5.7. Line of Force; Moment Labeling

These are 2 more graphical methods for planar problems.

Graphical method help ~~to~~ build intuition by giving full space of Solutions!

Numerical methods apply to ~~all~~ spatial problems, but give specific solutions. Additional computation can give "most" solutions.

Recap of Graphical Methods so far

Instantaneous Center – represents a ~~ray~~ vector in diff'l twist space by projection to the oriented plane

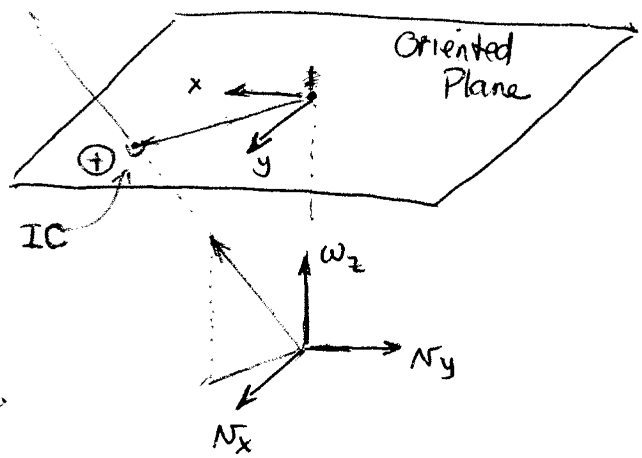
Reuleaux's Method – represents polyhedral convex cones in planar diff'l twist space by projecting them to the oriented plane.

Instantaneous Center

diff'l twist
OR
velocity twist

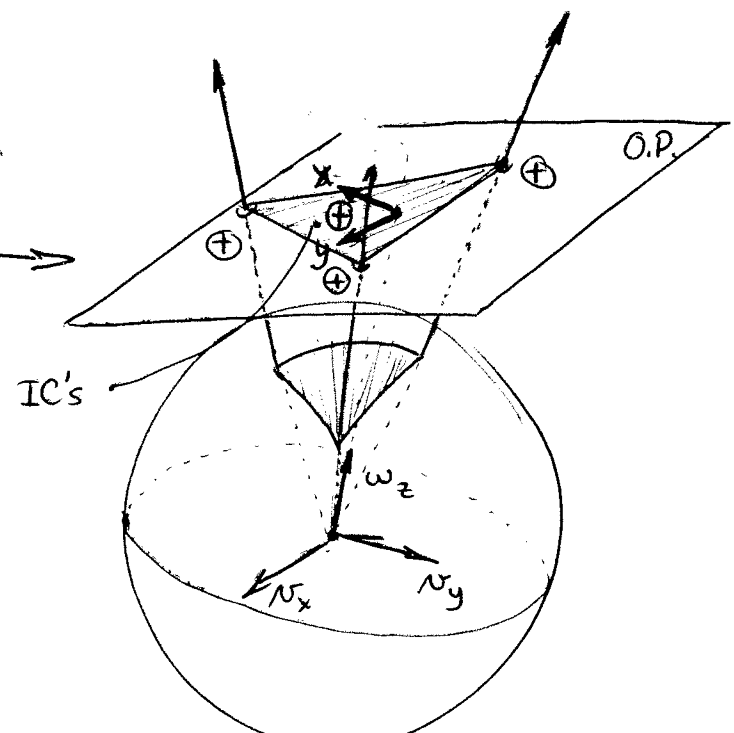
$$\nu\, dt = \begin{bmatrix} v_x \\ v_y \\ \omega_z \end{bmatrix} dt \longrightarrow \begin{bmatrix} -v_y/\omega_z \\ v_x/\omega_z \end{bmatrix} dt$$

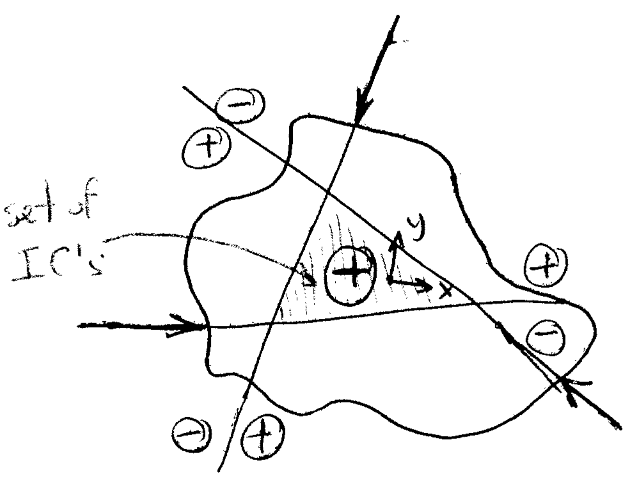
rotate (x,y) axes of twist frame $-90°$ & project onto Oriented Plane



Reuleaux's Method

Intersect the half spaces.
3 contacts gives rise to 3 half space constrs.





Only ⊕ triangle is set of possible IC's.

Note: For this picture, the frame must lie inside the triangle of IC's.

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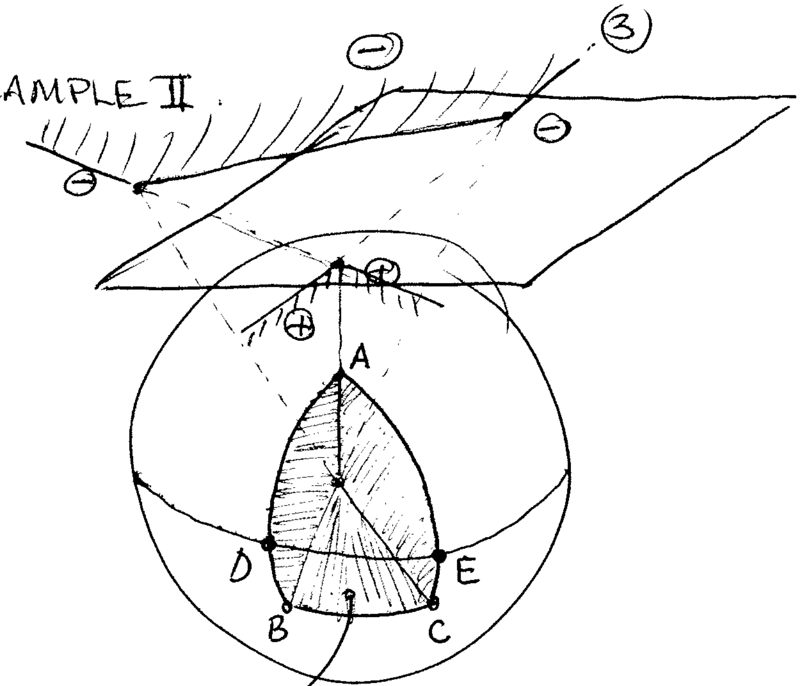
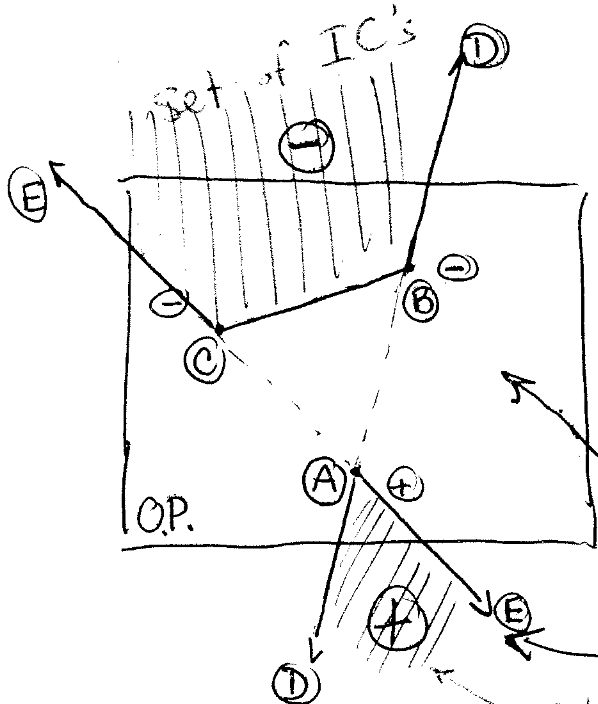
Reuleaux's Method - EXAMPLE II





PCC of IC's

⊕ part in the ⊕ plane

⊖ part in the ⊖ plane.

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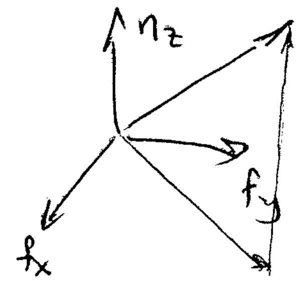
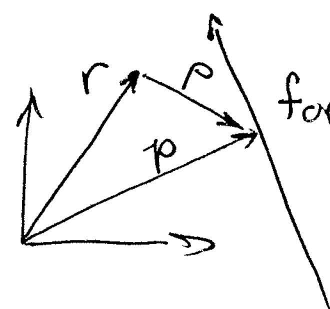
5.7 Moment Labeling

Line of force is another application of the oriented plane.

Given a wrench (f_x, f_y, n_z), what is the line of force?



It is the set of points for which the moment of the force is zero



We know $n_z = p \times f$.

Find all $r \ni \rho \times f = 0$.

$\rho = p - r$

$\therefore \quad (p - r) \times f = 0$

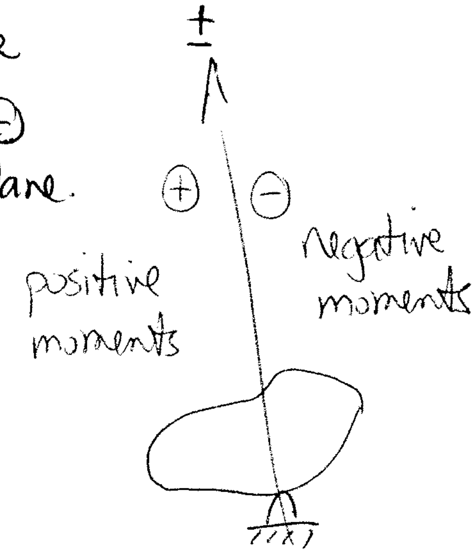
$\quad p \times f - r \times f = 0$

$$\boxed{n_z - r_x f_y + r_y f_x = 0}$$

Since f_x, f_y, n_z are given, the above equation describes an <u>undirected</u> line.

Moment Labeling (in Workspace)

All possible resultants must make positive moment wrt all pts in ⊕ half plane & neg. wrt ⊖ half plane.



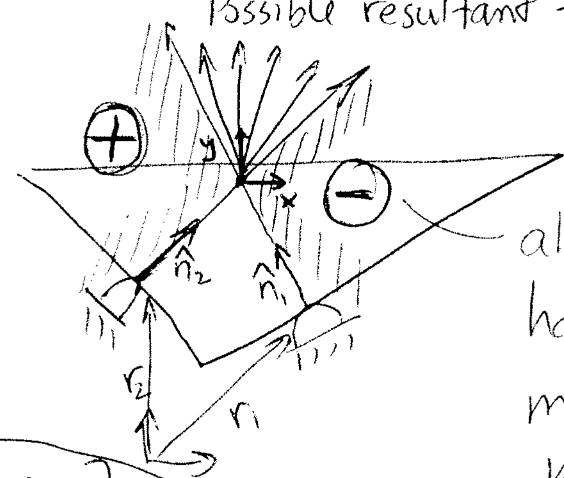
Use ful to characterize the set of forces that could be applied by a contact.

⇒ Set of forces that could be resisted, i.e. for stable grasping.

Triangle with 2 frictionless contacts

Possible resultant forces — magnitude is arbitrary

What else can be said?



all resultant forces have negative moments about these points

$$\underset{\text{wrench}}{w} = \begin{bmatrix} \hat{n}_1 & \hat{n}_2 \\ r_1 \times \hat{n}_1 & r_2 \times \hat{n}_2 \end{bmatrix} \begin{bmatrix} \lambda_{1n} \\ \lambda_{2n} \end{bmatrix}$$

$$\begin{bmatrix} \lambda_{1n} \\ \lambda_{2n} \end{bmatrix} \geq 0$$

Still more information

Chose frame ∋ $r_i \times \hat{n}_i = 0$

$$\begin{bmatrix} \hat{n}_1 & \hat{n}_2 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \end{bmatrix} \overset{?}{=} \begin{bmatrix} f_x \\ f_y \\ 1 \end{bmatrix}$$

No!

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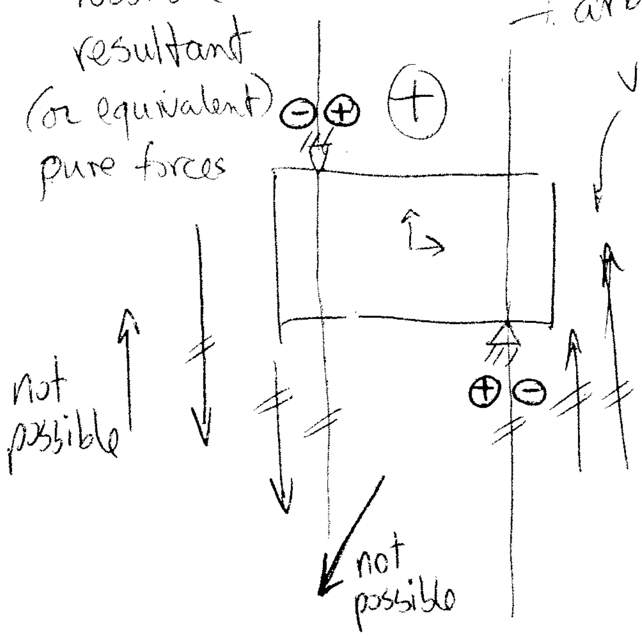


Can have

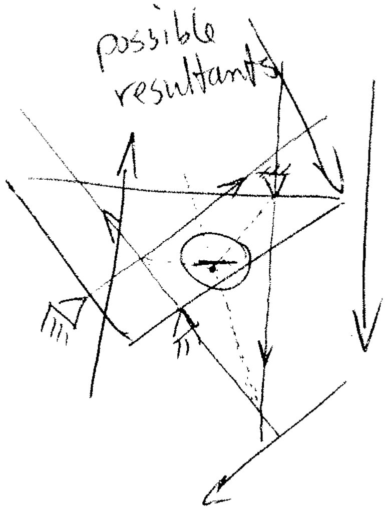
$$\begin{bmatrix} 0 & 0 \\ 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \lambda_{1n} \\ \lambda_{2n} \end{bmatrix} = w$$

$$\lambda_{1n}, \lambda_{2n} \geq 0$$

Is $\begin{bmatrix} 0 & 0 \\ 1 & -1 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} \lambda_{1n} \\ \lambda_{2n} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 1/2 \end{bmatrix}$ Possible? $\Rightarrow \begin{bmatrix} \lambda_{1n} \\ \lambda_{2n} \end{bmatrix} = \begin{bmatrix} 3/2 \\ -1/2 \end{bmatrix}$

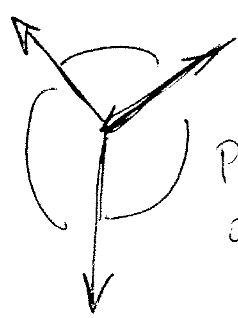
No!

Instead this



$$\begin{bmatrix} n_{1x} & n_{2x} & n_{3x} \\ n_{1y} & n_{2y} & n_{3y} \\ -1 & -1 & -1 \end{bmatrix} \begin{bmatrix} \lambda_{1n} \\ \lambda_{2n} \\ \lambda_{3n} \end{bmatrix} = w$$

$$\lambda_{1n}, \lambda_{2n}, \lambda_{3n} \geq 0$$



pos span of forces is $\mathbb{R}^2$

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6.1

Summarize

- A line of force represents a ray in planar wrench space by projecting its supplementary cone to the oriented plane.
- Moment labeling represents a polyhedral convex cone of wrenches by projecting its supplementary cone to the oriented plane.

Force Dual

Reuleaux's method represents cone of diff twists by central projection to the oriented plane.

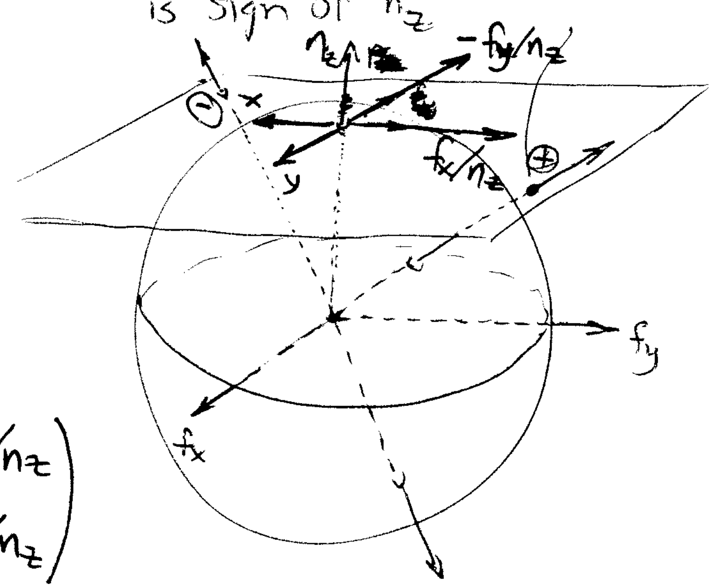
Moment labeling represents cone of wrenches by central projection of their ~~orient~~ supplementary cone to the oriented plane

Force-Dual Method represents a P.C.C. in wrench space by central projection to the oriented plane

define transformation from line (or ray) to point

$$\begin{pmatrix} f_x \\ f_y \\ n_z \end{pmatrix} \mapsto \begin{pmatrix} -f_y/n_z \\ f_x/n_z \end{pmatrix}$$

where sign of point is sign of n_z

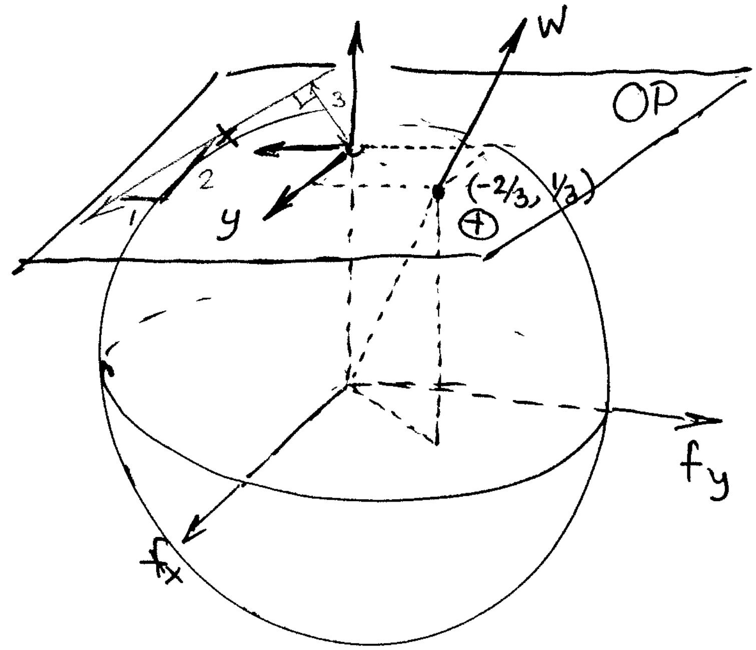


Independent of wrench magnitude

$$k\begin{pmatrix} f_x \\ f_y \\ n_z \end{pmatrix} \mapsto \begin{pmatrix} -f_y k/n_z k \\ f_x k/n_z k \end{pmatrix} = \begin{pmatrix} -f_y/n_z \\ f_x/n_z \end{pmatrix}$$

Example:

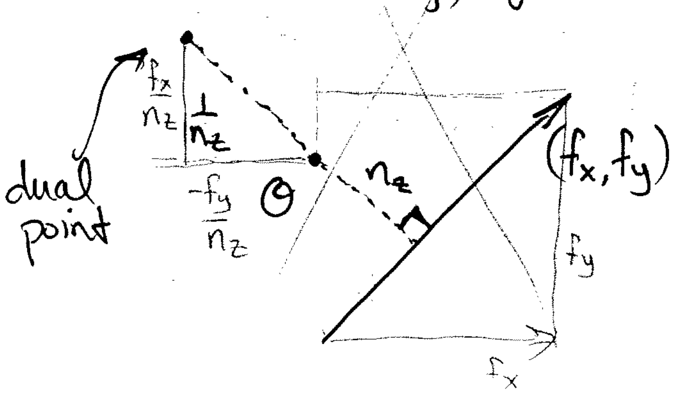
$$w = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} k \longmapsto \begin{bmatrix} -2/3 \\ 1/3 \end{bmatrix} k$$



Sign of point is the sign of the moment.

SKIP Interpretation of Dual Point of Force

Recall $\begin{pmatrix} f_x \\ f_y \end{pmatrix}$ give force direction, n_z gives location



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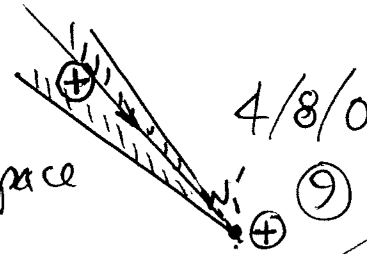
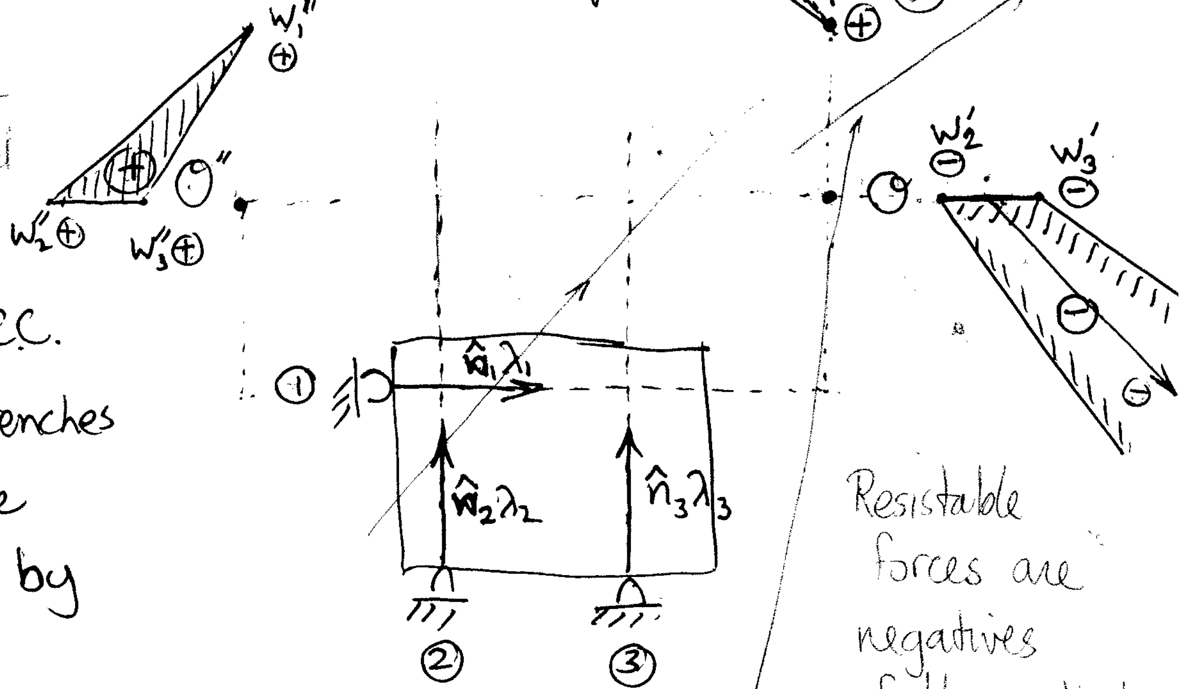
Overlaying Oriented Plane on Workspace





The " here represents a second choice of O, not the " in the text.

Points in P.C.C. represent wrenches that can be generated by contact

Resistable forces are negatives of those that can be produced. (Just change signs of regions for this)

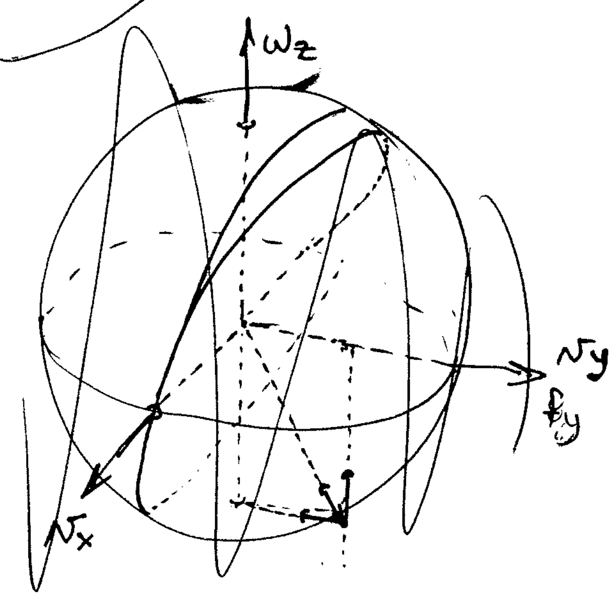
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & -2 & -1 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} = w$$

$$\lambda_1, \lambda_2, \lambda_3 \geq 0$$

Don't try to lay forces on this plane!

Moment Labeling Representation

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ \omega_z \end{bmatrix} \geq 0$$



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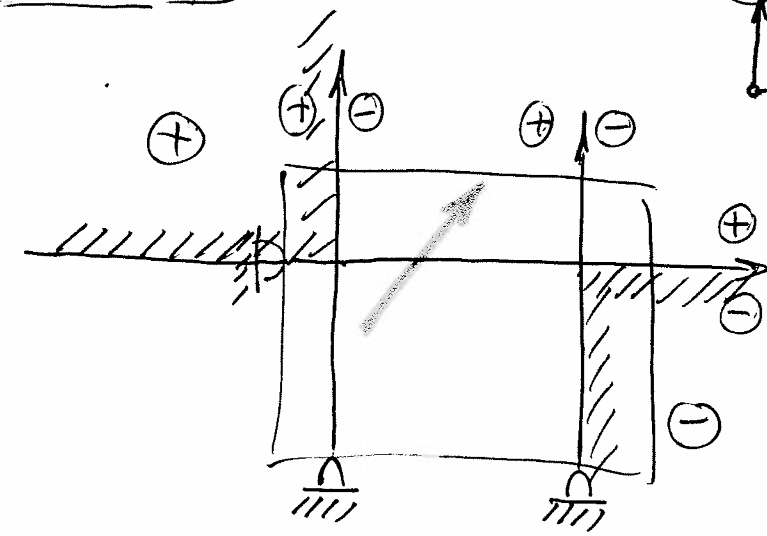
Moment Labeling Representation



$$\begin{array}{cc} & W_n \quad \lambda_n \\ W = & \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & -2 & -1 \end{bmatrix} \begin{bmatrix} \lambda_x \\ \lambda_y \\ \lambda_z \end{bmatrix} \cancel{\geq 0} \end{array}$$

$$\begin{array}{c} W_n^T \quad v \\ \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -2 \\ 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} v_x \\ v_y \\ \omega_z \end{bmatrix} \geq 0 \end{array}$$

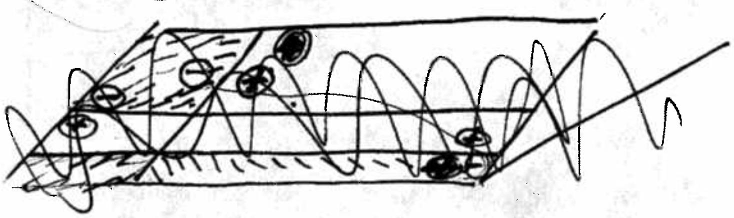
Reciprocal or Repelling twists are possible
Contrary twists are halted.

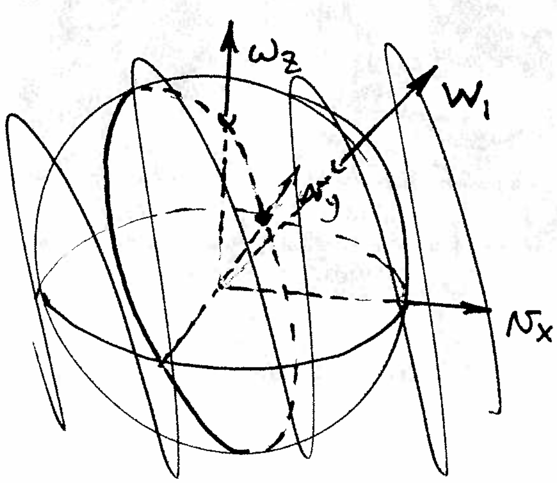
skip?

$v_x + \omega_z \geq 0 \Rightarrow v_x \geq -\omega_z$

$v_y - 2\omega_z \geq 0 \Rightarrow v_y \geq 2\omega_z$

$v_y - \omega_z \geq 0 \Rightarrow v_y \geq \omega_z$ redundant if $\omega_z > 0$





More on Moment Labeling

Rotate (N_x, N_y) $-90°$

~~Change Geometry~~

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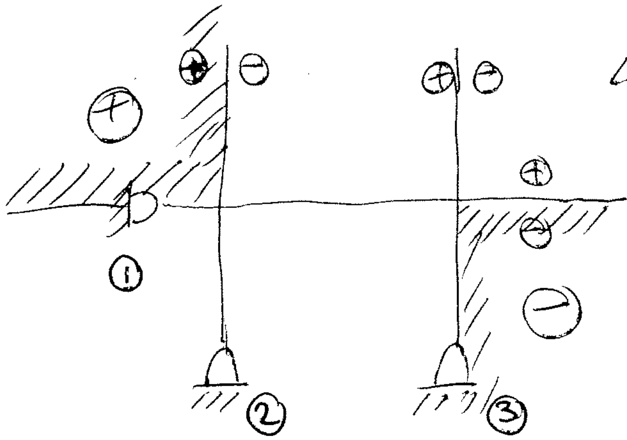
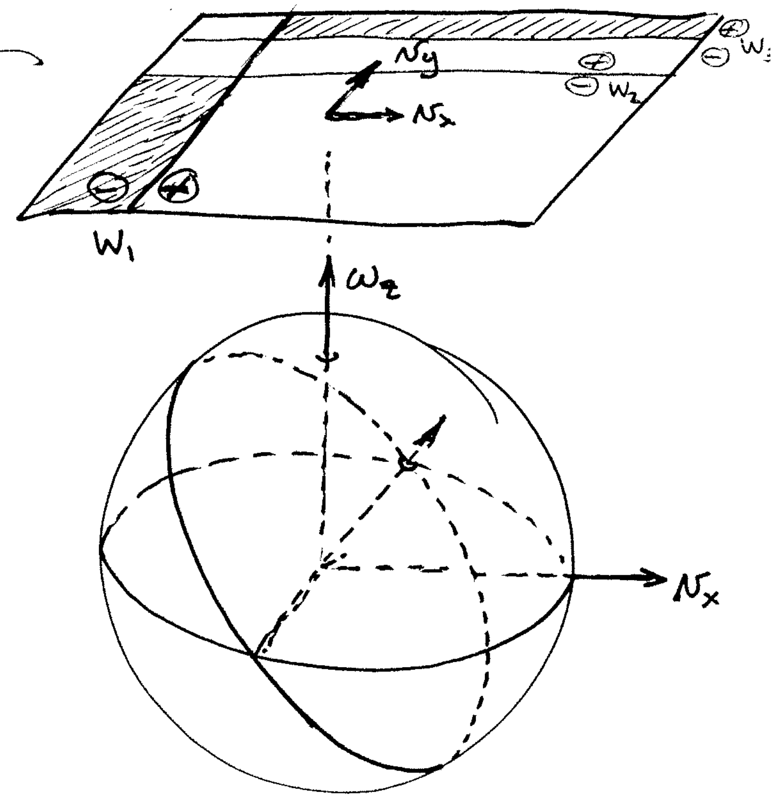
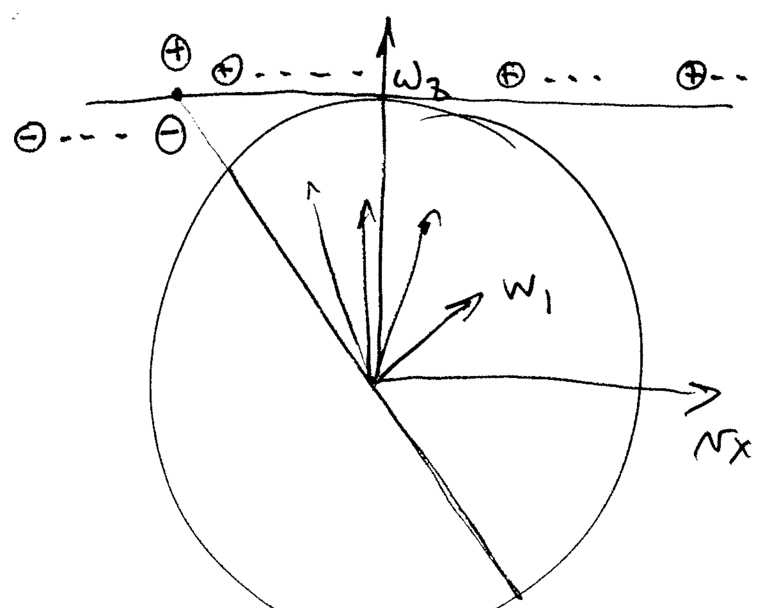
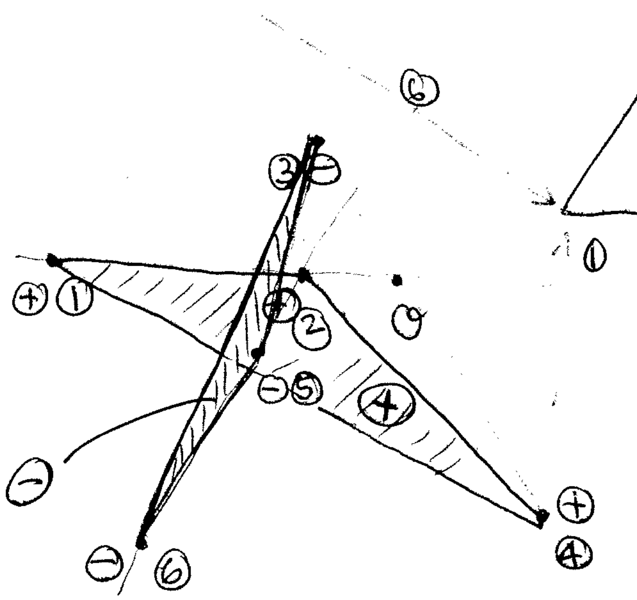


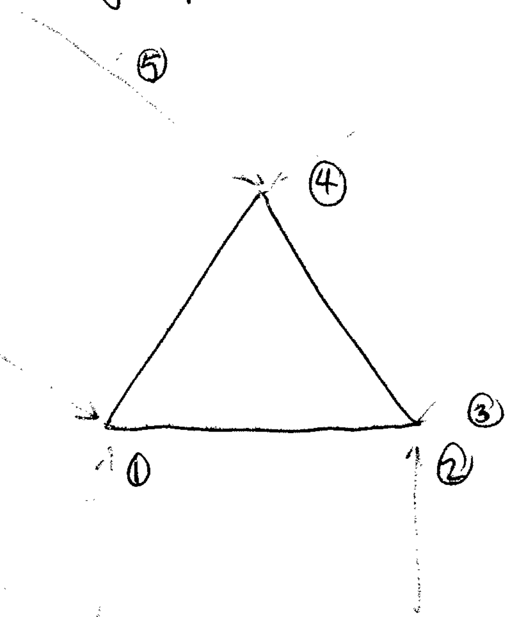


Side view for W_1



Suppose we have frictionless contact on all edge points



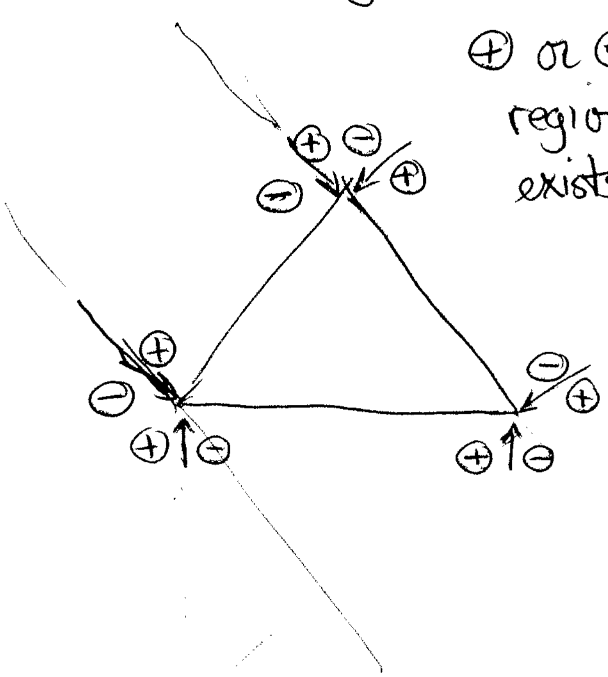


Since the + & − regions overlap non-trivially, we get that all of the O.P. is covered.

∴ all of wrench space can be generated.

Hence we have form cl.

Moment Labeling – shows no + or − region exists



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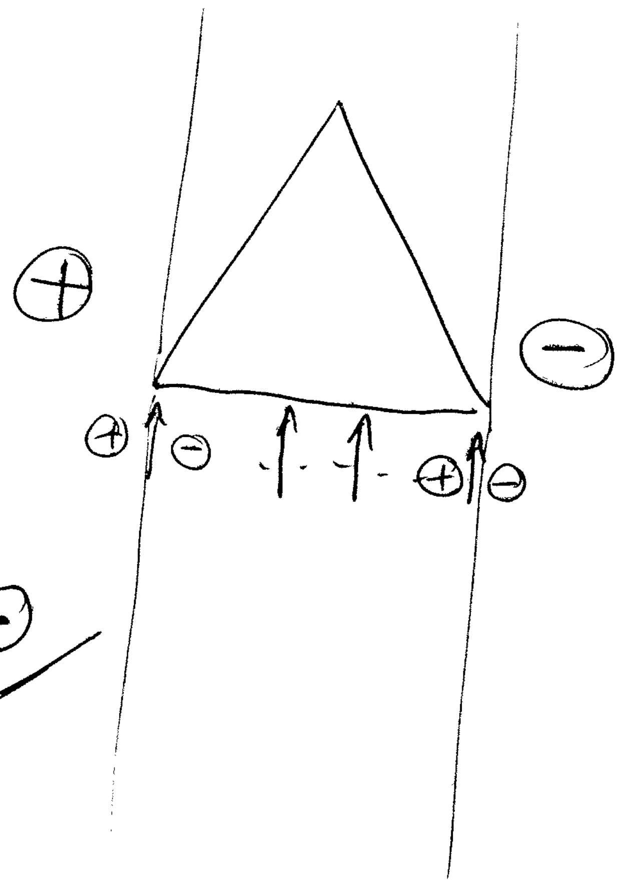
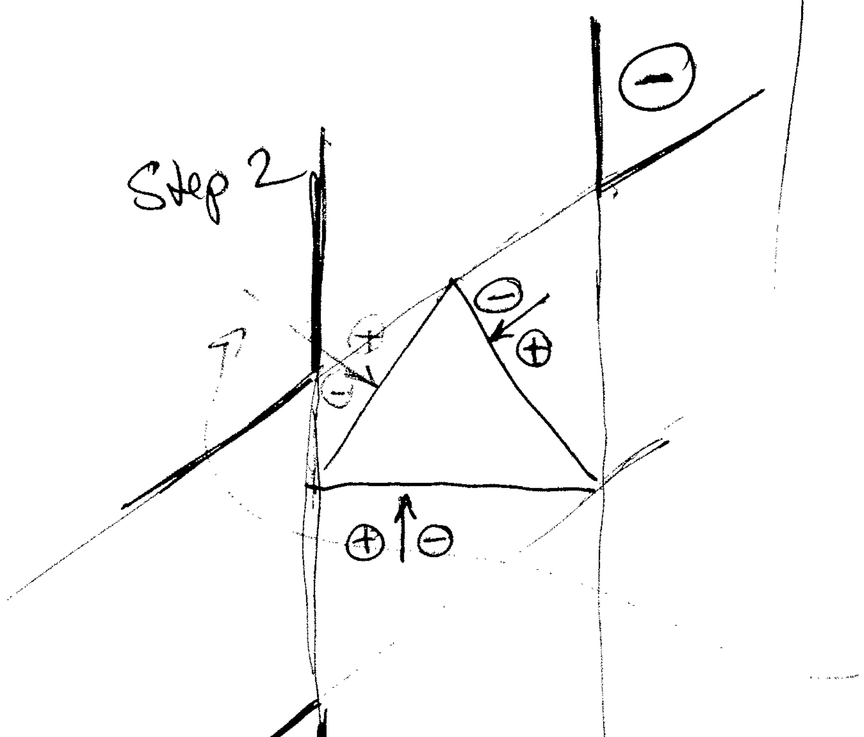
More Detail

Step 1
Edge 1



Step 2



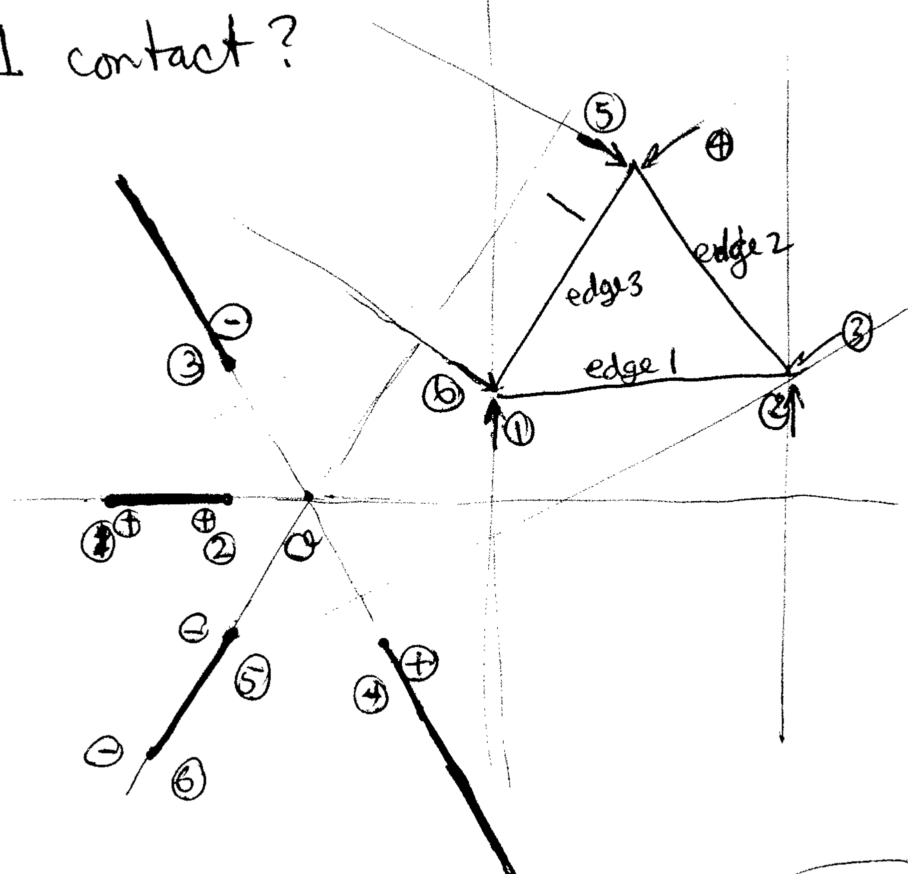
Step 3.

Pick any force on the other edge to eliminate $\oplus$ $\ominus$ regions.

So why bother with dual force method?

When we're interested in convex hulls & all possible forces & moments for a given set of contacts. Moment labeling is simpler.

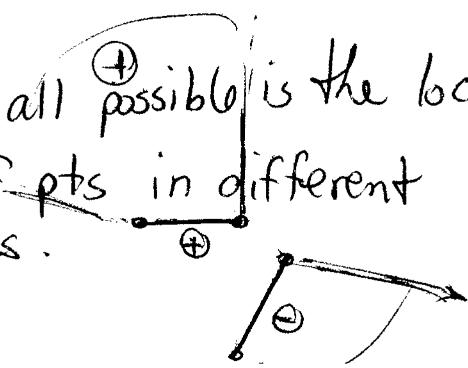
What if we want to know set of wrenches that could be applied to a frictionless part via 1 contact?



One contact ⇒ One point in the O.P., all possible is the locus.

Two contacts ⇒ conv. hull of pairs of pts in different parts of the locus.

edges 1 & 3 ⟹



Suppose we had one
contact with friction