

Probs_Chap3_Lavalle

Tuesday, April 28, 2009
9:13 AM

- 3.1) Make a nose with three linear inequalities. Assume origin is at the center of base of nose.

We want to exclude the interior of the nose.

Interior is given by:

$$y \leq -2x + 2 \Rightarrow$$

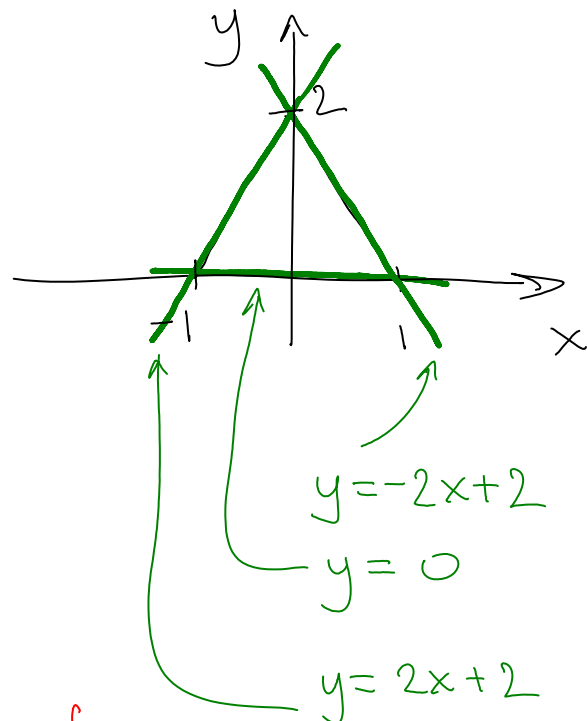
So we get:

$$y + 2x - 2 \leq 0$$

$$-y \leq 0$$

$$y - 2x - 2 \leq 0$$

Interior of
nose

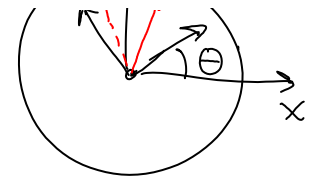


To complete this problem, put results in form of primitives that remove nose interior.

- 3.2) Consider rotation of



a disc primitive about its center.



$$x^2 + y^2 - 1 \leq 0$$

$$\begin{aligned} \begin{bmatrix} x_{\text{new}} \\ y_{\text{new}} \end{bmatrix} &= \begin{bmatrix} c_{\theta} & -s_{\theta} \\ s_{\theta} & c_{\theta} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \\ &= \begin{bmatrix} c_{\theta}x - s_{\theta}y \\ s_{\theta}x + c_{\theta}y \end{bmatrix} \end{aligned}$$

Substitute into primitive.

$$\begin{aligned} &(c_{\theta}x - s_{\theta}y)^2 + (s_{\theta}x + c_{\theta}y)^2 \\ &= c_{\theta}^2 x^2 - 2c_{\theta}s_{\theta}xy + s_{\theta}^2 y^2 \\ &\quad + (s_{\theta}^2 x^2 + 2s_{\theta}c_{\theta}xy + c_{\theta}^2 y^2) \\ &\hline &x^2 + 0 + y^2 \end{aligned}$$

\therefore if the original (x, y) satisfies the primitive, so will the rotated (x, y) .