Tuesday, April 07, 2009
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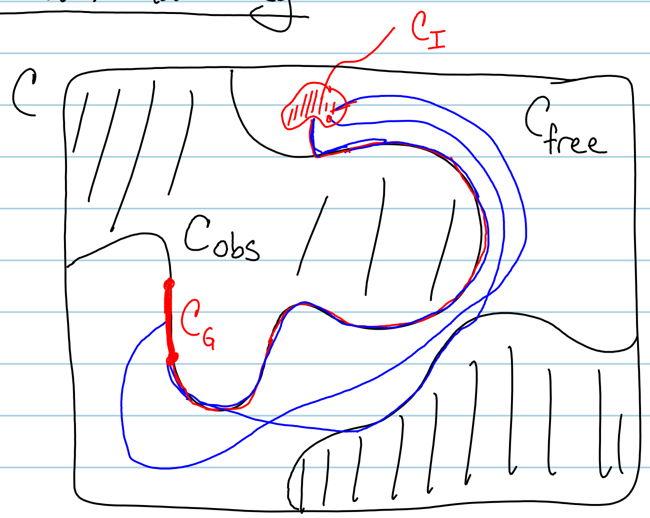
Sampling-Based Motion Planning

Geometric problems "live" in C-space

Some quasistatic problems can be cast in C-space too

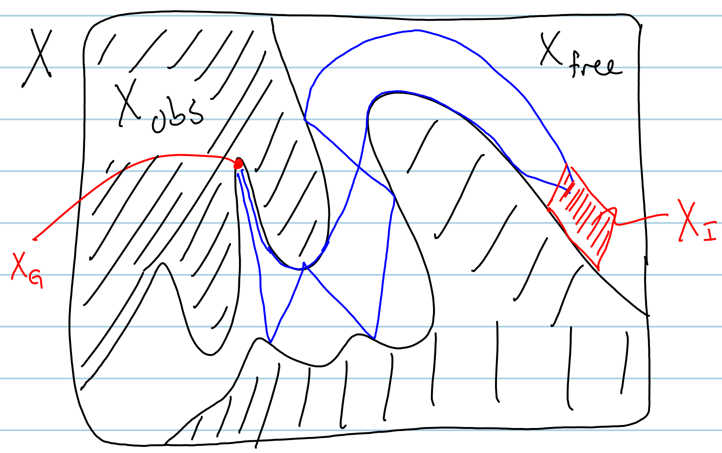


Basic approach to solving MP problems

Geometric & Kinematic Models → Collision Detection → Sampling-Based M.P. Algorithm (Discrete Search | C-space Sampling)

C-space isn't always the right space.

Dynamic systems live in state space



Geometric, Kinematic, → Collision Detection → Sampling-Based M.P. Algorithm

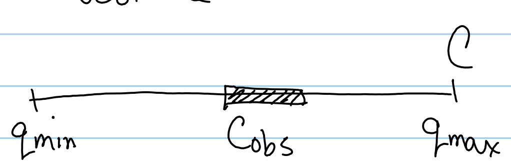
Geometric, Kinematic, Physical Models → Collision Detection & Response → Sampling-Based M.P. Algorithm

Discrete Search	C-space Sampling
Accurate Simulation	Input space Sampling

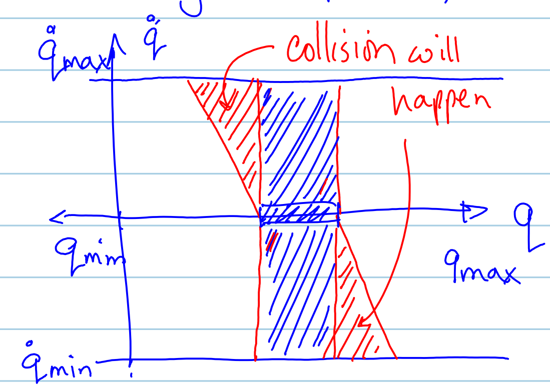
Beyond M.P. are parametric design problems.
Design problems live in $X \times P$, where P is the space of possible design parameters.

Imagine a 1-D Cspace:

bead on wire with obstacle



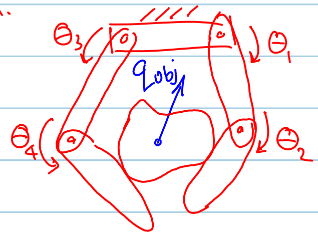
If velocity is important, ...



System Type	Search Space	Input Types	Validity Check
Geometric (w/ kinematics)	C	Path segment in C, control is abstracted away.	Collision detection
Dynamic (w/ kinematics) (a.k.a. kinodynamic)	X	Path in X if sufficient control authority available Force/torque trajectories from input space	Simulation with Collision Detection

(a.k.a. Kinodynamic)		from input space	Detection
Quasi-static	$\mathcal{C}$	paths of some bodies	Sim with Coll. Detect.
Kinematically Reducible	$\mathcal{C}$	paths of some bodies	Sim with Coll. Detect.

e.g. fix palm, rotate fingers, but don't specify object motion, q_{obj}.



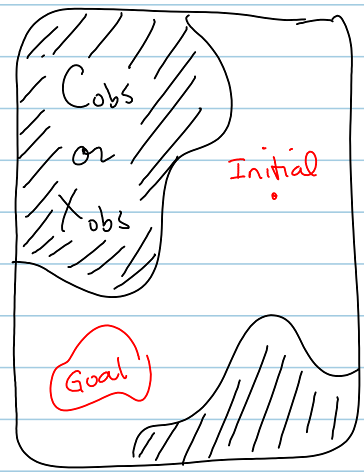
Basic Algorithm

Let $G(E,V)$ be a topological graph with edge and vertex sets, E and V, resp.

1. Initialize $G(E,V)$

   $E = \{\}$ $\quad V = q_I, q_{G_1}, q_{G_2} \cdots$

   or $V = x_I, x_{G_1}, x_{G_2}, \cdots$



2. VSM (vertex selection method)

   Need metric to choose points at desirable distances (far or near)

3. LPM (local planning method)

Choose vertex pair to connect

Solve 2 pt. b.v. problem to connect with edge.

(edge could be directed).

4. If LPM successful, insert edge into $\mathcal{G}$.

5. Search $\mathcal{G}$ for solution. If not found, go to step 2.

Pick points "carefully." Depends of difficulty of 2pt b.v.p.

1.) One-time problems (one (x_I, x_G) pair)

   a.) points easy to connect

   b.) $\mathcal{G}$ grows to absorb goal quickly

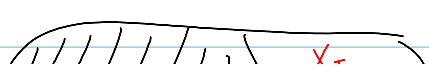
2.) Many problems in same space. (many (x_I, x_G) pairs)

   a.) points easy to connect

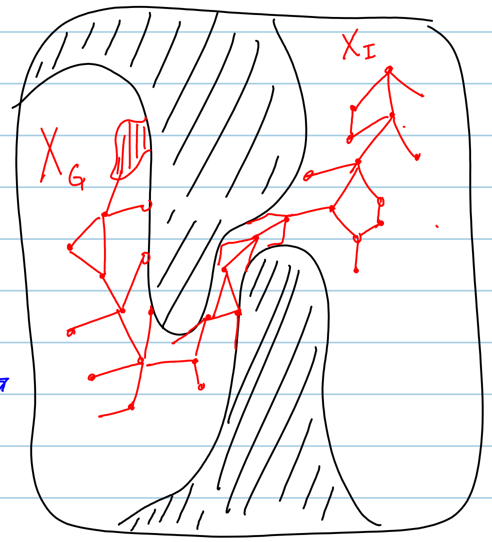
   b.) $\mathcal{G}$ grows to approximate X well

      i.) $\mathcal{G}$ & X have same # of conn. comp. w/ 1-1 correspondence

      ii.) Any point in X can be connected to $\mathcal{G}$ easily.



If 2 pt b.v.p. are easy to solve, then search can proceed just as for geometric problems. Otherwise →



Use input sampling.

Reachability is analogous to path existence in geometric problems.

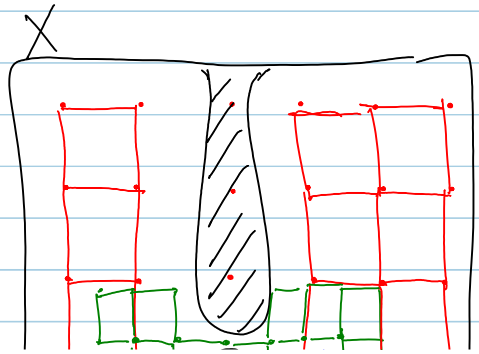
Alg. Completeness

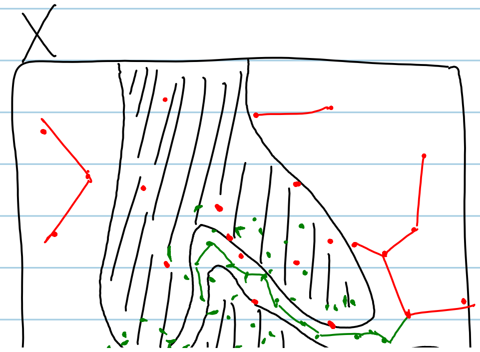
If a soln exists one will be found in finite time

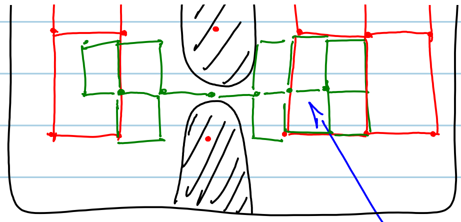
If no soln exists, this will be determined in finite time

Sample-Based MP Algs are NOT complete!

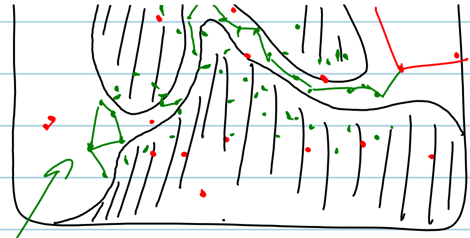
With "good" sample selection sequences, algs can be resolution complete or probabilistically complete.







Suppose LPM tries only adjacent pairs of points. Then need more resolution

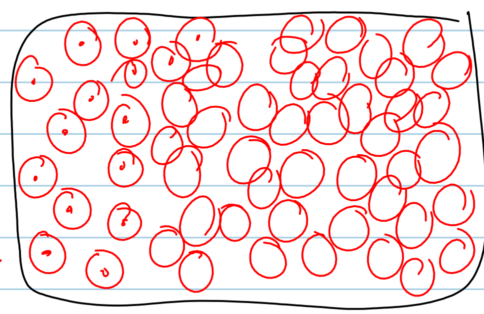


Need more points. If can detect narrow passage, concentrate sampling near it.

To guarantee prob. or resol. completeness, the sequence of samples must be dense in X.

A set U is dense in V if $cl(U) = V$

An infinite sequence of points is dense over X if the sequence contains a point arbitrarily close to every point in X.



1) choose pts at random
2.) center small circle on each point.
3) the union of the circles must eventually $= X$ regardless of how small the radius is.

Uniform random sampling of I and $S = I \backslash \sim$

is provably dense. $\therefore$ we can solve problems for planar robots $(X = (I^2 \times S^1)^n)$

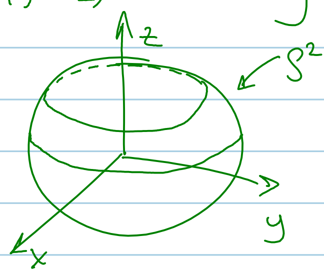
Fortunately $SO(3)$ can be sampled densely as follows:

Choose (u_1, u_2, u_3) uniformly at random on $[0,1]$.

$$h = \left(\sqrt{1-u_1}\sin(2\pi u_2), \sqrt{1-u_1}\cos(2\pi u_2), \ldots\right.$$
$$\left.\ldots \sqrt{u_1}\sin(2\pi u_3), \sqrt{u_1}\cos(2\pi u_3)\right)$$

The random quaternions formed this way are uniformly distributed on $SO(3)$.

This method is similar to choosing (u_1, u_2) uniformly at random on $[0,\pi]$ & $[0, 2\pi]$.



This gives a dense sequence on S^2, but it is not uniform, since points are denser near the poles.

Uniform random sampling of 3-param reps of orientation do not give dense samples on $SO(3)$!

So now we can sample uniformly at random in all relevant spaces for problems discussed so far.

We still cannot sample on "varieties"

so far.

$\boxed{\text{We still cannot sample on "varieties" defining valid configs of closed chains and contact spaces of arbitrary systems.}}$

There is one important implied assumption in res. & prob. completeness of sample-based algs:

if 2 pts are close enough, if a solution to the 2pt. b.v.p. exists, it can be found in finite time!

Since 1995 there have been research papers written on how to best choose points to solve probs. with narrow passages.

Note: all assembly/dissembly problems have narrow passages.

Section 5.1 – Distance & Volume

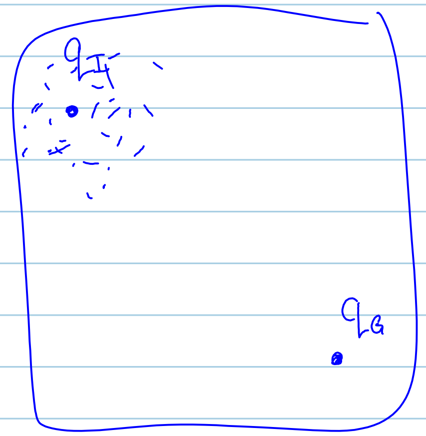
To make uniform sampling meaningful, we need a measure of distance (i.e., a metric) or volume (i.e., a measure).

Metric Space = a topological space w/ metric

Measure Space = a topological space w/ measure

without a measure or metric, you could do all your sampling in a "small" region

not defined



Properties of metrics:

Let ρ denote a metric.

Let $a, b, c,$ be points in the space, X

Nonnegativity: $\rho(a,b) \geq 0 \quad \forall\, a,b \in X$

Reflexivity: $\rho(a,b) = 0$ iff $a = b \in X$

Symmetry: $\rho(a,b) = \rho(b,a) \quad \forall\, a,b \in X$

Tri. Ineq.: $\rho(a,b) + \rho(b,c) \geq \rho(a,c) \quad \forall\, a,b,c \in X$

Some metrics

L_p metrics:
$$\rho = \left(\sum_{i=1}^{n} |a_i - b_i|^p \right)^{1/p} \qquad p \geq 1, \quad a,b \in \mathbb{R}^n$$

L_2 = the "usually" Euclidean distance

L_1 = Manhattan distance

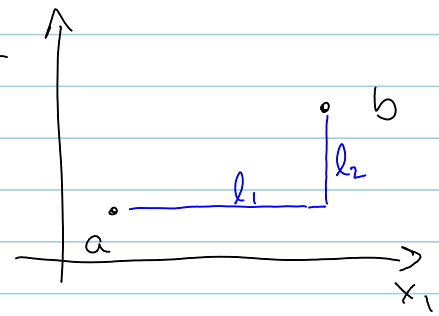
$$L_\infty = \lim_{p \to \infty} \rho = \max_i \{|a_i - b_i|\}$$

Example:

$$L_1 = l_1 + l_2$$

$$L_2 = \sqrt{l_1^2 + l_2^2}$$

$$L_\infty = l_1$$



Important! A product of metric spaces is a metric space.

Let (X_1, ρ_1) and (X_2, ρ_2) be metric spaces.

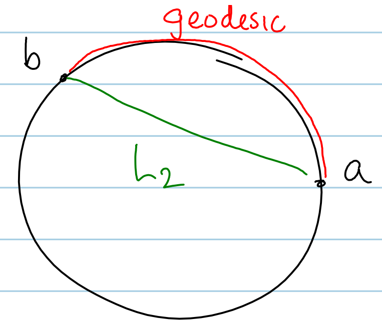
$(X_1 \times X_2,\ c_1\rho_1 + c_2\rho_2)$ with $c_1, c_2 > 0$

is a metric space.

If you have a metric on $\mathbb{R}^3$ and another on $SO(3)$, then you have one on $SE(3)$.

Metrics on $SO(2)$

$SO(2) = \{(a,b) \in \mathbb{R}^2 \mid a^2+b^2=1\}$



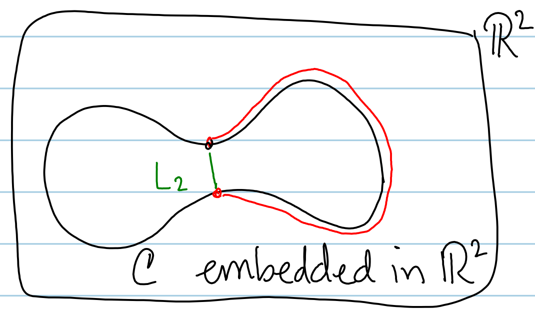
L_2 vs. geodesic

geodesic is shortest path in the space between two points

In arbitrarily curved spaces, geodesics can be hard to compute.

Also, in such spaces L_p can be very bad.

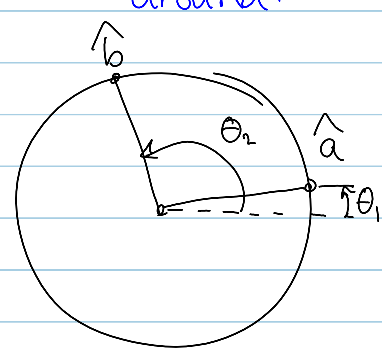
For example:



Good metric for S^1 is:

$$\rho = \min\{|\theta_1-\theta_2|,\ 2\pi - |\theta_1-\theta_2|\}$$ ← handles wrap-around.

this assumes $\theta_1, \theta_2 \in [0, 2\pi)$

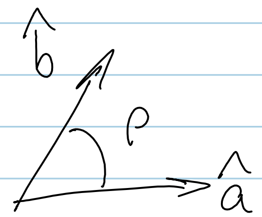


Suppose we want to use 4 parameters,

but instead of using cord length for the metric, use arc length?

Here's how....

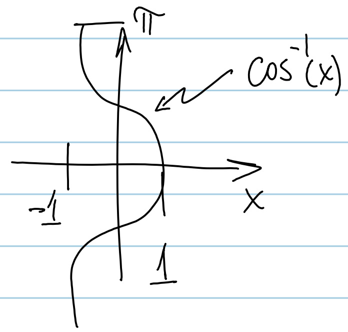
Alternative based on dot product in $\mathbb{R}^2$



$$\hat{a} \cdot \hat{b} = \underbrace{\cancel{\|\hat{a}\|}}_{1} \, \underbrace{\cancel{\|\hat{b}\|}}_{1} \cos(\rho)$$

$$\rho(a,b) = \cos^{-1}(a_1 b_1 + a_2 b_2)$$

must map to $[0, \pi)$



A metric on SE(2) embedded in $\mathbb{R}^4$

$$q = (x, y, a, b)$$

$$\rho = c_1 \sqrt{(x_1^2 - x_2^2) + (y_1 - y_2)^2} + c_2 \left[\cos^{-1}(a_1 a_2 + b_1 b_2)\right]$$

The usually Euclidean norm could be used too.

Note: c_1 & c_2 can be used to deal with units mismatch

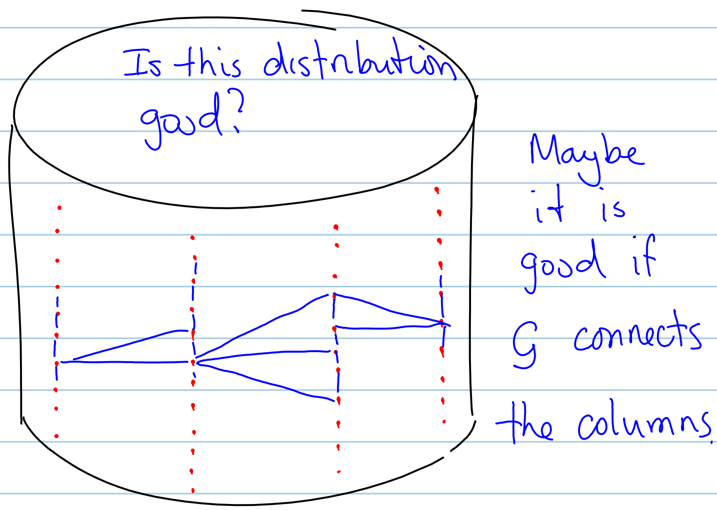
radians, degrees, ...

feet, millimeters, ...

Sometimes useful to choose c_1 w/o units and choose c_2 with units of length.

Then c_2 plays role of "characteristic length"

E.g. $S^1 \times I^1$



A Metric for SO(3):

Use quaternions, h_1, h_2.

$$\rho(h_1,h_2) = \min\left(\rho_s(h_1,h_2),\ \rho_s(h_1,-h_2)\right)$$

where

$$\rho_s(h_1,h_2) = \cos^{-1}(a_1a_2 + b_1b_2 + c_1c_2 + d_1d_2)$$

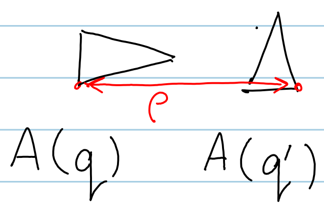
spherical linear interpolation.

This metric pushes the cord in $\mathbb{R}^4$ out onto the surface of the 3-sphere, S^3, on which the quaternions live. ρ_s is the length of the geodesic between 2 quaternions.

Preferably your metric represents a physical property important to your problem.

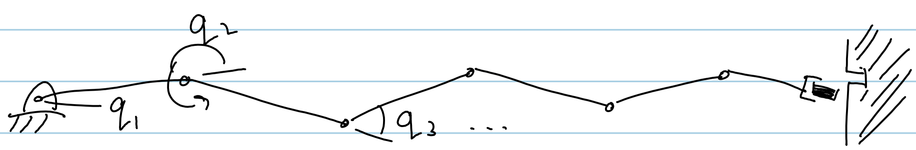
If collisions are an issue, maybe metric should be..

$$\rho(q, q') = \max_{a \in A} \{ a(q) - a(q') \}$$



Max displacement of any point on robot.

Maybe doing assembly with a highly articulated robot.

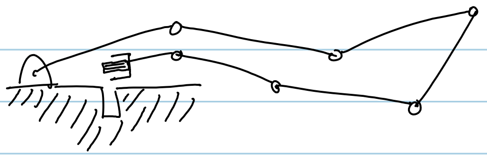


$$\rho = c_1 \sqrt{(q_1 - q_1')^2} + c_2 \sqrt{(q_2 - q_2')^2} + \cdots$$

Maybe $0 < c_1 < c_2 < c_3 < \cdots$

What if we have a different config?

Maybe c_is are



Maybe c_is are config dependent.

Metric based on Jacobian will provide good weighting.

$$\rho = \left(\mathrm{Det}\left(J(q)\, J^T(q)\right)\right)^{1/2}$$

if manipulator is not redundant, then this reduces:

$$\rho = \left| \mathrm{Det}(J(q)) \right|$$

Configuration dependence would adjust sampling density across various parts of C-space.