Measuring the Fitness of Evolving Networks

Section 6.3

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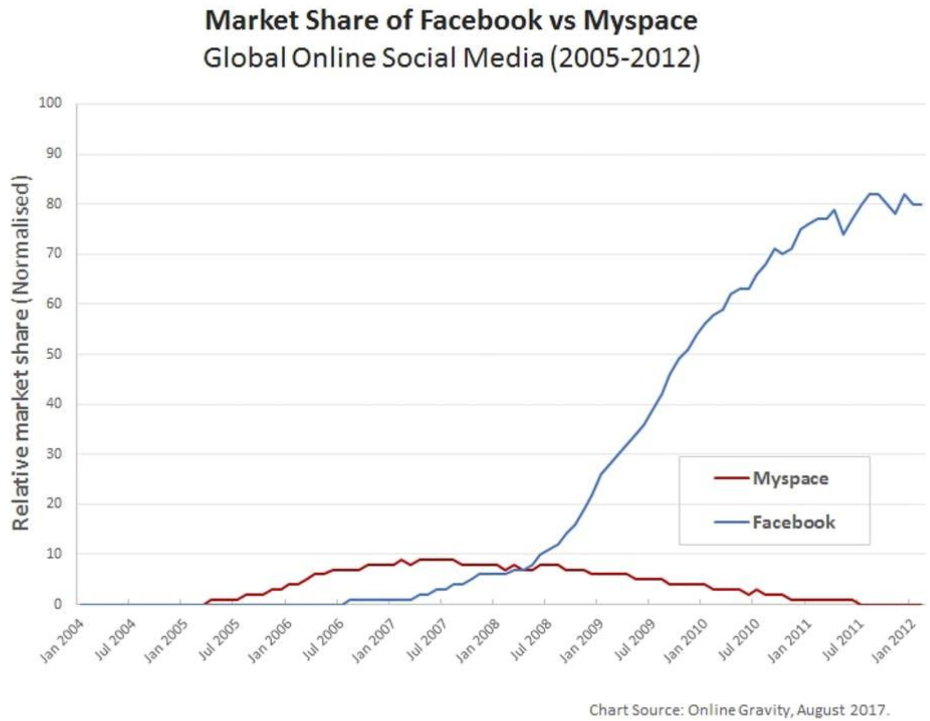
Overview

1. Recap of Evolving Networks
2. Bianconi-Barabási Model
3. Measuring Fitness
4. Examples of Measuring Fitness

Evolving Networks - Examples

- Real networks change

https://medium.com/@nikhilbd/how-did-google-surpass-all-the-other-search-engines-8a9fddc68631



http://theconversation.com/spotify-may-soon-dominate-music-the-way-google-does-search-this-is-why-81621

Evolving Networks - Motivation

- Our network models so far cannot express this evolution
  - In ER networks, largest node is random
  - In BA networks, largest node is oldest node
    - Preferential attachment
    - First mover advantage

$$\Pi(k_i) = \frac{k_i}{\sum_j k_j}$$

Horváth Árpád, https://en.wikipedia.org/wiki/File:Barabasi_Albert_model.gif, 2009

Fitness

- Fitness (η) = intrinsic ability for node to gain links
  - Ex. Ability for website to maintain users
  - Ex. Ability for person to make a friend
  - Ex. Ability for company to maintain customer

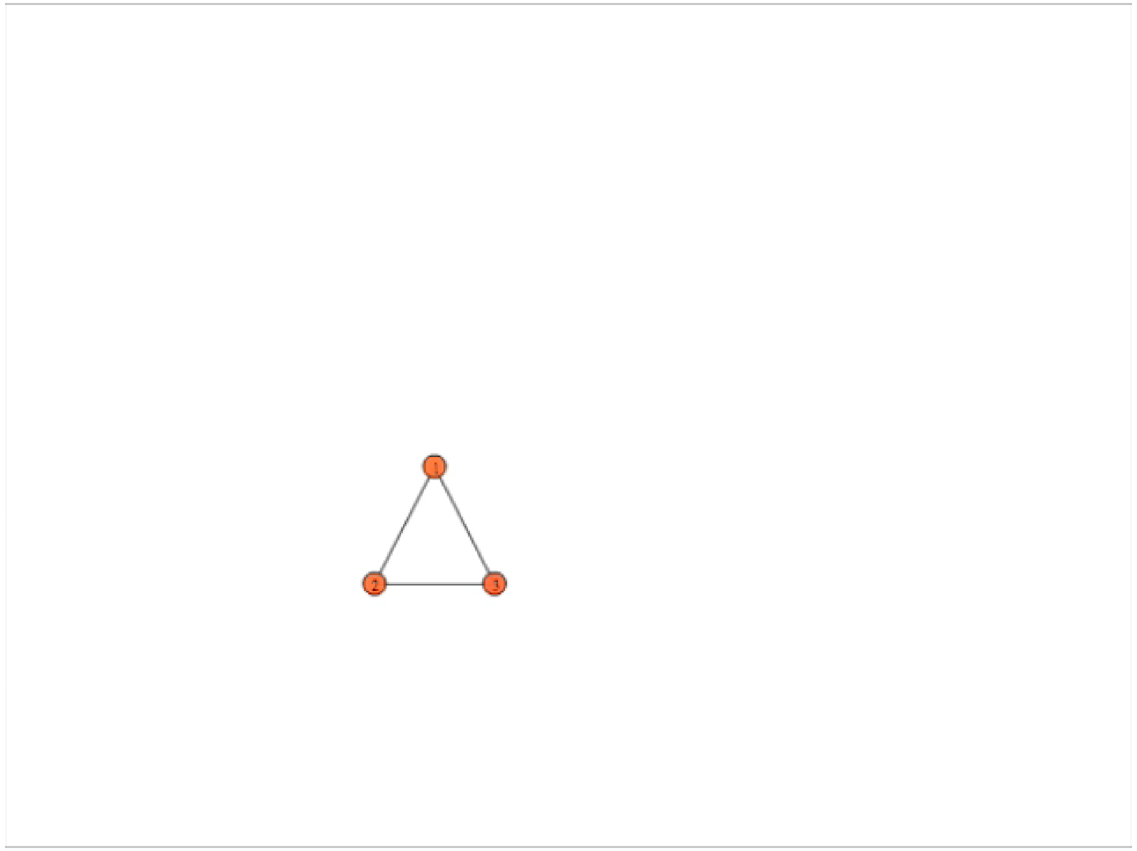
Bianconi-Barabási Model

- AKA "fitness model"
- Includes fitness parameter in growth rate
- Each node *j* gets random fitness η_j chosen from fitness distribution $\rho(\eta)$
- Fitness is fixed
- Probability that a link of a new node connects to node *i*:

$$\Pi_i = \frac{\eta_i k_i}{\sum_j \eta_j k_j}$$

Bianconi-Barabási - Example

Node label = Timestep created
Node color = Fitness (red is larger)
Node size = Num links

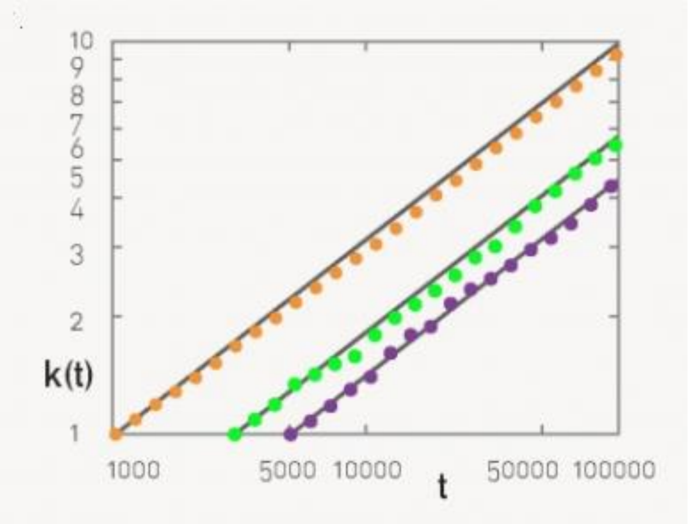


Dashun Wang, Albert-Lázló Barabási, Network Science, Cambridge University Press, 2016

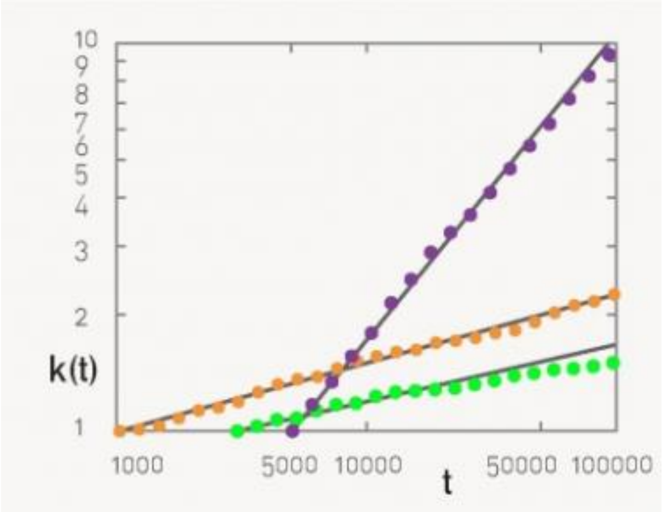
Bianconi-Barabási - Degree

- Time evolution of degree of node *i* joined at time t_i, at time *t:* $k(t, t_i, \eta_i) = m\left(\frac{t}{t_i}\right)^{\beta(\eta_i)}$
- Dynamical exponent based on fitness: $\beta(\eta) = \frac{\eta}{C}$ $C = \int \rho(\eta) \frac{\eta}{1-\beta(\eta)} d\eta$
- Avg over 100 runs, the degree of a node over time:

BA:



BB:



Measuring Fitness - Motivation

If we can accurately measure fitness

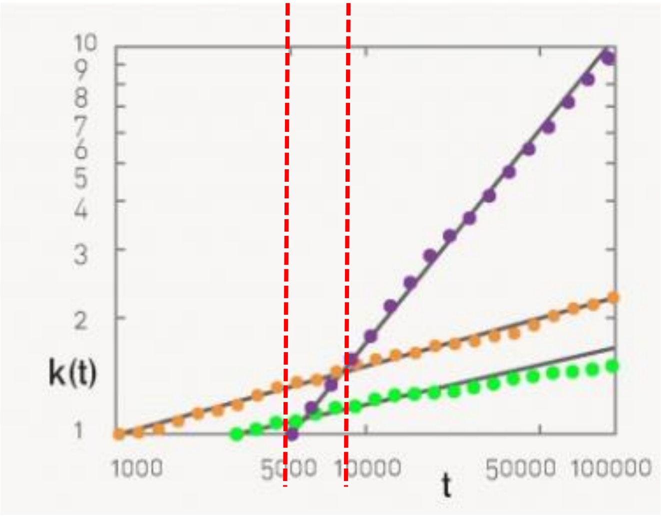
We can identify growth before it happens

Measuring Fitness - Method

- Fitness = "Network's collective perception of a node's importance **relative** to the other nodes"

$$\Pi_i = \frac{\eta_i k_i}{\sum_j \eta_j k_j}$$

- We can measure fitness by comparing a node's degree growth to the growth of other nodes in the network



Measuring Fitness - Method

$$k(t, t_i, \eta_i) = m\left(\frac{t}{t_i}\right)^{\beta(\eta_i)}$$

$\Big\downarrow$ ln

$$\ln k(t, t_i, \eta_i) = \beta(\eta_i) \ln t + B_i$$

$B_i = \ln\left(m/t_i^{\ \beta(\eta_i)}\right)$

1. Degree growth *k(t)* depends linearly on dynamical exponent
2. Dynamical exponent β depends linearly on fitness η

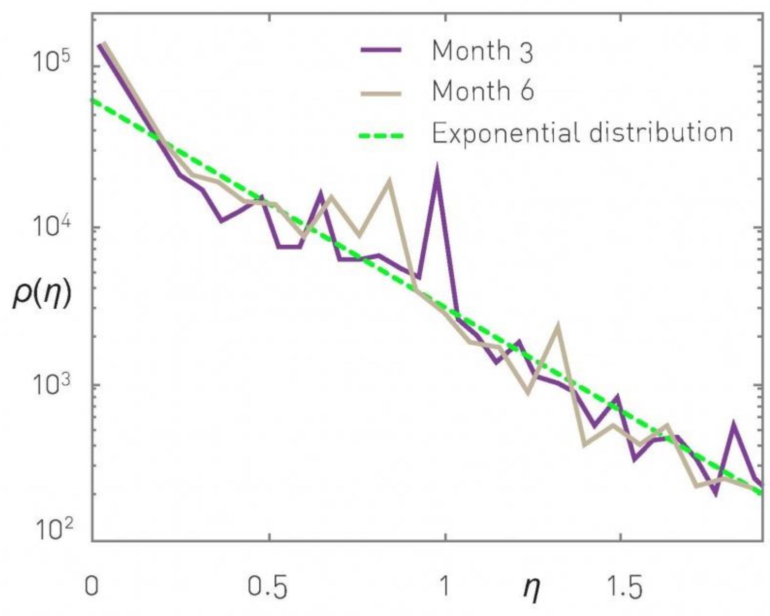
$$\beta(\eta) = \frac{\eta}{C}$$

Therefore, if we can track degree growth, we can track fitness

Distribution of $\beta(\eta_i) \approx \rho(\eta)$

Fitness of the Web

- Crawled 22 million websites over 13 months, looking for degree changes
- Slope of curve is $\beta(\eta_i)$, which equals fitness * constant
- Fitness distribution:



- Takeaways:
  - *ρ(η)* approximated by exponential
  - At different months, *ρ(η)* stayed same
    - Time independent
  - Fitness range is small
  - High fitness nodes are rare

Degree Amplification

- Small differences in fitness amplify degree over long periods of time

$$\frac{k_2 - k_1}{k_1} \sim t^{\frac{n_2 - n_1}{C}}$$

Fitness of Scientific Publications

- Some networks require more complex growth laws
- Bianconi-Barabási model can be adapted to different *ρ(η)*

- Scientific publication network:
  - Nodes are papers and links are citations
  - For a research paper, fitness measures novelty and importance of paper
  - Probability research paper *i* is cited at time *t* after publication

$$\Pi_i \sim \eta_i c_i^t P_i(t)$$

Total number of citations of paper *i*

Decaying probability – the further the time from publication, the less likely to be cited

Fitness of Scientific Publications

Solve for total number of citations of paper *i* at time *t*:

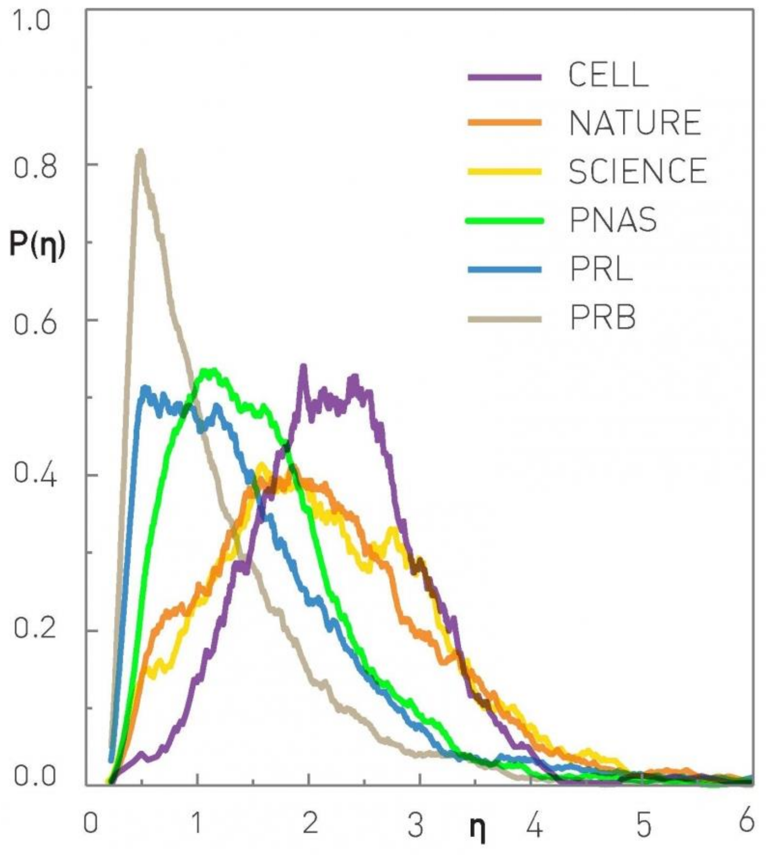
$$C_i^t = m\left(e^{\underbrace{\frac{\beta\eta_i}{A}}_{\text{Relative fitness}}\Phi\left(\frac{\ln t-\mu_i}{\sigma_i}\right)}-1\right)$$

Immediacy = Time to reach citation peak

Longevity = Decay rate

Fitness of Scientific Publications

- Fitness distributions of journals in 1990:



Conclusion

- Real networks evolve
- Evolving networks can be modeled by Bianconi-Barabási
- Fitness determines growth
- We can predict fitness values by measuring growth
- Small differences in fitness amplify degree
- Bianconi-Barabási can be adapted to different growth laws

Questions?