VelocityKinematicsOfGrasps

Friday, January 16, 2009
9:16 AM

Overall Goal for Grasp Modeling & Analysis

This goes beyond today's lecture!

Build model of grasping systems sufficient to sucessfully design, plan, and execute grasping actions and dexterous manipulation

Note that one cannot plan effectively without the ability to predict!

Questions we'll answer along the way:

- How does moving a finger joint affect object motion?
- How does locking a finger joint affect object motion?
- How does changing an actuator torque affect the contact forces?
- How can we design a grasp that is secure?
- What affects does contact friction have on the grasp?

Main analysis problems:

Forward & inverse velocity kinematics of a grasp

Dynamics of a grasp

Closure properties of a grasp

Velocity kinematics equation:

$$\underset{\text{a Jacobian matrix}}{A}\,\underset{\text{joint velocities}}{\dot{q}} = \underset{\text{velocity of grasped object}}{\nu}$$

$$\nu = \begin{bmatrix} \nu_x \\ \nu_y \\ \nu_z \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \underbrace{\left[\begin{array}{c|c|c} \quad & \quad & \quad \\ \quad & \quad & \quad \\ \quad & \quad & \quad \\ \quad & \quad & \quad \\ \quad & \quad & \quad \\ \quad & \quad & \quad \end{array}\right]}_{A} \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{bmatrix}$$

Each column of A represents the affect of the motion of the corresponding joint on ν

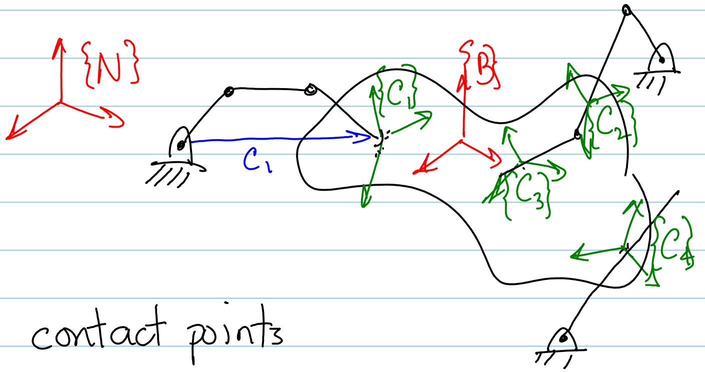
Forward kinematics: Given $\dot{q}$ compute ν

Inverse kinematics: Given ν compute $\dot{q}$

Our first goals:

Given ν compute velocity of contact points on body

Given $\dot{q}$ compute velocity of contact points on hand



Notation

n_c = # of contact points

n_b = # of grasped objects

n_u = # of params. to represent object position & orientation

n_q = # of joint d.o.f.

n_ν = # object dof = $6n_b$

$q \in \mathbb{R}^{n_q}$ = "vector" of joint displacements

$\dot{q} \in \mathbb{R}^{n_q}$ = vector of joint velocities

$\tau \in \mathbb{R}^{n_q}$ = vector of non-contact joint loads (from gravity, etc.)

$u \in \mathbb{R}^{n_u}$ = configuration of objects

$\nu \in \mathbb{R}^{n_\nu}$ = velocity twist of objects $\quad \nu_i = \begin{bmatrix} v_i \\ \omega_i \end{bmatrix} \forall i$

$g \in \mathbb{R}^{n_\nu}$ = noncontact load (or wrench) on objects

c_i = position of contact point i

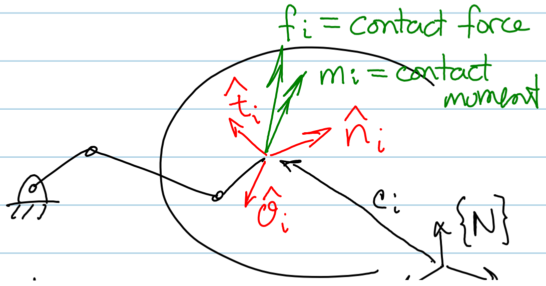
p_j = position of center of gravity of grasped object j

$\{C_i\}$ = coordinate frame at contact i

$\{B_j\}$ = coordinate frame at c.g. of object j

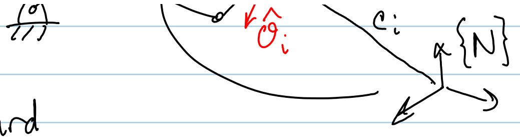
$\{N\}$ = inertial frame (non-accelerating frame)

Contact Frame →



$\hat{n}_i$ = unit vector normal to contact

$\hat{n}_i$ = unit vector normal to contact tangent plane. Inward w.r.t the object.



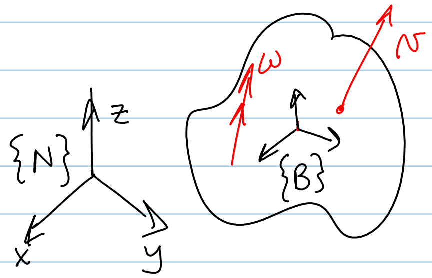
$\hat{t}_i$ = any unit vector in tangent plane

$o_i = \hat{n}_i \times \hat{t}_i$ = another vector in tangent plane

In 3D (spatial systems), $n_\nu = 6$.

For $n_b = 1$, we have:

$$\nu = \nu^N = \begin{bmatrix} \nu_x \\ \nu_y \\ \nu_z \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix}_{(6\times 1)}$$



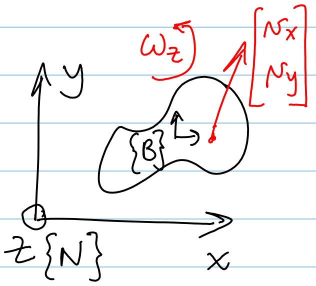
For $n_b > 1$, stack the ν vectors for each object:

$$\nu = \begin{bmatrix} \nu_1 \\ \vdots \\ \nu_{n_b} \end{bmatrix}, \text{ where } \nu_i = \begin{bmatrix} (\nu_j)_x \\ (\nu_j)_y \\ (\nu_j)_z \\ (\omega_j)_x \\ (\omega_j)_y \\ (\omega_j)_z \end{bmatrix} \quad j = 1, \dots, n_b$$

In 2D (planar systems), $n_\nu = 3$.

For $n_b = 1$:

$$\nu = \begin{bmatrix} \nu_x \\ \nu_y \\ \omega_z \end{bmatrix}$$

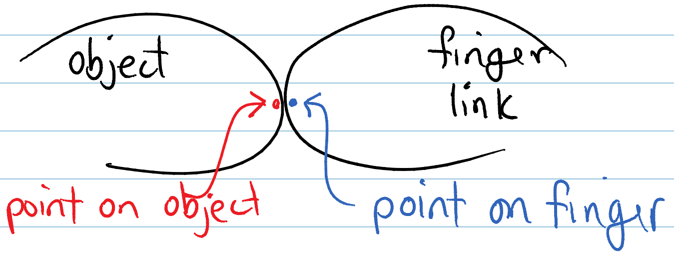


For $n_b > 1$

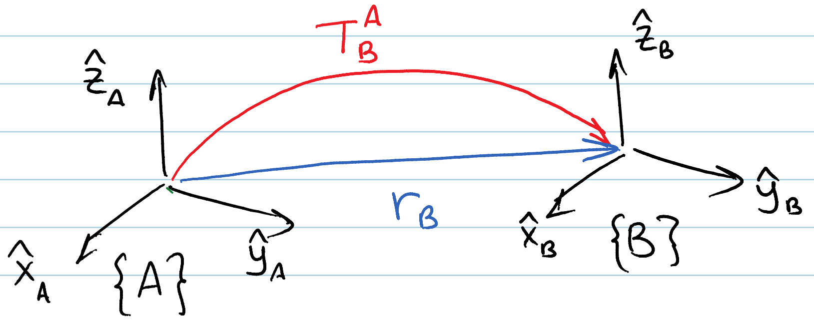
$$\nu = \begin{bmatrix}\nu_1 \\ \vdots \\ \nu_{n_b}\end{bmatrix}, \qquad \nu_j = \begin{bmatrix}(v_j)_x \\ (v_j)_y \\ (\omega_j)_z\end{bmatrix} \qquad j = 1, \dots, n_b$$

Let's go after the first goal for contact i.

Aside: There are really two i^{th} contact points; one on the object and one on the hand



First define homogeneous transformation



$$\underset{(4\times4)}{T_B^A} = \begin{bmatrix} \hat{x}_B^A & \hat{y}_B^A & \hat{z}_B^A & r_B^A \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} R_B^A & r_B^A \\ 0\,0\,0 & 1 \end{bmatrix}$$

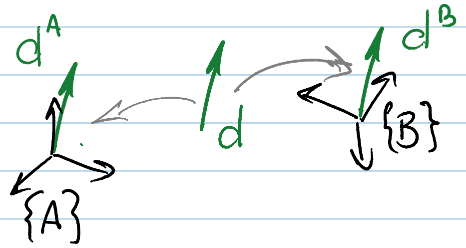
$$\underbrace{\hat{x}_B^A \quad \hat{y}_B^A \quad \hat{z}_B^A}_{\text{directions}} \quad \underbrace{r_B^A}_{\text{position}}$$

Given direction d expressed in $\{B\}$ (i.e., d^B is known);

Determine d^A.

Determine d^A.

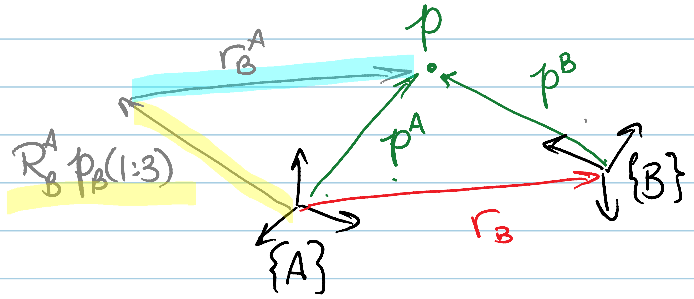
$$\begin{bmatrix} d^A \\ 0 \end{bmatrix} = T_B^A \begin{bmatrix} d^B \\ 0 \end{bmatrix} = \begin{bmatrix} R_B^A d^B \\ 0 \end{bmatrix}$$



Given a point p expressed in {B} (i.e., p^B is known);

Determine p^A

$$\begin{bmatrix} p^A \\ 1 \end{bmatrix} = T_B^A \begin{bmatrix} p^B \\ 1 \end{bmatrix} = \begin{bmatrix} R_B^A p^B(1:3) + r_B^A \\ 1 \end{bmatrix}$$



Contact Twist of Contact i on Object

$$\boxed{\nu_{i,obj} = \tilde{G}_i^T \nu}$$

(eq. 28.5 from Grasping chapter)

expressed in {N}

expressed in frame $\{C_i\}$

by definitions in Grasping chapter

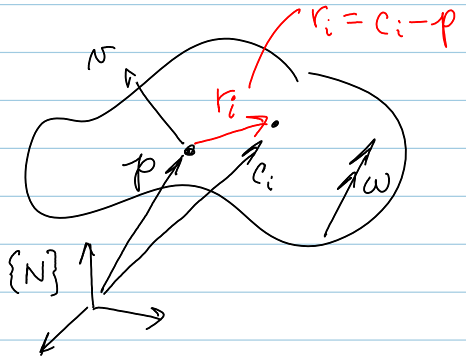
These superscripts are not used in the text in order to simplify the notation.

We want $\tilde{G}_i^T$!

We want G_i !

Let p be the center of mass of the object.

then $\dot{p}^N = v^N_{Borg}$



Given: $v^N \quad \omega^N$

Determine: $v_i^{c_i}, \ \omega^{c_i}$

Translational velocity varies from point to point.

First compute velocity of point c_i on object in frame $\{N\}$

$$v^N_{c_i} = v^N + \boxed{\omega^N \times r_i^N}$$

where $r_i^N = c_i^N - p_i^N$

$$\boxed{-r_i^N \times \omega^N}$$

$$v^N_{c_i} = v^N \boxed{- S(r_i^N)\,\omega^N}$$

The angular velocity of every point on a rigid body is identical

∴ simply change frame of expression

$$\omega^{c_i} = R^{c_i}_N \ \omega^N$$

where $S(a)$ is the cross product matrix of the vector $a \in \mathbb{R}^3$

$$S(a) = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix}$$

Note that $S(a) = -S(a)^T$.

Equivalently, $S(a)$ is skew-symmetric.

Write vel. twist at contact i in $\{N\}$ in matrix form:

$$\begin{bmatrix} v_i^N \\ \omega_i^N \end{bmatrix} = \begin{bmatrix} I_{3\times3} & S(r_i^N)^T \\ 0_{3\times3} & I_{3\times3} \end{bmatrix} \begin{bmatrix} v^N \\ \omega^N \end{bmatrix}$$

$$\begin{bmatrix} \\ \omega_i^N \end{bmatrix}_{(6\times1)} = \begin{bmatrix} & \\ 0_{3\times3} & I_{3\times3} \end{bmatrix}_{(6\times6)} \begin{bmatrix} \\ \omega^N \end{bmatrix}_{(6\times1)}$$

Now T'form velocity twist to $\{C_i\}$

Let $R_i = R^N_{c_i} = [\,\hat{n}_i^N \quad \hat{t}_i^N \quad \hat{o}_i^N\,]_{(3\times3)}$

Given a vector b expressed in $\{C_i\}$, we can express it in $\{N\}$ via: $b^N = R^N_{c_i}\, b^{c_i} = R_i b^{c_i}$

Back to the twist t'form ....

$$\nu_{i,obj} = \begin{bmatrix} v^{c_i}_{i,obj} \\ \omega^{c_i}_{i,obj} \end{bmatrix} = \underbrace{\begin{bmatrix} R_i^T & 0 \\ 0 & R_i^T \end{bmatrix}}_{\bar{R}_i^T} \underbrace{\begin{bmatrix} I & S(r_i^N)^T \\ 0 & I \end{bmatrix}}_{P_i^T} \underbrace{\begin{bmatrix} v^N \\ \omega^N \end{bmatrix}}_{\nu = \nu^N}$$

$$\nu_{i,obj} = \bar{R}_i^T P_i^T \nu$$

$$\boxed{\nu_{i,obj} = \tilde{G}_i^T \nu} \longleftarrow \text{(eq. 28.5 of Grasping)}$$

where $\underline{\underline{\tilde{G}_i^T = \bar{R}_i^T \; P_i^T}}$ (eq. 28.6)

↳ First step completed, we have $\tilde{G}_i^T$

$$\tilde{G}_i^T = \begin{bmatrix} R_i^T & R_i^T(S(r_i^N))^T \\ 0 & R_i^T \end{bmatrix} = \begin{bmatrix} \hat{n}_i^T & (r_i \times \hat{n}_i)^T \\ \hat{t}_i^T & (r_i \times \hat{t}_i)^T \\ \hat{o}_i^T & (r_i \times \hat{o}_i)^T \\ 0 & \hat{n}_i^T \\ 0 & \hat{t}_i^T \\ 0 & \hat{o}_i^T \end{bmatrix}_{(6\times6)}$$

Note: The matrix $\tilde{G}_i$ is identical to the t'form

T_v in Craig's 2nd edition page 181, where his $\{A\}$ and $\{B\}$ are replaced by our $\{C_i\}$ and $\{N\}$ respectively AND if our $\{B\}$ = our $\{N\}$.

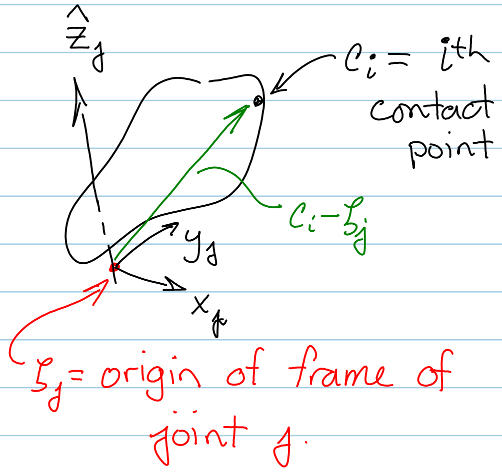
Now the second step with contact i, determine its velocity on the hand

Twist of Contact i on Hand

$$\boxed{\underset{(6\times 1)}{\nu_{i,hnd}} = \nu^{C_i}_{i,hnd} = \underset{(6\times n_q)}{\tilde{J}_i}\,\underset{(n_q\times 1)}{\dot{q}}} \longleftarrow \left(\text{eq 28.10 of Grasping}\right)$$

As for object twists, first map joint velocities to finger contact velocities in $\{N\}$. Then t'form velocities into $\{C_i\}$.

Let v^N_{ij} denote translational vel. of c_i induced by vel. of joint j.



$$v_{ij} = \begin{cases} 0 & \longleftarrow \text{joint } j \text{ distal to contact } i \\ \hat{z}^N_j \dot{q}_j & \longleftarrow \text{joint } j \text{ prismatic} \\ \underbrace{\hat{z}^N_j \dot{q}_j \times (c^N_i - \xi^N_j)} & \leftarrow \text{joint } j \text{ revolute} \end{cases}$$

$$= -(c_i^N - \xi_j^N) \times \hat{z}_j^N \dot{q}_j$$

$$= -S(c_i^N - \xi_j^N)\, \hat{z}_j^N \dot{q}_j$$

$$= S^T(c_i^N - \xi_j^N)\, \hat{z}_j^N \dot{q}_j$$

Let d_{ij} be the geometric part of v_{ij}^N, so that

$$v_{ij}^N = d_{ij}^N \dot{q}_j$$

Then we see that $$d_{ij}^N = \begin{cases} 0_{(3\times1)} \longleftarrow & j \text{ distal to } i \\ \hat{z}_j^N \longleftarrow & j \text{ prismatic} \\ S^T(c_i^N - \xi_j^N)\, \hat{z}_j^N \leftarrow & j \text{ revolute} \end{cases}$$

Denote by l_{ij} the angular velocity induced at point c_i by joint velocity $\dot{q}_j$.

$$\omega_{ij}^N = l_{ij}^N \dot{q}_j$$

$$l_{ij}^N = \begin{cases} 0_{(3\times1)} \longleftarrow & j \text{ distal to } i \\ 0_{(3\times1)} \longleftarrow & j \text{ prismatic} \\ \hat{z}_j^N \longleftarrow & j \text{ revolute} \end{cases}$$

Putting d_{ij} and l_{ij} together ...

$$v_{ij}^N = \begin{bmatrix} d_{ij}^N \\ l_{ij}^N \end{bmatrix} \dot{q}_i$$

Combine all joints ...

$$v_{i,hnd}^N = \underbrace{\begin{bmatrix} d_{i,1}^N & \cdots & d_{i,nq}^N \\ l_{i,1}^N & & l_{i,nq}^N \end{bmatrix}}_{Z_i} \begin{bmatrix} \dot{q}_1 \\ \vdots \\ \dot{q}_{nq} \end{bmatrix}$$

Now t'form to contact frame i, $\{C_i\}$, as before

$$\nu_{i,hnd} = \nu^{C_i}_{i,hnd} = \underbrace{\bar{R}_i^T Z_i}_{\tilde{J}_i} \dot{q}$$

$$\boxed{\nu_{i,hnd} = \tilde{J}_i \tilde{q}}$$ (eq 28.10 in Grasping Chap.)

Now we've reached the second goal for contact i:

$$\tilde{J}_i = \bar{R}_i^T Z_i$$

Put all contacts together...

$$\nu_{c,hnd} = \underset{(6n_c \times 1)}{\begin{bmatrix} \nu_{1,hnd} \\ \vdots \\ \nu_{n_c,hnd} \end{bmatrix}} \qquad \nu_{c,obj} = \underset{(6n_c \times 1)}{\begin{bmatrix} \nu_{1,obj} \\ \vdots \\ \nu_{n_c,obj} \end{bmatrix}}$$

$$\boxed{\nu_{c,hnd} = \tilde{J} \dot{q} \quad (28.12) \qquad \nu_{c,obj} = \tilde{G}^T \nu \quad (28.13)}$$

where $\tilde{J} = \underset{(6n_c \times n_q)}{\begin{bmatrix} \tilde{J}_1 \\ \vdots \\ \tilde{J}_{n_c} \end{bmatrix}}$, $\tilde{G}^T = \underset{(6n_c \times 6)}{\begin{bmatrix} \tilde{G}_1^T \\ \vdots \\ \tilde{G}_{n_c}^T \end{bmatrix}}$

First step completed!

Given the positions of the contact points, the kinematic structure of the hand, the object's velocity twist, and the joint velocities, we can construct the Jacobian matrices $\tilde{G}^T$ and $\tilde{J}$ and compute the velocities of the contact points on the object $\nu_{c,obj}$ and the velocities on the hand $\nu_{c,hnd}$.