2 \\ \sqrt{2}/2 \end{bmatrix} 1.121 + \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\nu^2 \simeq \begin{bmatrix} 1 - .8 \\ 0.8 \end{bmatrix} = \begin{bmatrix} 0.2 \\ 0.8 \end{bmatrix}$$

$$u^2 = u^1 + \nu^2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 1.2 \\ -0.2 \end{bmatrix}$$



u(1) & u(2) are penetrating the boundary.