Improving the Resilience of Mutualistic Networks

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Introduction to Research
- Motivation
- What is Resilience?
- Modeling the Problem

Theoretical Basis
- Modifying existing equation
- Strategies to improve resilience
- Projected stats that will improve

Current Results
- Change in $\beta_{eff}$
- Degree Distributions
- Bifurcations
- Overall graphs

Future Work
Research focuses on improving a pollination network’s resilience via node additions.

To qualify and find resilience, heavily draw upon work done by Gao, Barzel, and Barabasi [1].

- Current research uses much of the derived framework.
- One aspect of the research focused on mutualistic networks; my research uses those dynamics!

Work focuses on positive side-effects of node addition.
How we will be modeling the network!
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We will be modifying the main network, which will change the resilience of the projection networks.
What is Resilience?

Resilience Definition

Resilience is the ability of a system to adjust its activity to retain its basic functionality when errors, failures, or disruptions occur. It is a dynamical property. Networks can be more or less resilient to node or link perturbations depending on their dynamics.
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Can capture resilience using a 1-d function:

\[
\frac{dx}{dt} = f(\beta, x)
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Can capture resilience using a 1-d function:

\[
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\]

More complex problems involve a multi-dimensional system:

\[
\frac{dx}{dt} = f(A_{ij}, x)
\]
Modeling Multi-Dimensionality is hard...

Resilience of a single component:
\[ \frac{dx}{dt} = f(\beta, x) \]

Resilience of a four-component system:
\[ \frac{dx}{dt} = f(A_{ij}, x), \quad x = (x_1, x_2, x_3, x_4)^T \]
\[
\frac{dx_i}{dt} = F(x_i) + \sum_{j=1}^{N} A_{ij} G(x_i, x_j)
\]

\(x_i \rightarrow\) time dependent activities of all \(N\) nodes

\(F(x_i), G(x_i, x_j) \rightarrow\) dynamics of systems interactions

\(A_{ij} \rightarrow\) rate at which \(j\) impacts \(i\)
\[
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\(x_i\) → time dependent activities of all \(N\) nodes
\(F(x_i), G(x_i, x_j)\) → dynamics of systems interactions
\(A_{ij}\) → rate at which \(j\) impacts \(i\)

But...can we reduce this equation even further?
**Reduced Equation**

\[
\frac{dx_i}{dt} = F(x_i) + \beta_{\text{eff}} G(x_i, x_j)
\]

With this, we can get rid of the adjacency matrix!

Also defining a new variable:

\[
x_{\text{eff}} = <s_{\text{out}} > x_{s_{\text{in}}}
\]
\[
\frac{dx_i}{dt} = F(x_i) + \beta_{\text{eff}} G(x_i, x_j)
\]

\[
\beta_{\text{eff}} = \frac{\langle s_{\text{out}} \rangle \langle s_{\text{in}} \rangle}{s}
\]

With this, we can get rid of the adjacency matrix!
\[ \frac{dx_i}{dt} = F(x_i) + \beta_{\text{eff}} G(x_i, x_j) \]

\[ \beta_{\text{eff}} = \frac{\langle s_{\text{out}} \rangle \langle s_{\text{in}} \rangle}{s} \]

With this, we can get rid of the adjacency matrix! Also defining new variable \( x_{\text{eff}} \)

\[ x_{\text{eff}} = \frac{\langle s_{\text{out}} \rangle x}{\langle s \rangle} \]
Mutualistic networks have specific dynamical equations

\[
\frac{dx_i}{dt} = B_i + x_i \left(1 - \frac{x_i}{K_i}\right) \left(\frac{x_i}{C_i} - 1\right) + \sum_{j=1}^{N} A_{ij} \frac{x_i x_j}{D_i + E_i x_i + H_j x_j}
\]

Term on left replaces \( F \), while term on right \( G \) deals with the dynamics
Mutualistic networks have specific dynamical equations

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Now...let’s apply the new formalism to the above equation!
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Now...let’s apply the new formalism to the above equation!

\[
\frac{dx_{\text{eff}}}{dt} = B + x_{\text{eff}} (1 - \frac{x_{\text{eff}}}{K}) (\frac{x_{\text{eff}}}{C} - 1) + \beta_{\text{eff}} \frac{x_{\text{eff}}^2}{D + (E + H)x_{\text{eff}}}
\]
• $\beta_{\text{eff}}$ is what we care about!
• $x_{\text{eff}}$ relates to overall low/high state of system
• Can adapt above equation to any dynamic; $\beta_{\text{eff}}$ will change depending on system
• Want to have $\beta_{\text{eff}}$ be greater than 7!
Modeling in $\beta_{eff}$ space
Quick Summary!

- Taking M bi-partite matrix matrix and separating it into projection networks A and B.
Taking $M$ bi-partite matrix matrix and separating it into projection networks $A$ and $B$.

Then find $\beta_{\text{eff}}$ of each projection

Can then understand resilience!
Okay....we have this cool framework, what does this have to do with your research?
Main research question!

How will modifying the $A_{ij}$ matrix affect the bipartite networks?
Now, have to understand how $A_{ij}$ is formulated to keep going!
Modifying $A_{ij}$

\[
A_{ij} = \sum_{k=1}^{m} \frac{M_{ik} M_{jk}}{\sum_{s=1}^{N} M_{sk}} \sum_{s=1}^{N} M_{sk}
\]

\[
A_{ij} = \sum_{k=1}^{m} \frac{\sigma(M_{ik} M_{jk})(M_{ik} + M_{jk})}{\sum_{s=1}^{N} M_{sk}} \sum_{s=1}^{N} M_{sk}
\]

\[
\sigma = \begin{cases} 
0 & M_{ik} \neq M_{jk} \\
1 & M_{ik} + M_{jk} = 2
\end{cases}
\]

Now, what occurs when adding species?

\[
M^* = \begin{bmatrix} M & \lambda \end{bmatrix}
\]

\[
M^* = \begin{bmatrix} M \\ \lambda \end{bmatrix}
\]
Modifying $A_{ij}$

Remember, $A_{ij}$ is the projection matrix!

$M \to A, B$

$A^* = A_0 + f(\lambda)$

$B^* = B_0 + f(\lambda)$

When adding pollinator

$f(\lambda) = [a\ b\ c\ d]$

When adding flower

$f(\lambda) = \begin{bmatrix} \cdot \\ \cdot \\ \cdot \end{bmatrix}$
Steps for Implementation

1. Classify locations
2. Adding pollinator
3. Adding flower
4. Analyzing effects of additions
Location Analysis

- Analyzed 143 locations
- About 50 unique locations
- Classified location by size
- Compiled tables of all species interactions as well as all each location’s interactions
Locations and Species

Pollinators and Flowers per Location

- Pollinators
- Flowers

#Species

Location ID

0 20 40 60 80 100 120 140

0 100 200 300 400 500
Figure 1: Projection Network Distributions
$\beta_{\text{eff}} = D + H$
\[ \beta_{\text{eff}} = D + H \]
Figure 2: Top, Adding Flower. Bottom, Adding Pollinator
Figure 3: Top, Adding Flower. Bottom, Adding Pollinator
Figure 4: Top, Adding Flower. Bottom, Adding Pollinator
Figure 5: Top, Overall Changes for Small Locations. Bottom, Overall Changes for Medium Locations
Adding species benefits opposite projection network. 
Improvement of $\beta_{\text{eff}}$ varies depending on location size 
Analyze how degree affects $\Delta \ beta_{\text{eff}}$
Figure 6: Top, Bifurcating Flower. Bottom, Bifurcating Pollinator
Figure 7: Top, Bifurcating Flower. Bottom, Bifurcating Pollinator
Why do these bifurcations occur?
Why do these bifurcations occur?
Maybe nearest neighbor degree has something to do with it?
From bifurcation graph, we saw that a bifurcation of some type was occurring.

Graphed three degrees:
- $k_{NN}$ → degree of all nearest neighbors
- $k_{Proj}$ → degree in projection network
- $k_{deg}$ → normal degree

For now, only have graphed these degrees after adding a flower.
Figure 8: Small Degrees
Figure 9: Medium Degrees
Future Work

1. Fixing input errors in the code
2. Analyzing *why* bifurcations occur
3. Graphing how changing $\beta_{\text{eff}}$ affects $H, D$
4. Finding optimal species, optimal k
5. Generating theoretical framework
Thank you!