Chapter 4, LaValle

Saturday, May 03, 2008
11:36 AM

5. C-space of cylindrical rod is $\boxed{\mathbb{R}^3 \times S^2}$

Imagine pinning rod end to center of sphere with a ball-in-socket joint.

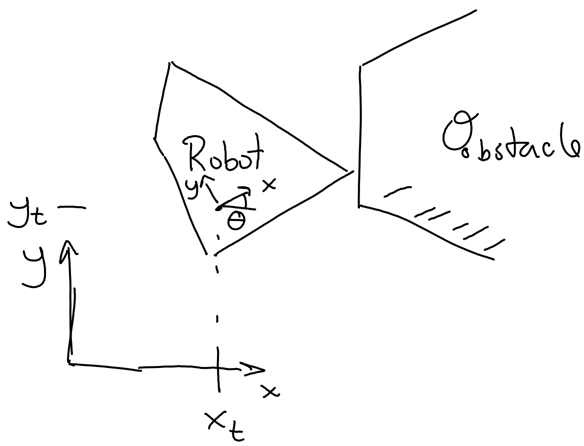
Rod can point in any direction, hence S^2

Base of rod can translate, hence $\mathbb{R}^3$

Rotation of rod about its axis makes no difference

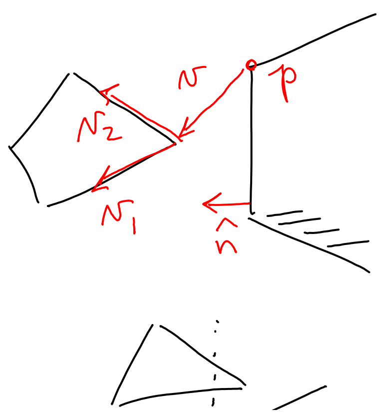
9. Derive H_A for type VE contact

vertex of robot contacts edge of obstacle



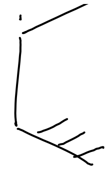
Let $q = (x_t, y_t, \theta)$

Sufficient conditions for $q \in C_{free}$



$$\boxed{\begin{aligned} N(q) \cdot \hat{n} &> 0 \\ N_1(\theta) \cdot \hat{n} &\geq 0 \\ N_2(\theta) \cdot \hat{n} &\geq 0 \end{aligned}}$$

Note: conditions are not necessary



Let $H_f = \{ q \mid v(q)\cdot\hat{n} > 0,\ v_1(\theta)\cdot\hat{n} \geq 0,\ v_2(\theta)\cdot\hat{n} \geq 0 \}$

$H_f \subset C_{free}$

Let $H_A = C \setminus H_f \quad \Rightarrow \quad H_A = H_1 \cup H_2 \cup H_3$

where

$$H_1 = \{ q \mid \hat{n}\cdot v(q) \leq 0 \}$$

$$H_2 = \{ q \mid \hat{n}\cdot v_1(\theta) \leq 0 \}$$

$$H_3 = \{ q \mid \hat{n}\cdot v_2(\theta) \leq 0 \}$$

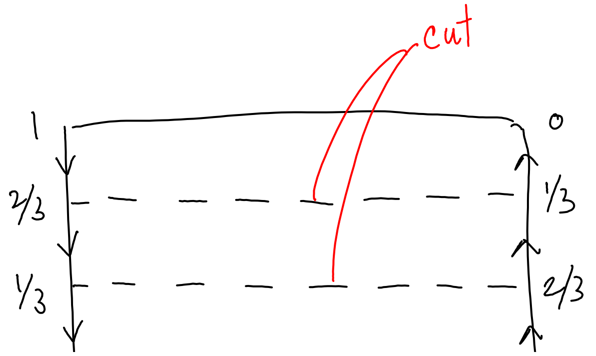
The main difference between EV & VE contacts are the details of the formulas in H_i, $i=1,2,3$, when expanded as functions of x_t, y_t, θ.

(16.) 5 bodies free to move in $\mathbb{R}^3$ gives

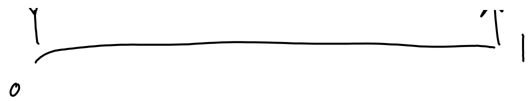
$$C = SE(3) \times \cdots \times SE(3) = (SE(3))^5$$

C is 30-dimensional

(18) Form mobius band then cut along



dashed line.



What results?

One gets a mobius band of length 1 from the center of the strip.

The outer ⅓-wide strips stay connected to each other & form a hoop like a mobius band with two twists.

Also the mobius band and double twist band are connected like links in a chain.

Question: Is the double twist band homeomorphic to a cylindrical surface, i.e., $S \times I$?

Is the result a manifold?

Yes, it is a manifold w/ boundary.

Every point not on the edge has a nbhd. that is Euclidean ($\mathbb{R}^2$). Every point on the edge has a nbhd that is a half-plane.

(21.) Tetrahedron & Cube. ~~How~~ many facets on the C_{obs} if they are free to move in $\mathbb{R}^3$?

$VF = 8 \times 4 = 32$

$$\left.\begin{array}{l} FV = 6 \times 4 = 24 \\ EE = 12 \times 6 = 72 \end{array}\right\} \Rightarrow 128 \quad \text{2D facets.}$$

Chapter 5: LaValle

Saturday, May 03, 2008
12:02 PM

(3.) Is $\rho(x,x') = \begin{cases} 1 & \forall\ x \neq x' \\ 0 & \text{if } x = x' \end{cases}$ a metric?

Requirements:

Non neg: $\rho(x,x') \geq 0$ ✓ Yes

$\rho(x,x) = 0$ ✓ Yes

Reflex: $\rho(x,x') = \rho(x',x)$ ✓ Yes

Tri Ineq.: $\rho(x,x') + \rho(x',x'') \geq \rho(x,x'')$ ✓ Yes

for Triangle Ineq. there are 3 cases:

① x, x', x'' are distinct

② all are equal

③ two are distinct

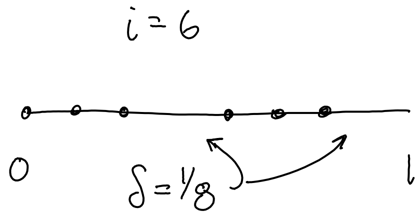
Triangle Ineq. is satisfied in all cases.

$\boxed{\text{Yes, } \rho \text{ is a metric}}$

(6.) Determine dispersion as a fcn of samples of a vander Corput sequence on the circle of a vander Corput sequence on the circle

$$\delta(P) = \sup_{x \in X} \left\{ \min_{p \in P} \{ \rho(x,p) \} \right\}$$

Note: disp. is determined by the largest interval



One can derive δ as:

$$\delta = \left(\frac{1}{2}\right)^{N+1}$$

floor.

where $N = \lfloor \log_2(i) \rfloor$

i = # of samples

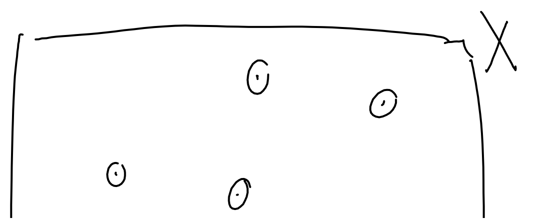
(13) Show that for any set of points in $[0,1]^n$ a range space $\mathcal{R}$ can be designed so that the discrepancy is as close to 1 as desired

cardinality measure of R

$$D(P,\mathcal{R}) = \sup_{R \in \mathcal{R}} \left\{ \frac{|P \cap R|}{k} - \frac{\mu(R)}{\mu(X)} \right\}$$

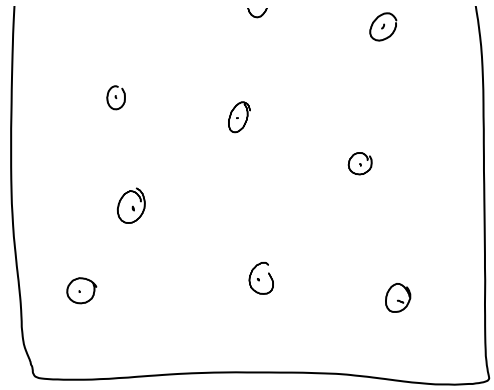
#samples measure of X (area)

R is any collection of subsets



of subsets

P is points →



R is open circles containing $p \in P$

Let radii of circles $\to 0$

$\Rightarrow \quad |P \cap R| = k$ and $\mu(R) \to 0$

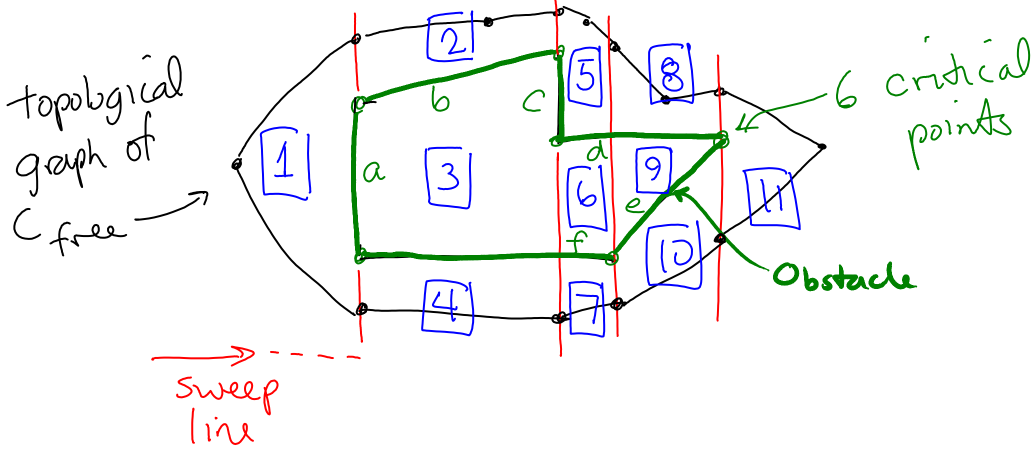
$\therefore \quad D \to 1$

Chapter 6 : LaValle

Saturday, May 03, 2008

2:05 PM

1.) When edges lie parallel to the vertical direction, one has to modify the way cell boundaries are entered into the list of bndries



$L = \{\emptyset\}, \{b, f\}, \{d, f\}, \{d, e\}, \{\emptyset\}$

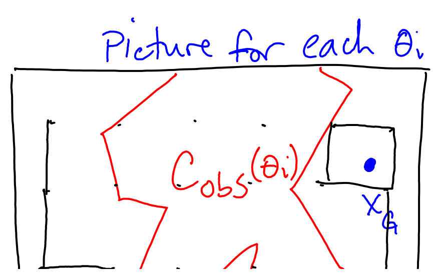
Note a & c are never in the list of bndries.

7) Resolution complete alg for planning motion of polygon robot in field of polygonal obstacles.

1. Create an $n \times n$ grid in (x,y) space

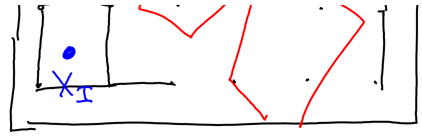
2. Choose a v.d. Corput seq. of orientations, Θ.

3. For next $\theta_i \in \Theta$:



a.) construct $C_{obs}(\theta_i)$

b.) connect adjacent grid points using straight-line local planner.



4. For each θ_i & θ_j that are adjacent in the sequence:

   a) Identify $(x_k, y_k) \notin C_{obs}(\theta_i) \cup C_{obs}(\theta_j)$

   b.) Attempt to connect (x_k, y_k, θ_i) to (x_k, y_k, θ_j) by sweeping robot around reference point from θ_i to θ_j. Connect if no collisions.

5. For each new θ_i check for solution using a usual method.

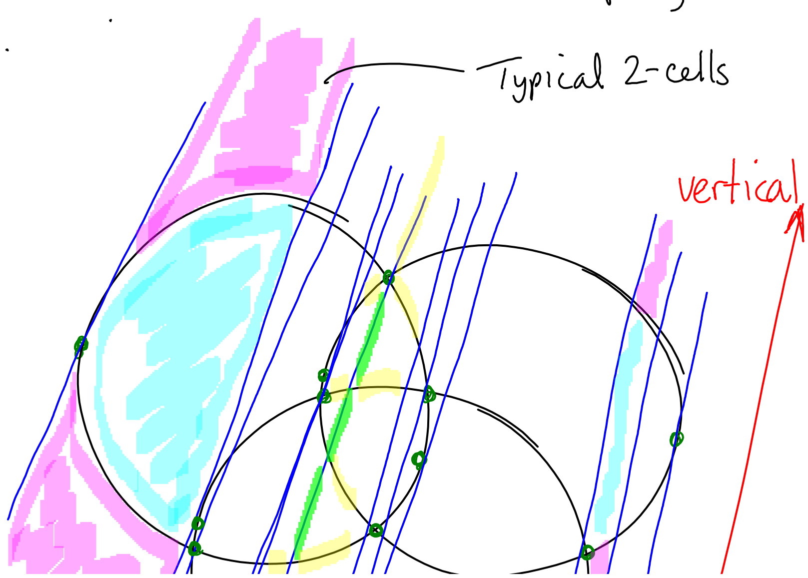
6. If no solution:

   a. increase grid resolution and restart

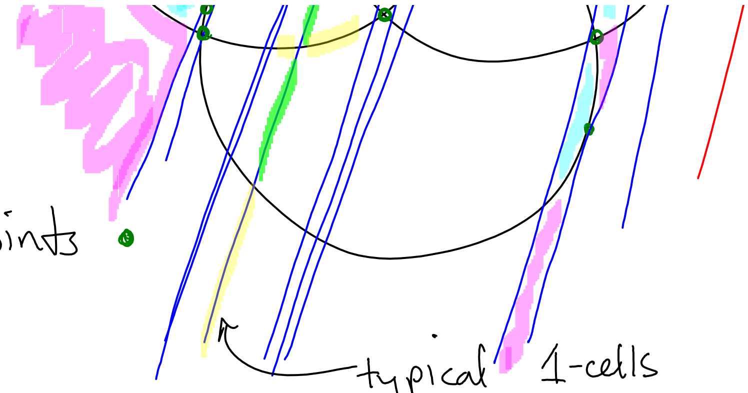
   b. select next θ_i and continue (Go to step 3)

   c. give up.

9. Cylindrical algebraic cell decomposition into 0-cells



into 0-cells,
1-cells, 2-cells.

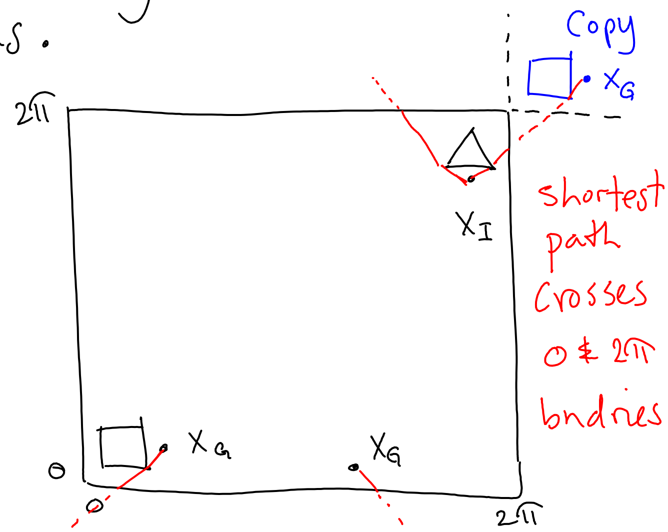


12 critical points

≈100 2-cells
≈60 1-cells
≈30 0-cells

(12) Develop shortest path alg on flattened torus.

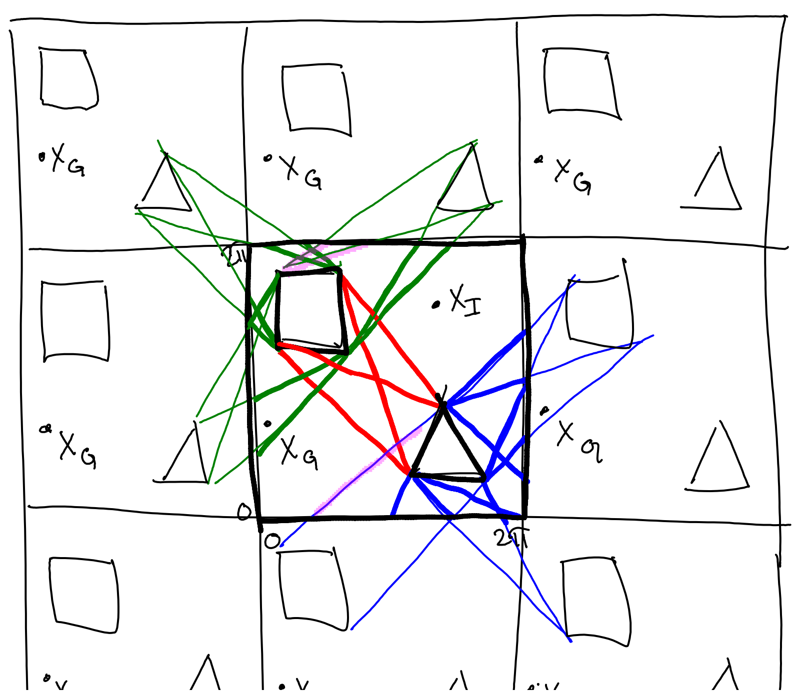
Be sure to handle identifications correctly.

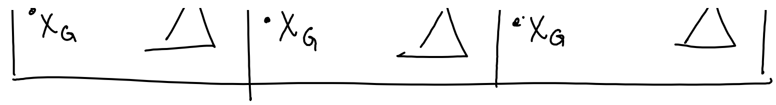


1. Make 8 adjacent copies

2. Create visibility graph between △ & □ on all tiles.

3. Make 8



copies of x_G 

4. Find shortest path to each of the 9 x_G's

5. Measure lengths of 9 paths. Keep shortest.