LaVallle_Ch4

Thursday, March 19, 2009
11:11 AM

The Configuration Space (C-space)

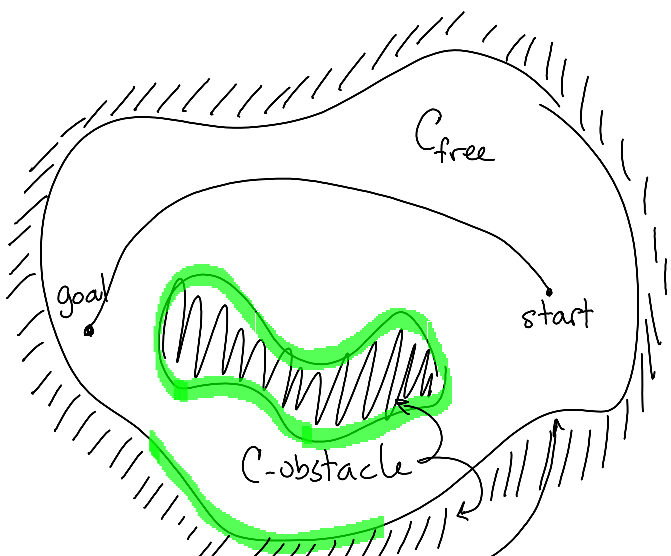
C-space is a (non-unique) space in which the robot is represented as a point

C-space is the set of points corresponding to every possible configuration of the robot, even those requiring overlap of bodies.

Robot motions correspond to continuous paths in

.

$C_{obstacle}$ = set of configs. corresponding to overlap



C_{free} = set of configs. corresp. to no overlap or contact

$C_{contact}$ = set of configs where one or more objects touch but don't overlap.

Let C-space be denoted by $\mathcal{C}$. Then

$$\mathcal{C} = \mathcal{C}_{obs} \cup \mathcal{C}_{cont} \cup \mathcal{C}_{free}$$

where $\mathcal{C}_{obs}$, $\mathcal{C}_{free}$, $\mathcal{C}_{cont}$ are mutually where C_{obs}, C_{free}, C_{cont} are mutually exclusive.

All motion planning problems can be transformed into finding a continuous path in ($C_{free} \cup C_{contact}$) connecting the starting and goal configs.

If we can approximate C as a discrete set of points with paths between them, then we can use discrete search methods!

For noncontact problems, it's usually best if the path is smooth (easier on actuators and structure of system).

Manipulation planning problems require a portion of the path to be in C_{cont}.

When dynamics is important planning may need to be done in state space X.

$$X = \mathcal{C} \times T$$

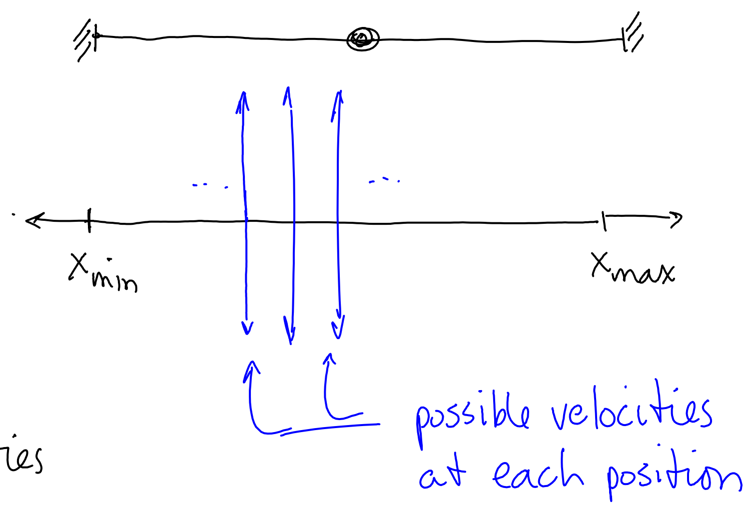
space of velocities

Cartesian product

At every point in $\mathcal{C}$, T is a "fiber" that is

At every point in C, T is a "fiber" that is a space of velocities.

Bead on wire

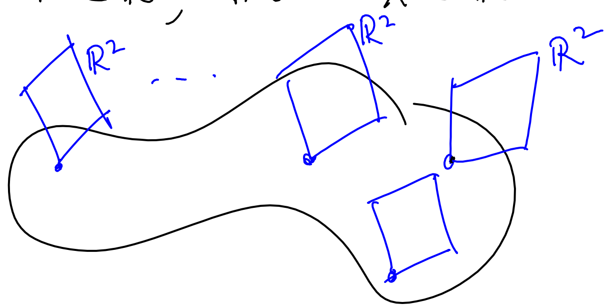


$X = I^1 \times \mathbb{R}^1$

if velocities are bounded,

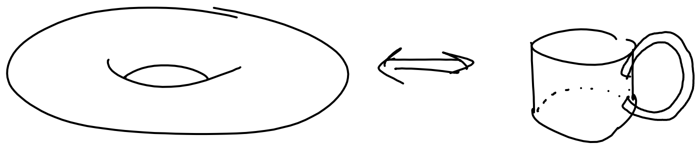
then $X = I^1 \times I^1 = D^2 =$ the disk in $\mathbb{R}^2$

If $C \subset \mathbb{R}^2$, $T \subset \mathbb{R}^2$, then $X \subset \mathbb{R}^4$



A bit of topology

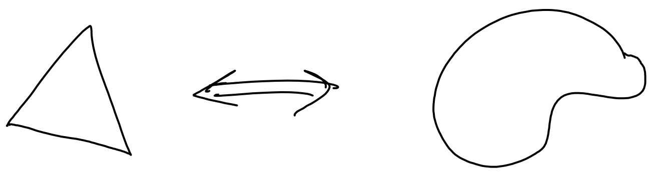
Homeomorphic – two shapes are homeomorphic if one can be deformed smoothly into the other.



torus mug

both have one hole!

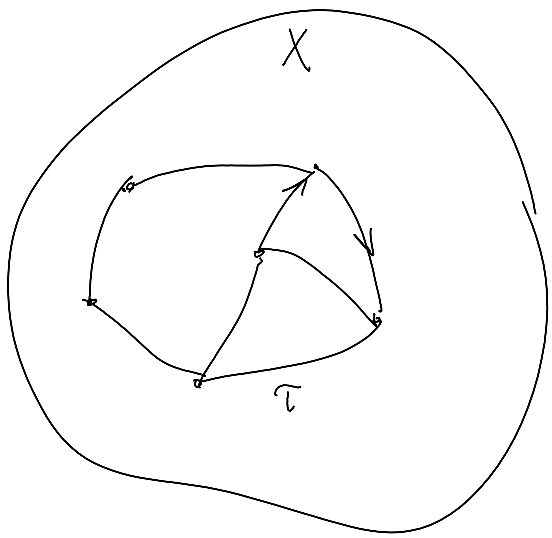
solid sphere $\longleftrightarrow$ solid cube



Topological Graphs

every vertex is in X

every edge is in X.



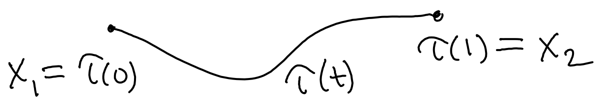
More precisely

every edge map I' onto a curve in X

$$\tau : [0,1] \longrightarrow X$$

(i.e., think of τ as $\tau(t)$), such that

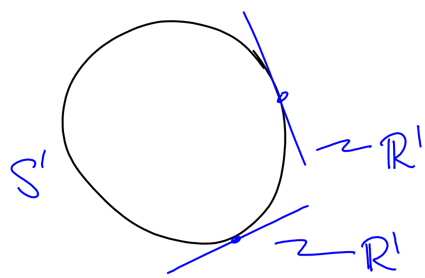
$\tau(t) \in X \quad \forall t)$



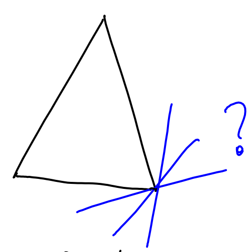
Manifolds – Most C-spaces are manifolds

A set M is a manifold if $\forall x \in M$, M is locally Euclidean.

This means there is a well-defined tangent plane at each point of M



Circle is a 1D manifold



Triangle is not a 1D manifold

You need coord. frame on M for motion planning.

For S^1, you could use a single parameter $\theta \in [0, 2\pi)$

Sometimes it is more convenient to embed M in a higher-dimensional space. e.g.

S^1 can be embedded in $\mathbb{R}^2$

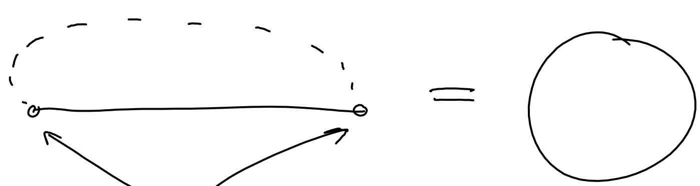
$$S^1 = \{(x,y) \in \mathbb{R}^2 \mid x^2 + y^2 = 1\} \Longleftarrow$$ 2 dof reduced to 1 by 1 eq.

One could also embed in $\mathbb{R}^3$, but then you need 2 eqs. Here's a trivial example:

$$S^1 = \{(x,y,z) \in \mathbb{R}^3 \mid x^2 + y^2 = 1,\ z = 14\}$$

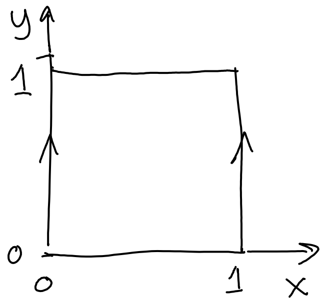
Identifications - must handle these carefully in motion planning.

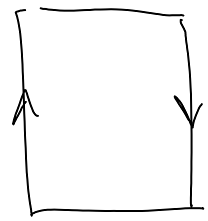
$S^1 = I^1$ with endpoints "identified"



Treat those points as if they were identical.

Some well-known identifications of $I^1 \times I^1$



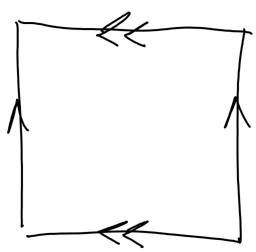


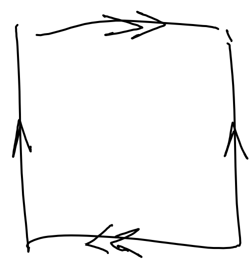
$(0,y) \sim (1,y)$

Cylinder

$(0,y) \sim (1, 1-y$

Mobius Strip





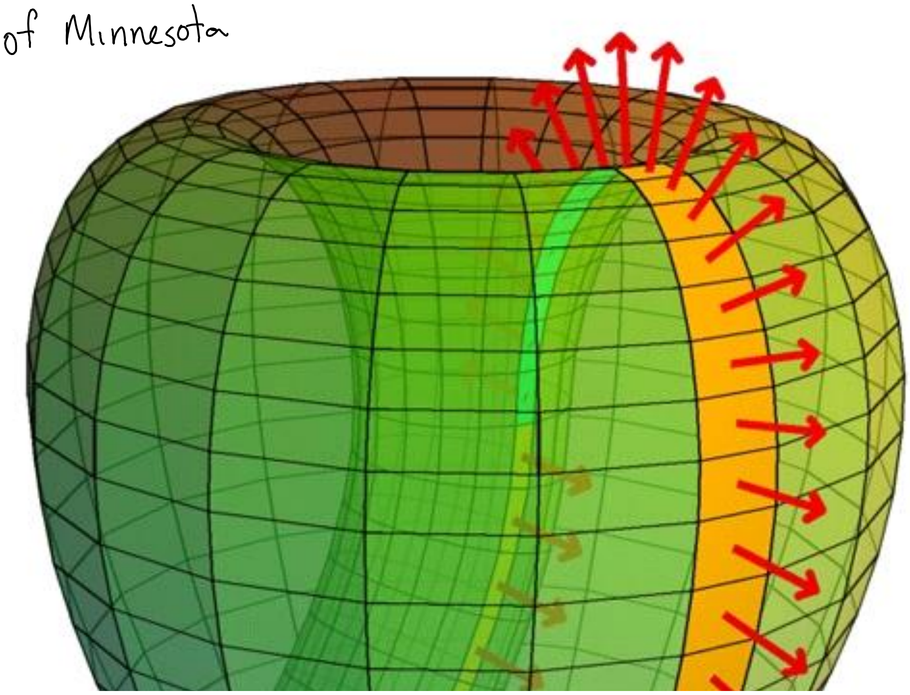
$(0,y) \sim (1,y)$

$(x,0) \sim (x,1)$

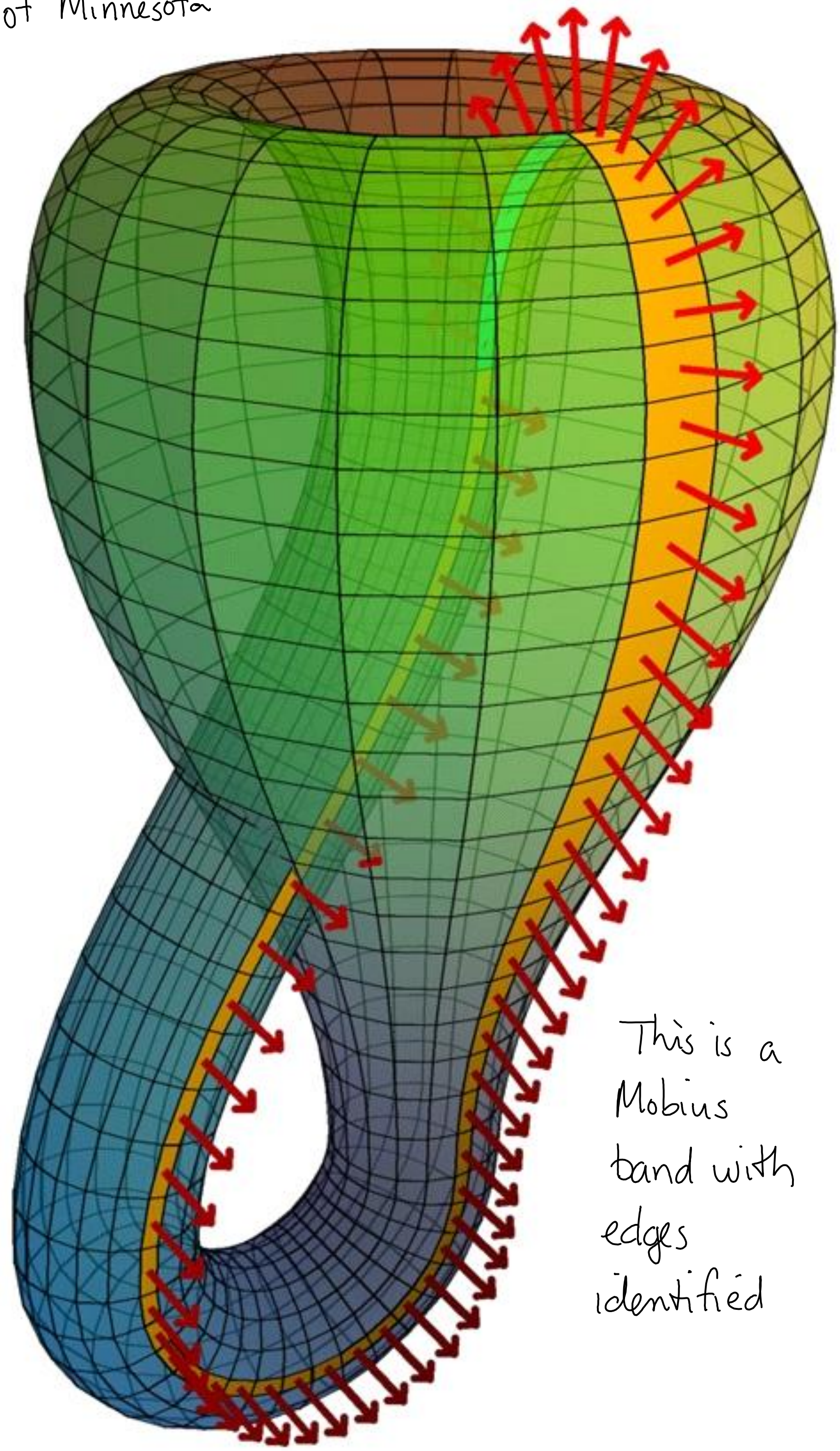
Torus

Klein Bottle

From Institute for Mathematics and its Application
Univ of Minnesota



Univ of Minnesota

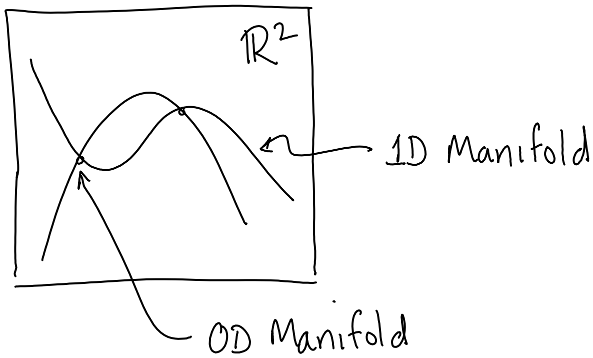


This is a Mobius band with edges identified

Higher-dimensional Manifolds

e.g. $S^n = \{ x \in \mathbb{R}^n \mid \|x\| = 1 \}$

Typically m simultaneous equations in $n > m$ variables yields a manifold



Intersection is a manifold of 2 distinct points (a point is considered a manifold of dimension zero)

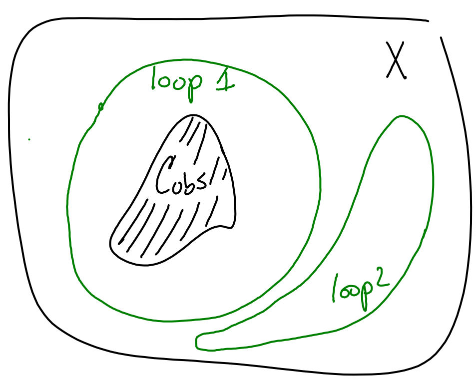
Simply connected space

Every loop can be contracted to a point

Loop 1 cannot be contracted

Loop 2 can be

X is not simply connected!

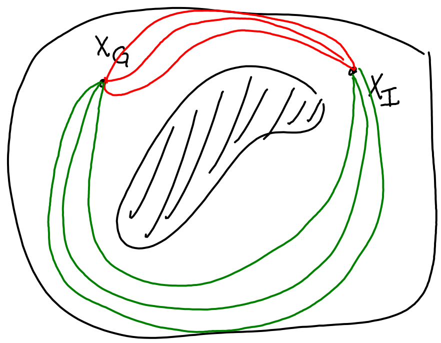


In a simply connected space all paths can be

In a simply connected space all paths can be morphed into any other path.

i.e. All paths are homotopic
" " " in the same homotopy class

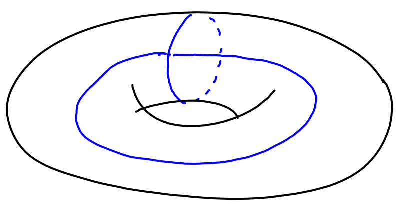
Green paths cannot be smoothly morphed into the red paths



Are there more classes of paths?

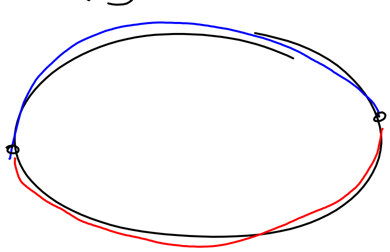
what about paths that encircle the obstacle?

Is the torus simply connected?



No. Not simply connected.

Is S^1 simply connected?



No!

An example of homotopic paths:

$$\tau_1(s) = \begin{bmatrix} s - \frac{1}{2} \\ (s - \frac{1}{2})^2 \end{bmatrix} \qquad \tau_2(s) = \begin{bmatrix} s - \frac{1}{2} \\ \frac{1}{2} - (s - \frac{1}{2})^2 \end{bmatrix}$$

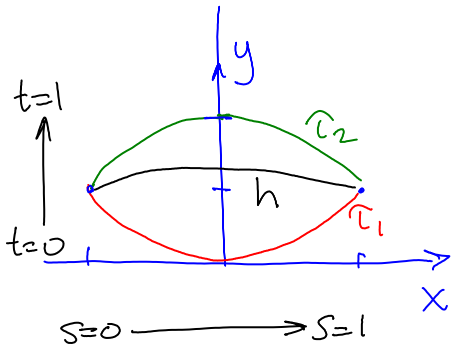
$\tau_1(s) = \begin{bmatrix} \cdots \\ (s-\frac{1}{2})^2 \end{bmatrix}$ $\tau_2(s) = \begin{bmatrix} \cdots \\ \frac{1}{2}-(s-\frac{1}{2})^2 \end{bmatrix}$

$0 \le s \le 1$

$\tau_1(0) = \begin{bmatrix} -\frac{1}{2} \\ \frac{1}{4} \end{bmatrix}$ $\tau_2(0) = \begin{bmatrix} -\frac{1}{2} \\ \frac{1}{4} \end{bmatrix}$

$\tau_1(1) = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{4} \end{bmatrix}$ $\tau_2(1) = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{4} \end{bmatrix}$

$\tau_1(\frac{1}{2}) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $\tau_2(\frac{1}{2}) = \begin{bmatrix} 0 \\ \frac{1}{2} \end{bmatrix}$



Define the morphing function, $h(s,t)$

$$h(s,t) = (1-t)\,\tau_1(s) + t\,\tau_2(s)$$

This function must be continuous in t and s,

i.e.: • $h(\alpha,\beta)$ must exist at every α, β

For each α, β, we must have:

- $\lim_{\alpha,\beta \to s,t} h(s,t)$ must exist at every α,β
- $\lim_{\alpha,\beta \to s t} h(s,t) = h(\alpha,\beta)$

LaValle 4.2 Defining C-space

2D rigid bodies – $C = \mathbb{R}^2 \times S^1 = SE(2)$

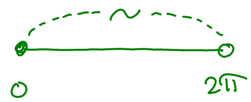
2D rotations

special Euclidean

(x,y) θ

space space

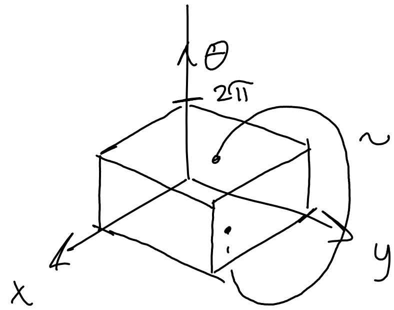
$S^1 = [0,\pi)/\sim$



$$u = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$$

three parameters represent three degrees of freedom

A chunk of $\mathcal{C}$ with bounds on (x, y). Note identification of top & bottom surfaces.



Other representations:

base on $T = \left[\begin{array}{cc|c} a & c & x \\ b & d & y \\ \hline 0 & 0 & 1 \end{array}\right]$

$u = [x\ y\ a\ b\ c\ d]^T \Rightarrow u \in \mathbb{R}^6$

but constraints: $a^2 + b^2 = 1$

$c^2 + d^2 = 1$

$ac + bd = 0$

$\therefore$ $\mathcal{C}$ is a 3D "variety" embedded in $\mathbb{R}^6$

You could also take advantage of the fact that $a = d$ and $b = -c$,

i.e. $R \in SO(2)$

$$R = \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

and $a^2+b^2=1$

Determinant must $= +1$

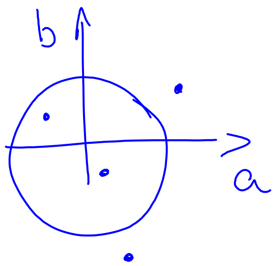
special orthogonal group of 2×2 matrices.

Columns (and rows) are orthogonal unit vectors

$$u = [x \ \ y \ \ a \ \ b]^T$$

$\mathcal{C}$ is a 3-D surface embedded in $\mathbb{R}^4$

Which representation of C-space should you use?



Choosing points in (a,b)-space requires subsequent adjustment to satisfy $a^2+b^2=1$

Sampling θ directly is simpler and can easily provide uniformly distributed values of θ.

3D Rigid Bodies

$$\mathcal{C} = SE(3) = \mathbb{R}^3 \times SO(3)$$

$\mathbb{R}^3$: (x,y,z) $\quad$ $SO(3)$: $(\ ?\ )$

Elements of $SO(3)$

$$\begin{bmatrix} a & d & g \end{bmatrix} \qquad \begin{cases} a^2+b^2+c^2=1 \end{cases} \qquad ad+be+cf=0$$

$$\begin{bmatrix} b & e & h \\ c & f & i \end{bmatrix} \Longleftarrow \begin{cases} d^2+e^2+f^2=1 & dg+eh+fi=0 \\ g^2+h^2+i^2=1 & ga+hb+ic=0 \end{cases}$$

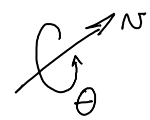
Determinant must equal +1

SO(3) is a 3-dimensional space embedded in $\mathbb{R}^9$

Alternative representation:

① Euler angles or roll-pitch-yaw: 3 angles

② Quaternions: scalar & 3-vector



Quaternions are prefered despite extra parameter, because mathematical properties are superior – i.e. no singularities, which improves uniformity of sampling

Let h denote a quaternion

$$h = \underbrace{a}_{\text{scalar}} + \underbrace{bi + cj + dk}_{\text{vector}} \; ; \quad a,b,c,d \in \mathbb{R}$$

$$i^2 = j^2 = k^2 = ijk = -1$$

$$ij = k, \; jk = i, \; ki = j$$

quaternion multiplication ≡ product of rotation matrices

∴ not commutative

Let h_1 & h_2 be quaternions

$h_1 = a_1 + v_1 \qquad h_2 = a_2 + v_2$

(eq. 4.19)

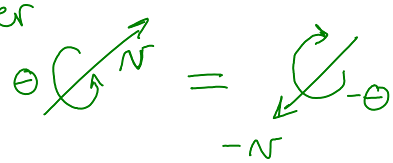
$$h_1 \cdot h_2 = (a_1 a_2 - \underbrace{v_1 \cdot v_2}_{\text{dot product}}) + (a_1 v_2 + a_2 v_1 + v_1 \times v_2)$$

Compare computational cost

$R_1 \cdot R_2 \Rightarrow$ 27 mults + 18 adds

$h_1 \cdot h_2 \Rightarrow$ 16 mults + 12 adds

Note that the extra parameter gives some duplication:



(Think of the quaternion as an operation on points, NOT as an axis v fixed in a body which is rotated about v)

Given v, θ, determine h (eq. 4.21):

$$h = \underbrace{\cos\left(\tfrac{\theta}{2}\right)}_{a} + \underbrace{\left(v_x \sin\left(\tfrac{\theta}{2}\right)\right)}_{b} i + \underbrace{\left(v_y \sin\tfrac{\theta}{2}\right)}_{c} j + \underbrace{\left(v_z \sin\tfrac{\theta}{2}\right)}_{d} k$$

Determine the rotation matrix (eq. 4.20):

$$R(h) = \begin{bmatrix} 2(a^2+b^2)-1 & 2(bc-ad) & 2(bd+ac) \\ 2(bc+ad) & 2(a^2+c^2)-1 & 2(cd-ab) \\ 2(bd-ac) & 2(cd+ab) & 2(a^2+d^2)-1 \end{bmatrix}$$

Note that $R(h)$ is quadratic in the elements of h.

Convert R to h (eqs. 4.24 – 4.30)

$$R = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}$$

eqs. (4.24 – 4.27)

$$\boxed{a = \frac{1}{2}\sqrt{r_{11} + r_{22} + r_{33} - 1} \qquad b = \frac{r_{32} - r_{23}}{4a} \qquad c = \frac{r_{13} - r_{31}}{4a} \qquad d = \frac{r_{21} - r_{12}}{4a}}$$

Not valid if $a = 0$

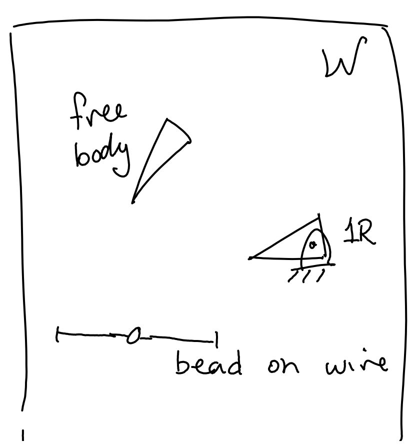
If $a = 0$, then use equations (4.28 – 4.30)
These also have possible zero denominators, but
the problem can be corrected (eqs. 3.39 – 3.41).

C-space for multiple bodies

Let C_i be the C-space of body i

$$C = C_1 \times C_2 \times \dots \times C_n$$

where n is # bodies



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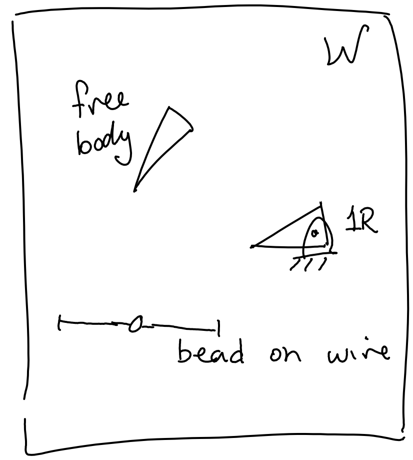
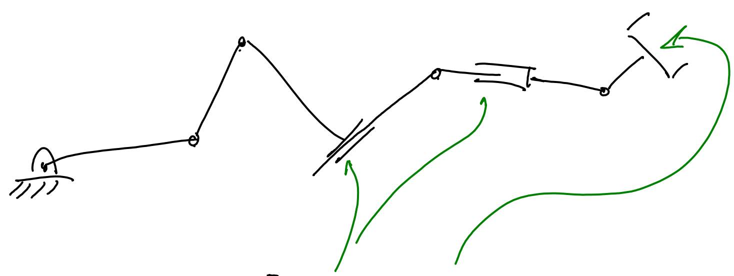


$$C = I^1 \times S^1 \times SE(3)$$

Note: for every robot, it is practical to treat nearly every revolute joint as having C-space $= I^1$. The hard part is dealing with arbitrary orientations of free bodies and bodies connected by spherical joints.



$$C = S^5 \times I^2 \times I^1$$

Assuming revolute joints have limits,

$$C = I^8$$

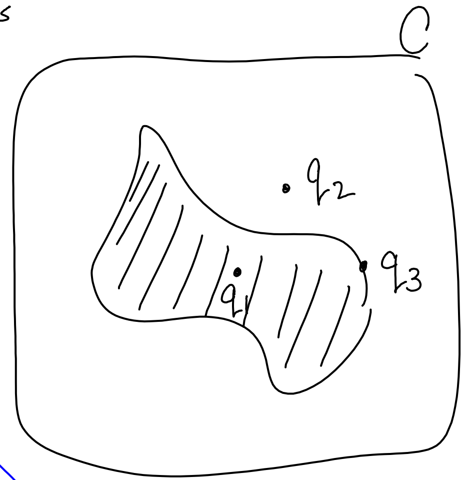
$\therefore$ for planning purposes, just sample 8 intervals to get a point in C-space.

If base of robot is floating, then

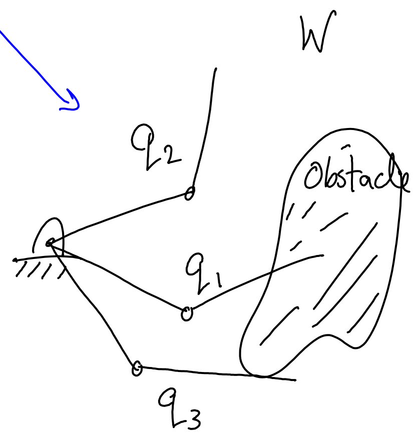
$$C = SE(3) \times I^8$$

4.3 The C-space Obstacle, C_{obs}

Let $q \in C$ denote a config. of the robot



Let $A(q)$ be the robot in config q.



$$C_{obs} = \{ q \in C \mid A(q) \cap \mathrm{Int}(\mathcal{O}) \neq \emptyset \}$$

$$C_{cont} = \{ q \in C \mid A(q) \cap \mathcal{O} \neq \emptyset, \; A(q) \cap \mathrm{Int}(\mathcal{O}) = \emptyset \}$$

equivalently,

$$C_{cont} = \partial C_{obs}$$

$$C_{free} = C \setminus (C_{obs} \cup C_{cont}) = C \setminus cl(C_{obs})$$

closure of a set

4.3.2 Explicit representations of $C_{cont} = \partial C_{obs}$

(LaValle treats C_{obs} and $C_{obs} \cup C_{cont}$)

Definition: Minkowski difference of two sets in $\mathbb{R}^n$

$$X \ominus Y = \{ x - y \in \mathbb{R}^n \mid x \in X, \; y \in Y \}$$

Definition: Minkowski sum

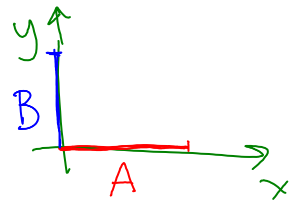
$$X \oplus Y = \{ x + y \in \mathbb{R}^n \mid x \in X, y \in Y \}$$

Note: $X \ominus Y = X \oplus (-Y)$ where $-Y$ is the set of negated elements of Y.

Example in $\mathbb{R}^2$

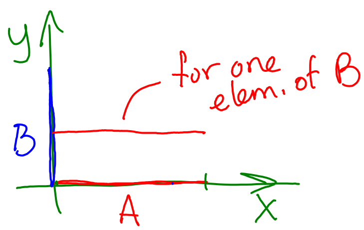
Let $A = I = ([0,1], 0) =$ interval on x-axis

$B = I = (0, [0,1]) =$ interval on y-axis

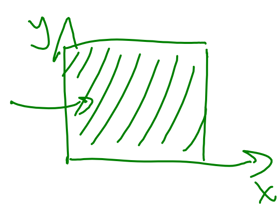


What is $A \oplus B$?

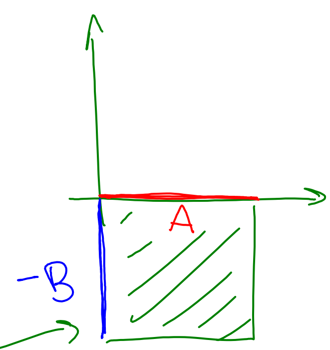
Add every element of A to every element of B



$A \oplus B = I^2$



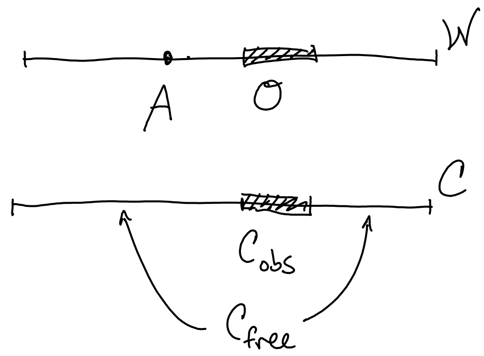
$A \ominus B = ?$



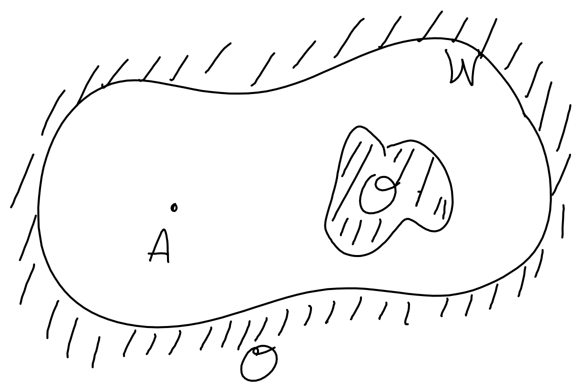
Using $\mathcal{O}$ to determine C_{free} & C_{obs}

Simplest case: A is a particle

1D world



2D world



Again $C_{obs} = \mathrm{Int}(\mathcal{O})$

$C_{cont} = \partial \mathcal{O}$

$C_{free} = W \setminus \mathrm{cl}(\mathcal{O})$

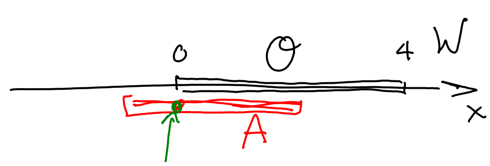
Same for 3D worlds

Robots of finite extent:

1D:

Let $A = [-1, 2]$

$\mathcal{O} = [0, 4]$



To create $C_{obs}, C_{cont}, C_{free}$,

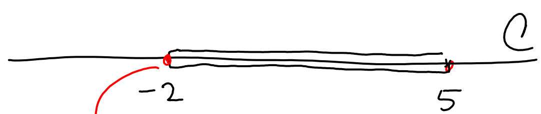
we need a ref. pt. on A.

(Choose the zero point)

$\mathcal{C}$ represents possible positions of A by possible positions of the ref. point.

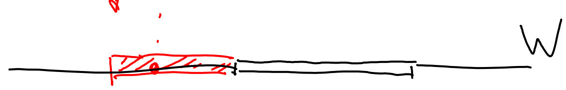
$$cl(C_{obs}) = \mathcal{O} \oplus -A = [0,4] \oplus [-2,1]$$

$$cl(C_{obs}) = [-2, 5]$$



$C_{free} = \{x \mid x < -2 \vee x > 5\}$

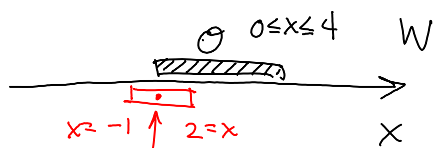
$C_{cont} = \{x \mid x = -2 \vee x = 5\}$



ref pt at -2
just touching

Why do we define C_{obs} by $\mathcal{O} \oplus (-A)$ instead of $\mathcal{O} \oplus A$?

Consider 1D case:



reference point.

Noncollision constraint on left side of $\mathcal{O}$

$$x + a < o, \ \forall a \in A(x), \ \forall o \in \mathcal{O}$$

$$\Rightarrow x < \underbrace{o - a, \ \forall a \in A(x), \ \forall o \in \mathcal{O}}_{\text{This is Definition of Minkowski difference}}$$

Worst case: $x < -2$

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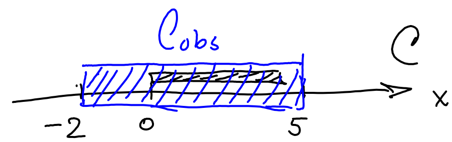
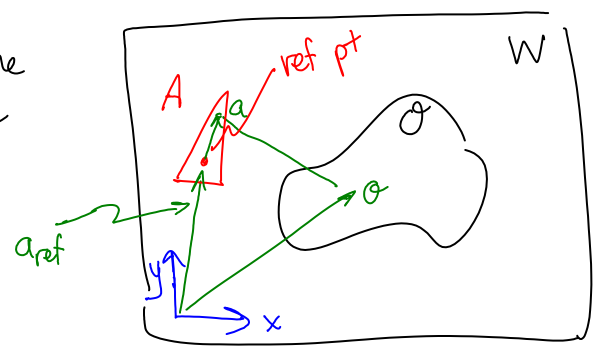


You can apply on right also

$$x > o - a, \quad \forall a \in A, \quad \forall o \in \mathcal{O}$$

worst case: $x > 5$

2D example works the same way, but constraints are more complicated



pt on robot, v

$$v = a_{ref} + a, \quad a \in A(q)$$

$q \in C_{obs}$ if $a_{ref} + a = o$ for some $o \in \mathcal{O}$
for some $a \in A$

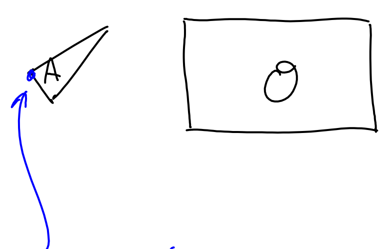
$\Downarrow$

$q \notin C_{obs}$ if $a_{ref} \neq o - a, \ \forall a \in A(q), \ \forall o \in \mathcal{O}$

Minkowski difference.

Planar Translation Only Robot.

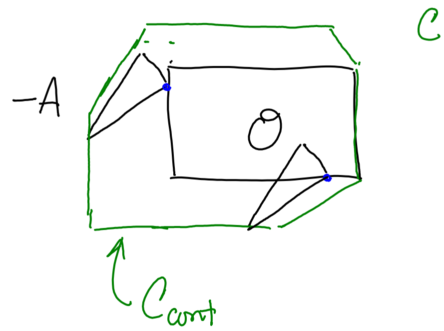
C_{free} is set of positions of ref. point $\ni$ $A \cap \mathcal{O} = \emptyset$



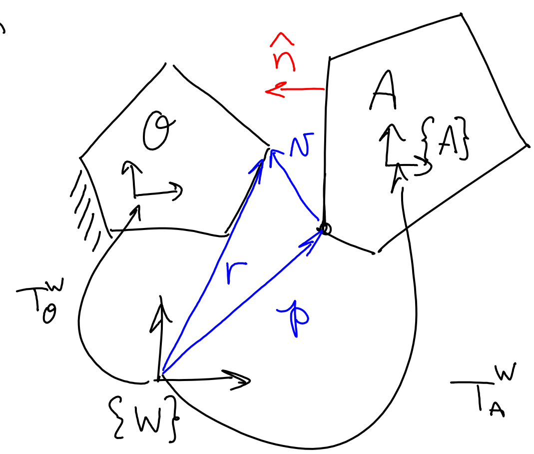
Choose ref. pt.

$cl(C_{obs}) = \mathcal{O} \oplus -A$

Add $-A$ to every point in $\mathcal{O}$



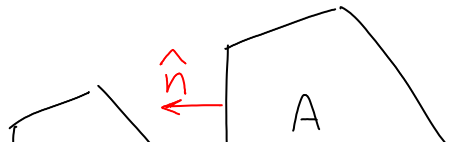
Formulas for the boundary of C_{obs} for planar world with $\mathcal{O}$ & A defined as polygons

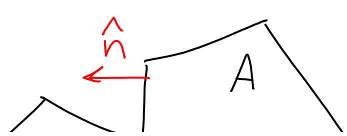


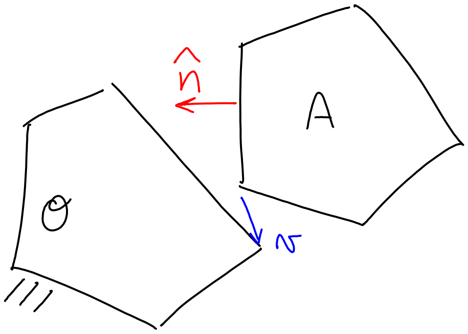
EV Contact – edge of robot in contact with vertex of $\mathcal{O}$.

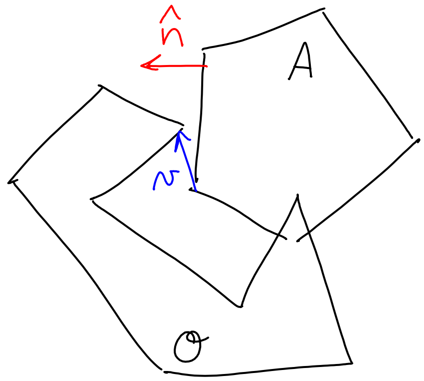
$\hat{n} \cdot v = 0$	potential contact
$\hat{n} \cdot v < 0$	potential penetration
$\hat{n} \cdot v > 0$	local separation

Why the qualifiers?









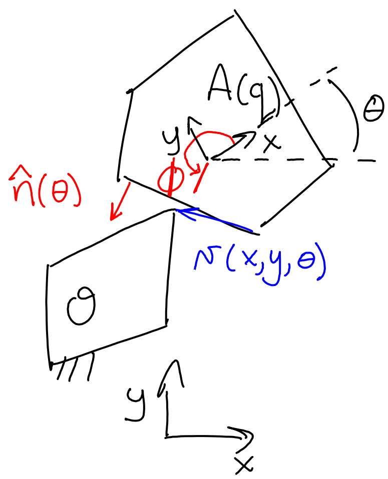
$\hat{n}\cdot v \le 0$,
but no contact
or penetration

$\hat{n}\cdot v \ge 0$, but
penetration occurs
(non-locally).

To obtain manifold of a given facet, write contact condition as a function of $q = \begin{bmatrix} x \\ y \\ \theta \end{bmatrix}$

Necessary condition
for contact :

$$\hat{n}(\theta) \cdot v(x,y,\theta) = 0$$



Let ϕ denote the
angle from the body-fixed
x-axis to the normal
direction

$$\hat{n}(\theta) = \begin{bmatrix} \cos(\phi+\theta) \\ \sin(\phi+\theta) \end{bmatrix}$$

position of p in
frame A

$$v^W = r^W - p^W = r^W - T_A^W\, p^A$$

$$\mathcal{N}(x,y,\theta) = \begin{bmatrix} \underbrace{r_x}_{\text{position of obstacle vertex in W}} - C_\theta p_x^A + S_\theta p_y^A - \underbrace{x}_{\text{position of origin of robot frame in W}} \\ r_y - S_\theta p_x^A - C_\theta p_y^A - y \end{bmatrix}$$

Substitute into $\hat{n}(\theta) \cdot \mathcal{N}(x,y,\theta) = 0$

$$0 = \cos(\theta+\phi)[r_x - C_\theta p_x^A + S_\theta p_y^A - x] + \sin(\theta+\phi)[r_y - S_\theta p_x^A - C_\theta p_y^A - y]$$

(This eq. defines a 2D "variety" in 3D C-space (x,y,θ)).

Nonlinear trigonometric polynomial in θ (e.g., $\cos(\theta+\phi)\cdot\cos(\theta)$)

Linear in (x,y)

$$\boxed{ax + by + c = 0}$$

where $a = -\cos(\theta+\phi)$, $b = -\sin(\theta+\phi)$,

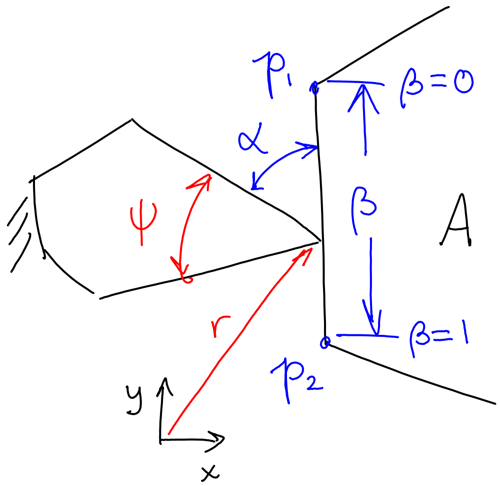
and $c = \cos(\theta+\phi)[r_x - C_\theta p_x^A + S_\theta p_y^A]$
$\quad + \sin(\theta+\phi)[r_y - S_\theta p_x^A - C_\theta p_y^A]$

Setting this contact eq. = 0 yields a 2D manifold in a 3D space ($\mathbb{R}^2 \times S^1$).

Pieces of this manifold appear in Brost's Cobs

How do we get the pieces?

Another useful formulation ...



$0 \le \alpha \le \pi - \psi$

$0 \le \beta \le 1$

r must be on the line segment $\overline{p_1 p_2}$

$$\begin{bmatrix} p_{ix}^W \\ p_{iy}^W \\ 1 \end{bmatrix} = \begin{bmatrix} c_\theta & -s_\theta & x \\ s_\theta & c_\theta & y \\ 0 & 0 & 1 \end{bmatrix} \underbrace{\begin{bmatrix} p_{ix}^A \\ p_{iy}^A \\ 1 \end{bmatrix}}_{\text{given in geom. model}}$$

$i = 1, 2$

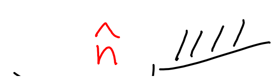
Substitute p_1^W, p_2^W into:

$$\boxed{\begin{aligned} &(1-\beta)\, p_1^W(x,y,\theta) + \beta\, p_2^W(x,y,\theta) = r \\ &0 \le \beta \le 1 \\ &0 \le \alpha \le \pi - \psi \end{aligned}}$$

If A & O are convex, then this is a patch of C_{obs}

If A or O are nonconvex, then some of this patch may be cut away by nonlocal interpenetrations!

Other contact types.



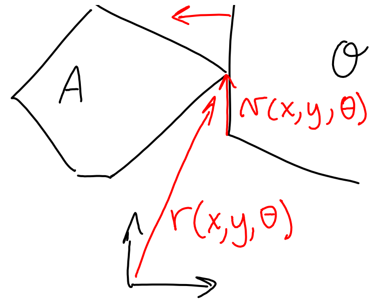
VE contact:

Same procedure, but

$\hat{n}$ is constant

v is $v(x, y, \theta)$

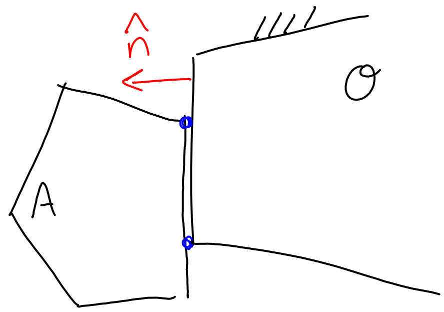
r is $r(x, y, \theta)$



VE ⟹ 2D facet of C_{obs}

EE contact:

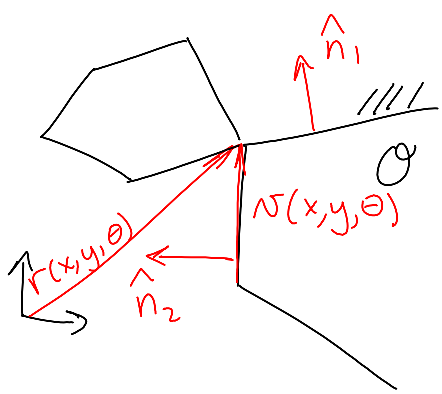
Treat as two contacts of type EV or VE



EE ⟹ 1D facet of C_{obs}

VV contact:

Treat as two contacts of type EV or VE

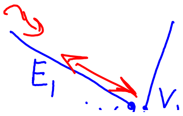


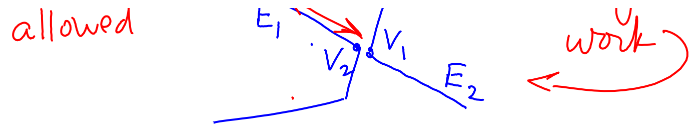
VV ⟹ 1D facet of C_{obs}

You can choose (EV, EV) or (VE, VE) or ~~(EV, VE)~~

← doesn't always work

1 Dof transl allowed





Collision Checking for Polygons

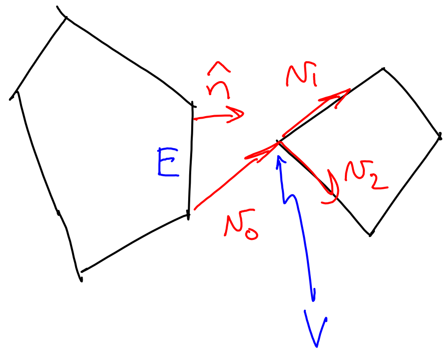
Assume polygons are convex
(can partition non-convex ones)

A pair of convex polygons are separated iff $\exists$ a vertex/edge pair $(E, V) \ni$

$\hat{n} \cdot \nu_0 > 0$

$\hat{n} \cdot \nu_1 > 0$

$\hat{n} \cdot \nu_2 > 0$



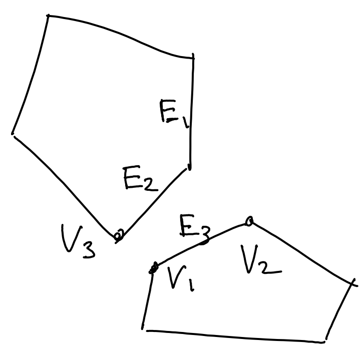
Example: →

(E_1, V_1) fails test

(E_2, V_2) fails

(E_3, V_3) succeeds

(E_2, V_1) succeeds too

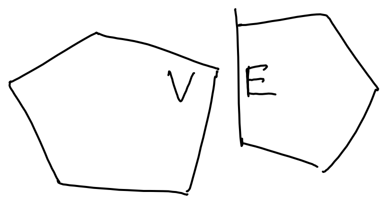


Fast collision checking - intuition

If relative config does not change much, and polygons were not in collision at previous check, it should be "easy" to find the definitive vertex/edge pair.

Exact collision checking w/o "hot start"

Consider 1 EV pair $\longrightarrow$



Let $H_A = H_1 \cup H_2 \cup H_3$

where
$$\left.\begin{aligned} H_1 &= \{q \in C \mid \hat{n} \cdot v_0 \leq 0\} \\ H_2 &= \{q \in C \mid \hat{n} \cdot v_1 \leq 0\} \\ H_3 &= \{q \in C \mid \hat{n} \cdot v_2 \leq 0\} \end{aligned}\right\} \text{negations of previously used inequalities}$$

H_A is a superset of C_{obs}, i.e.

if $q \in H_1$ or H_2 or H_3, then q might be an element of C_{obs}

if $q \in \overline{H}_1$ and $\overline{H}_2$ and $\overline{H}_3$, then $q \in C_{free}$

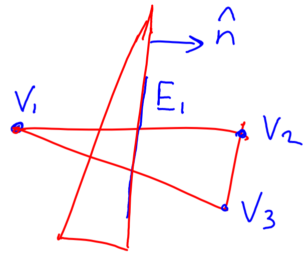
However, if $q \in H_A$ for every EV and VE pair, then $q \in C_{obs}$

Testing: start checking all EV and VE pairs until one is not satisfied. Then $q \in C_{free}$.

If all are satisfied, the $q \in C_{obs}$.

Does this work for this case?

$V_j E_i$, $\left.\begin{array}{l} i=1,2,3 \\ j=1,2,3 \end{array}\right\}$ satisfied



$V_j E_i$, $\left.\begin{array}{l} i=4,5,6 \\ j=4,5,6 \end{array}\right\}$ satisfied

Yes! It works!

Simple exhaustive collision checking is $O(m^2)$ where m is the maximum # of vertices on each polygon.

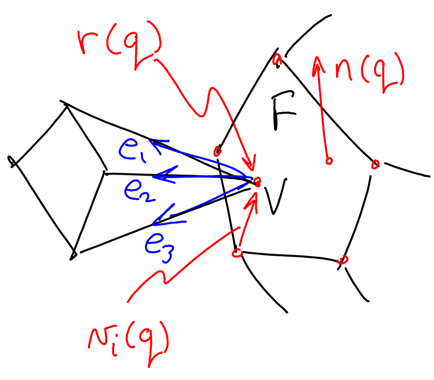
More precisely # of predicates is $2 n_A n_O$ where

n_A = # vertices on robot

n_O = # vertices on obstacle

What about 3D?

Extension is analogous but a bit more complicated.



eg. Collision free q if:

$$\hat{n}_i(q) \cdot \nu_i(q) \geq 0$$

$$\hat{n}(q) \cdot e_i(q) \geq 0$$

Contact facet of C_{obs} defined by

$r(q)$ on plane of facet & within
bounds defined by facet edges.

Collision checking can be formed by checking
inequalities of all VF, FV, & EE pairs.