Frontiers of Network Science 4250 (undergraduates)
Assignment 2, due before Lecture 26, noon on Monday, December 3rd 2018

Problem for Assignment 2: Clustering in Networks

Part A. Consider a network of N round and N square nodes. The probability that there is a link between nodes of the same shape is p_{in} and the probability that there is a link between nodes of different shape is p_{out}. A network has associative clusters if $p_{in} > p_{out}$ capturing a tendency to connect to nodes with the same shape. For $p_{out} = 0$ the network has at least two components, containing nodes with the same shape.

(a) Calculate the average degree of the square cluster made of only square nodes, and the average degree in the entire network.
(b) Determine the minimum p_{in} and p_{out} required to have, with high probability, just one component.
(c) Show that for large N even with very highly associative clusters ($p_{in} \gg p_{out} > 0$) the network displays the small-world property.

Part B. Consider the following balance variant of the above model, in which we have total $2N$ nodes and three clusters, two clusters of equal size containing round and square nodes and the third cluster with fraction f of all nodes, with hexagon nodes. Round and square nodes do not connect to each other (their $p_{out} = 0$) while they connect with probability p_{in} to nodes of the same shape. Hexagon nodes connect with the same probability $q_{out} = p_{in}$ to round nodes and with the same probability to the square nodes, but not to any of hexagon nodes (their $q_{in} = 0$).

(a) We call the round and square clusters *interactive* if a typical square node is just three steps away from a round node and vice versa. Evaluate the fraction of hexagon nodes required for the clusters to be interactive.
(b) Comment on the size of the hexagon cluster if the average degree of round (or square) nodes is $\langle k \rangle \geq 1$.
(c) Discuss the implications of this model for the structure of social (and other) networks.

Grading. Part A 20 pts: (a) 5 (b) 8 (c) 7; Part B 20 pts: (a) 8 (b) 7 (c) 5