Frontiers of Network Science 4250 (undergraduates)
Assignment 2, due before Lecture 26, noon on Thursday, December 7th 2017

3.11.5-6. Snobbish Network (textbook, p. 102)

3.11.5. Consider a network of N red and N blue nodes. The probability that there is a link between nodes of identical color is p and the probability that there is a link between nodes of different color is q. A network is snobbish if $p > q$, capturing a tendency to connect to nodes of the same color. For $q = 0$ the network has at least two components, containing nodes with the same color.

(a) Calculate the average degree of the "blue" subnetwork made of only blue nodes, and the average degree in the full network.
(b) Determine the minimal p and q required to have, with high probability, just one component.
(c) Show that for large N even very snobbish networks ($p \gg q>0$) display the small-world property.

3.11.6. Consider the following variant of the above model, in which we have $2N$ nodes with equal number of blue and red nodes and fraction f of purple nodes. Blue and red nodes do not connect to each other ($q=0$) while they connect with probability p to nodes of the same color. Purple nodes connect with the same probability p to red and blue nodes.

(a) We call the red and blue communities *interactive* if a typical red node is just two steps away from a blue node and vice versa. Evaluate the fraction of purple nodes required for the communities to be interactive.
(b) Comment on the size of the purple community if the average degree of blue (or red) nodes is $\langle k \rangle \geq 1$.
(c) Discuss the implications of this model for the structure of social (and other) networks.

Clarifications:
Can you prove (convince yourself) that indeed with q=0 the network has at least two components? Under what conditions it can have 0 components? Why it is impossible for the network to have 1 component with q=0?

You can do it this assignment purely analytically, what in case of 3.11.4 you can accomplish by

- For (a) expressing the average degree of the "blue" subnetwork and in the full network as functions of independent variables *p, q* and *N*.
- For (b) expressing the minimal *p* and *q* as functions of independent variables *p, q* and *N*, based on ER networks giant component existence condition depending on network connectivity level.
- For (c) demonstrating that as *N* goes to infinity growing linearly, the distance between nodes increases much slower than *N*.

Alternatively, you can just experiment with the snobbish networks to demonstrate the above conclusion experimentally.
In experimental solution, in all three cases 3.11.5-6 (a), (b), and (c) plot the functions that you computed.