# THE IMPACT OF VARIABLE COMMITMENT IN THE NAMING GAME ON CONSENSUS FORMATION

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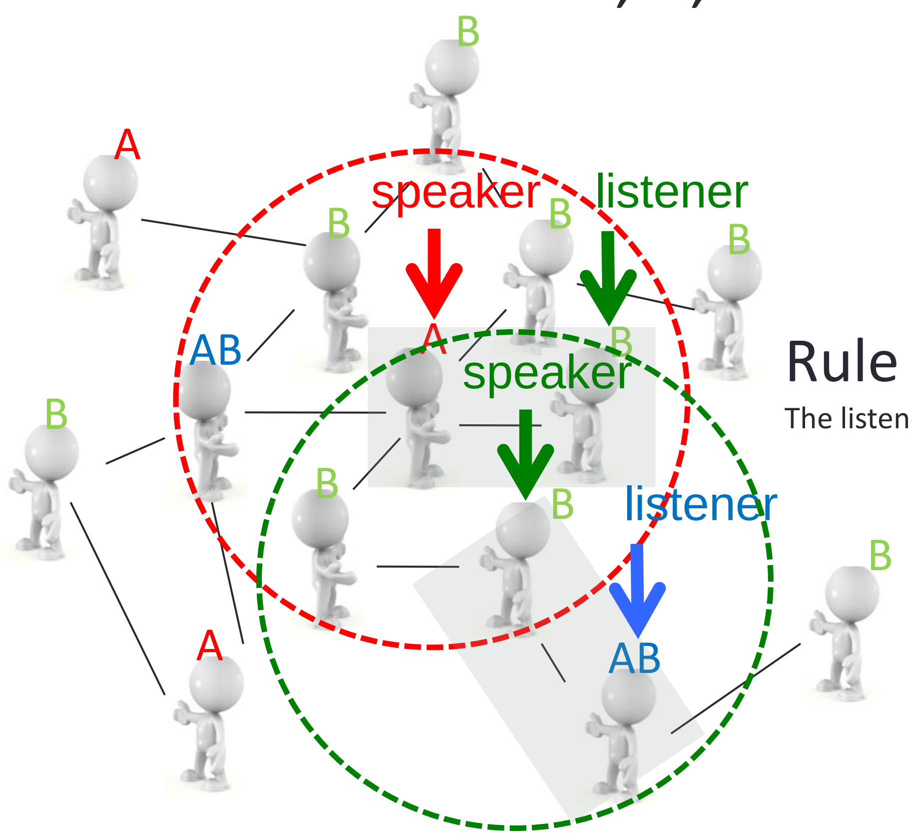
# Outline

- Introduction
- Waning Commitment
- Increasing Commitment
- Distributed Commitment Strengths
- Conclusion

# Binary Agreement Model (BAM)

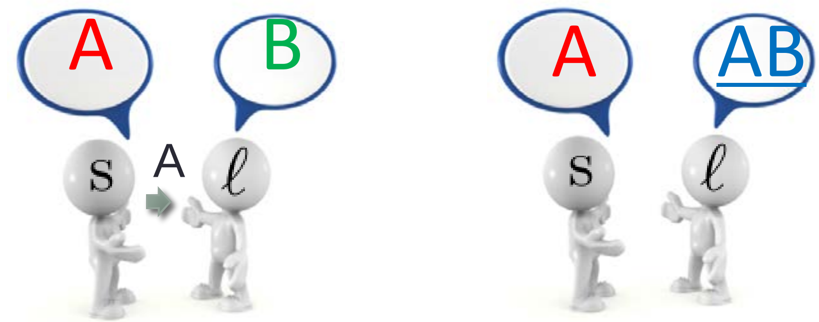
There are two opinions A, B,

three states A, B, AB.



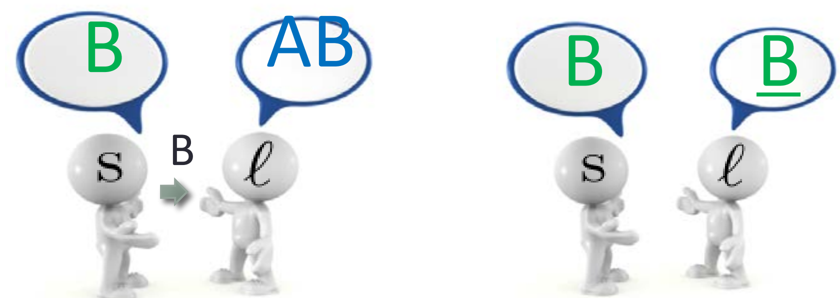

## Rule 1: Accept new opinion (for listener)

The listener does **not** have this opinion, it adds the sent opinion to his list.




## Rule 2: Reinforce common opinion (for both)

The listener already **has** this opinion, **both** retain **only** this opinion.




A<=> AB <=> B

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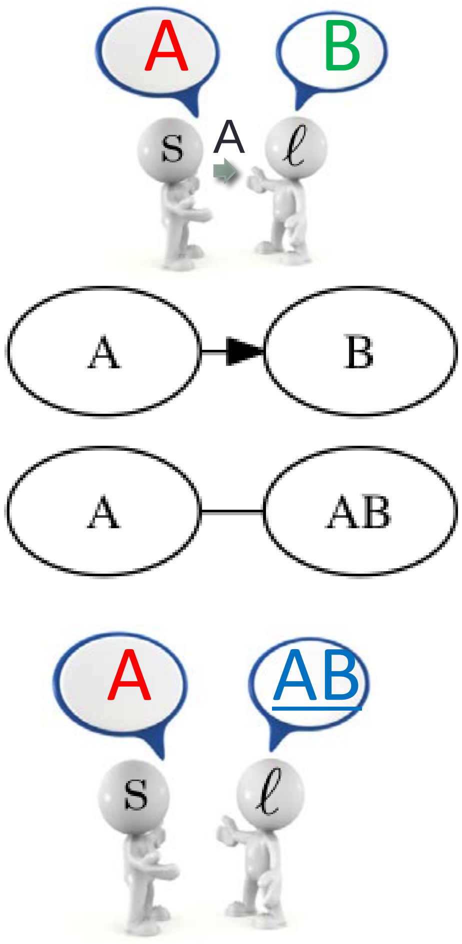
# Mean Field




- In a complete network, the fraction of agents
  - at state A is a,
  - at state B is b,
- The probability of this interaction is x=ab.
- After the interaction, B is changed to AB.
- dB/dt = -x, dAB/dt = x

# Interactions

| Prob. x | Before interaction | | Message | After interaction | | Updated Value |
|---|---|---|---|---|---|---|
| | Speaker | Listener | | Speaker | Listener | |
| aa | A | A | A | A | A | |
| ab | A | B | A | A | AB | dB=-x, dAB=x |
| ac | A | AB | A | A | A | dAB=-x, dA=x |
| ab | B | A | B | B | AB | dA=-x, dAB=x |
| bb | B | B | B | B | B | |
| bc | B | AB | B | B | B | dAB=-x, dB=x |
| 0.5ac | AB | A | A(50% chance) | A | A | dAB=-x, dA=x |
| 0.5bc | AB | B | A(50% chance) | AB | AB | dB=-x, dAB=x |
| 0.5cc | AB | AB | A(50% chance) | A | A | dAB=-2x, dA=2x |
| 0.5ac | AB | A | B(50% chance) | AB | AB | dA=-x, dAB=x |
| 0.5bc | AB | B | B(50% chance) | B | B | dAB=-x, dB=x |
| 0.5cc | AB | AB | B(50% chance) | B | B | dAB=-2x, dB=2x |

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# Commitment Naming Game

Rule 3: Committed agents do not change states.



- Critical value $p=p_c \sim 0.098$
- $p>p_c$ : 1 fixed points
  - the consensus state : the system reaches a consensus
- $p<p_c$ : 3 Fixed points:
  - the consensus state : the system reaches a consensus
  - saddle point: no consensus.
  - active steady state: no consensus but stable.

Jierui Xie, et al. Social consensus through the influence of committed minorities. 2011

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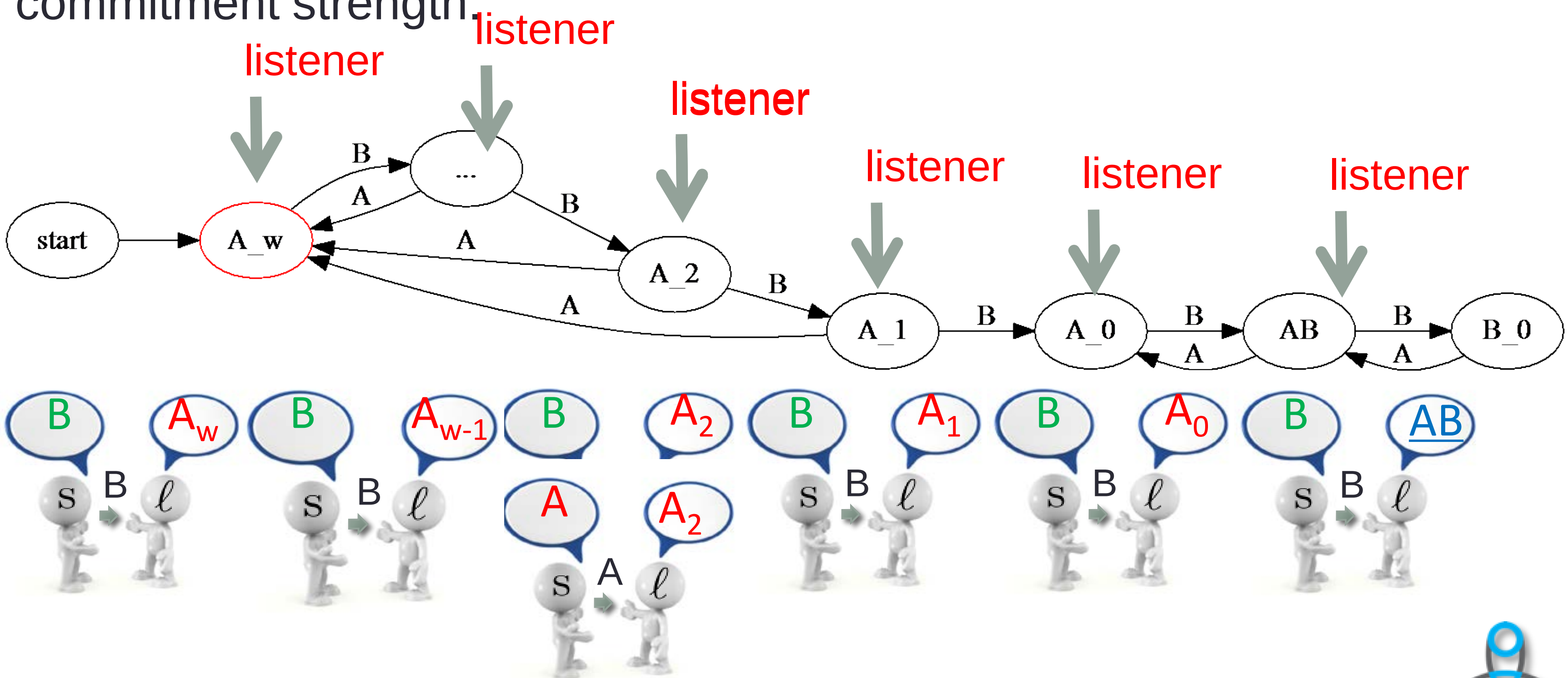
# Waning Commitment

Waning committed agent: a committed agent will become uncommitted when consecutively receiving w opposite opinion, where w is the commitment strength.



# Example of Possible Interactions

| Before Interaction | Message | After Interaction | Mean Field |
|---|---|---|---|
| $A\text{-->}A_0$ | A | $A\text{--}A_0$ | |
| $A\text{-->}A_i$ (i=1~w-1) | A | $A\text{--}A_w$ | $dA_i = -A*A_i, dA_w = A*A_i$ |
| $A\text{-->}A_w$ | A | $A\text{--}A_w$ | |
| $A\text{-->}B_0$ | A | $A\text{--}AB$ | $dB_0 = -A*B_0, dAB = A*B_0$ |
| $A\text{-->}B_1$ | A | $A\text{--}B_0$ | $dB_1 = -A*B_1, dB_0 = A*B_1$ |
| $A\text{-->}B_{j'}$ (j'=2~w') | A | $A\text{--}B_{j'-1}$ | $dB_{j'} = -A*B_{j'}, dB_{j'-1} = A*B_{j'}$ |
| $A\text{-->}AB$ | A | $A\text{--}A_0$ | $dAB = -A*AB, dA_0 = A*AB$ |
| … | … | … | … |

# $a_w r^w=0$

- w : the number of consecutive interactions to become uncommitted
- $a_w$ : the fraction of committed agents at state A.
- r = b+c/2 : the probability of a listener receiving opinion B
  - b : the fraction of all agents at state B
  - c : the fraction of all agents at state AB
- $r^w$ : the probability of an listener consecutively receiving B w times
- $a_w r^w=0$ : no more committed agent becoming uncommitted

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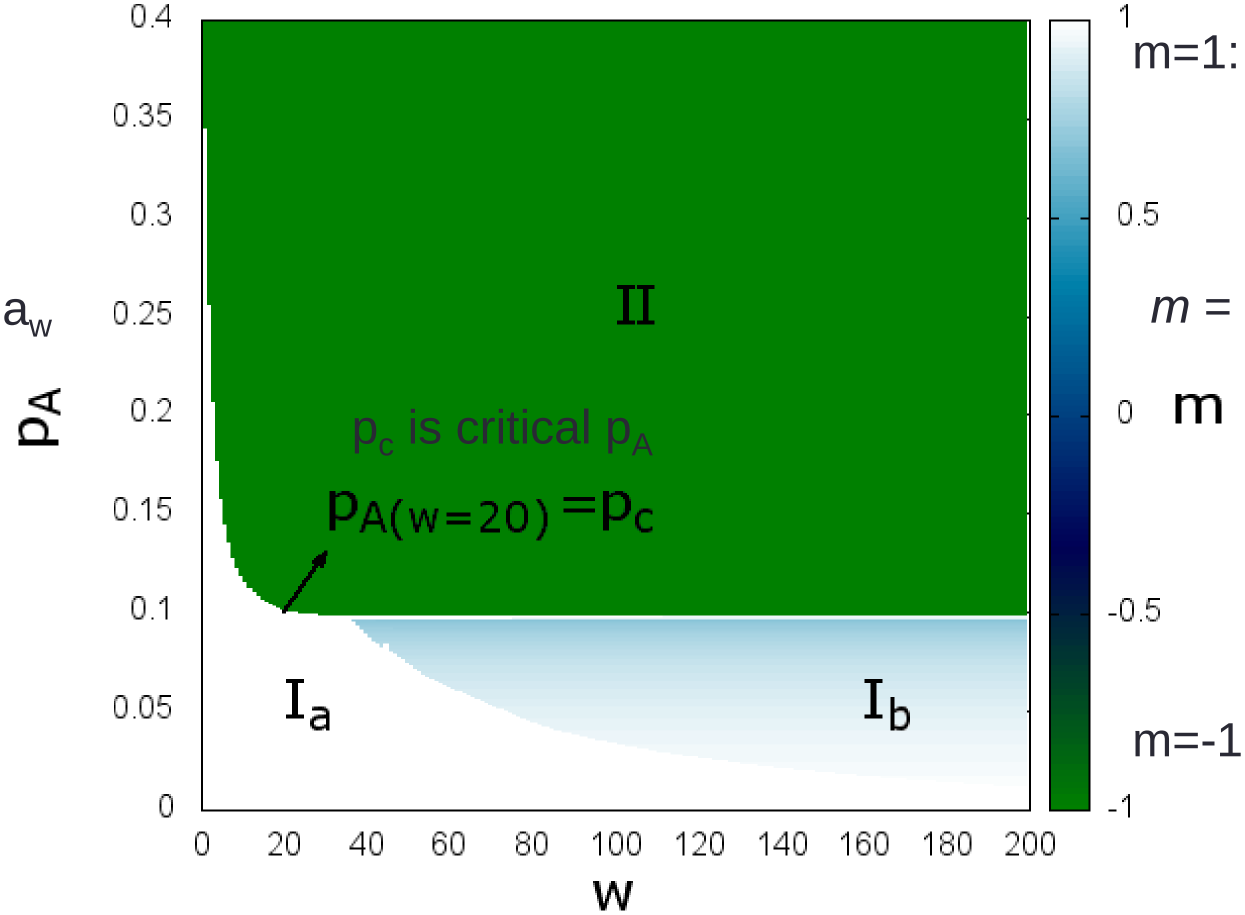
# $a_w r^w = 0$

- When w is finite:
  - r=0 (a=1, b=0, c=0): consensus state A. All agents reach a consensus on opinion A.
  - r=1 (a=0, b=1, c=0): consensus state B. All agents reach a consensus on opinion B.
  - 0<r<1 (a=b, $a_w$=0): unstable state.

# Single Committed Group

Assuming the committed group commits to A, the uncommitted agents are initialized at state B

$p_A$ is the initial $a_w$




m=1: consensus B

$m = (b - a)/(1 - a_w)$

m=-1: consensus A

# Critical Value Function

- Committed agents are lost at a rate of $da_w/dt = -a_w r^w$
- After $t$ interactions the total number of committed agents will be approximately $a_w - a_w r^w t$
- Hence, we can assume that after times $t_w$ , $t_{w+1}$, systems with commitment strengths $w$ and $w + 1$ will be with the similar number of remaining committed agents

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# Critical Value Function

$$
\begin{aligned}
& p_c(w) - p_c(w) r^w t_w \\
& \approx p_c(w+1) - p_c(w+1) r^{w+1} t_{w+1} \approx \dots \\
& \approx p_c(\infty) - p_c(\infty) r^{\infty} t_{\infty} \\
& \approx p_c(\infty)
\end{aligned}
$$

$$
p_c(w) \approx p_c(\infty) + p_c(w) r^w t_w
$$

# Critical Value Function

- After plugin in the value of r, $t_w$ and $p_c(\infty)$

$$p_c(w) \approx p_c(\infty) + p_c(\infty) r^w t_w$$
$$\approx 0.1 + 0.3 * 0.815^w$$

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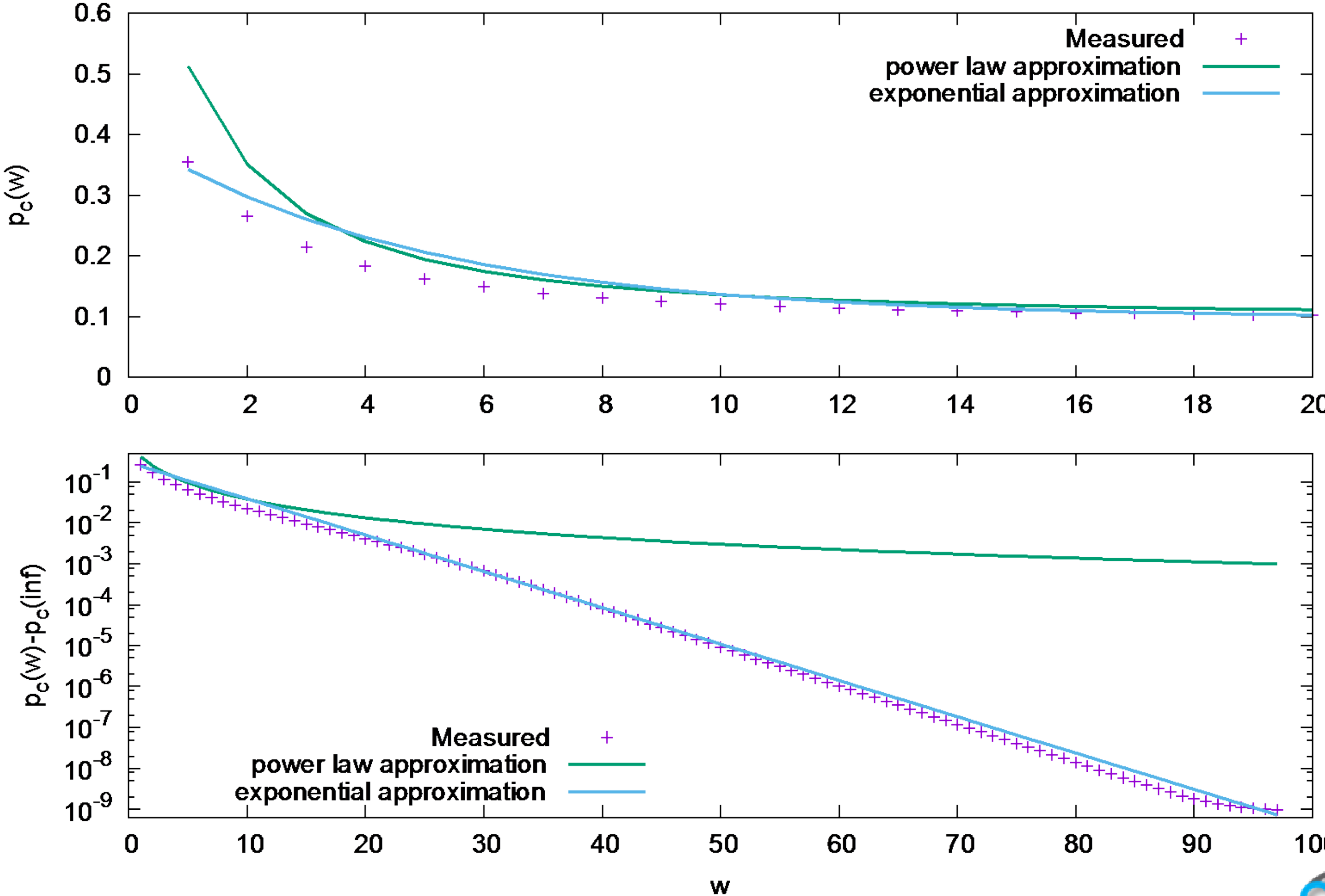
# Critical Value Function

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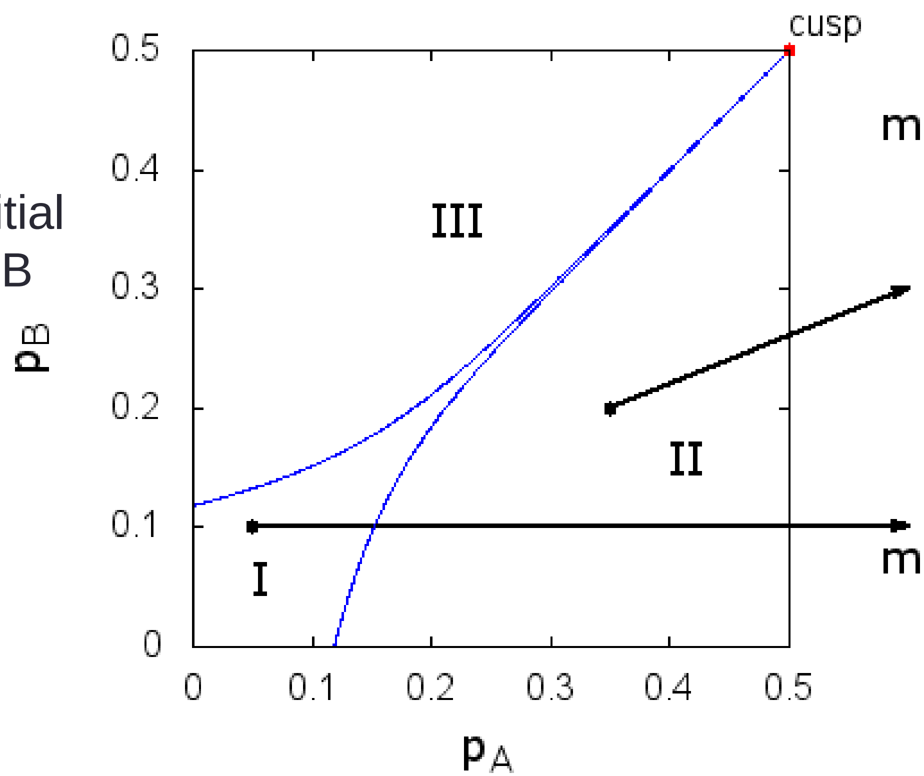
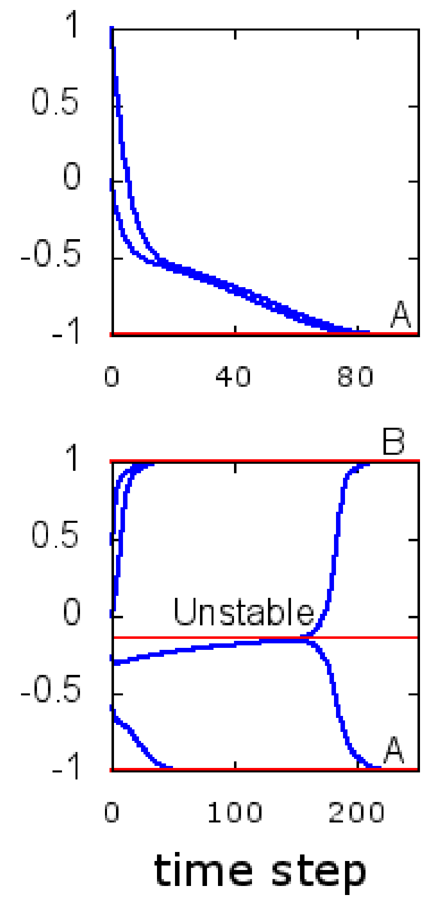
# Two competing committed groups

$p_B$ is the initial committed B

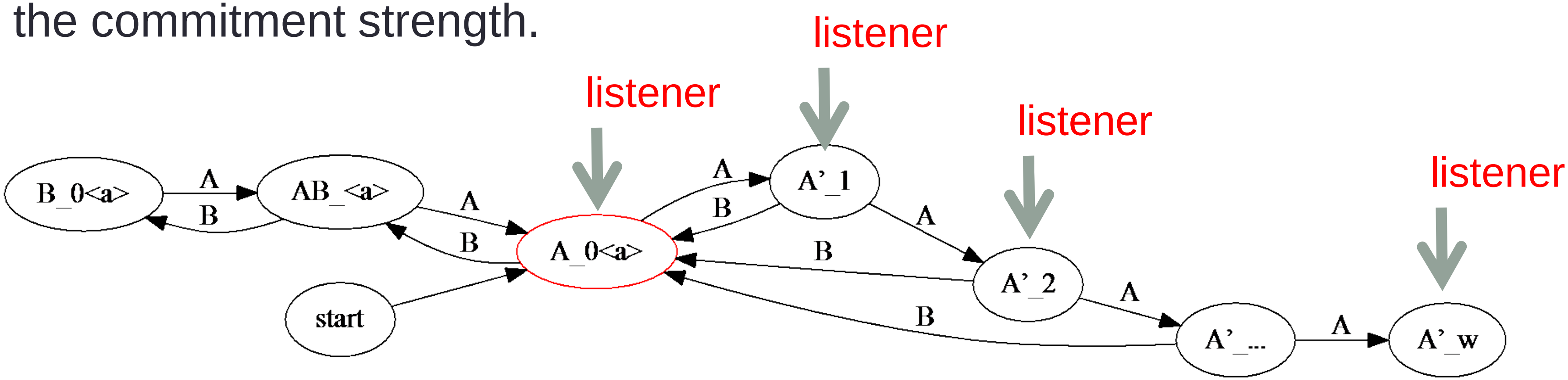


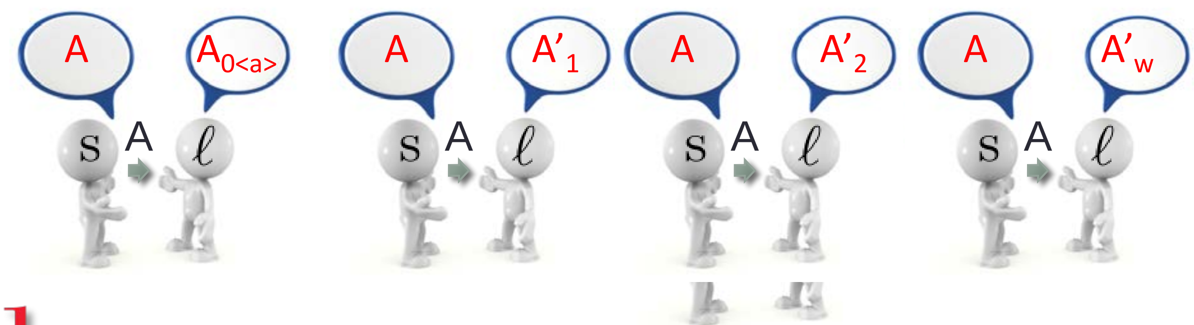




$p_A$ is the initial committed A

# Increasing Commitment

Potential committed agent: an uncommitted agent will become committed when consecutively receiving w same opinion, where w is the commitment strength.



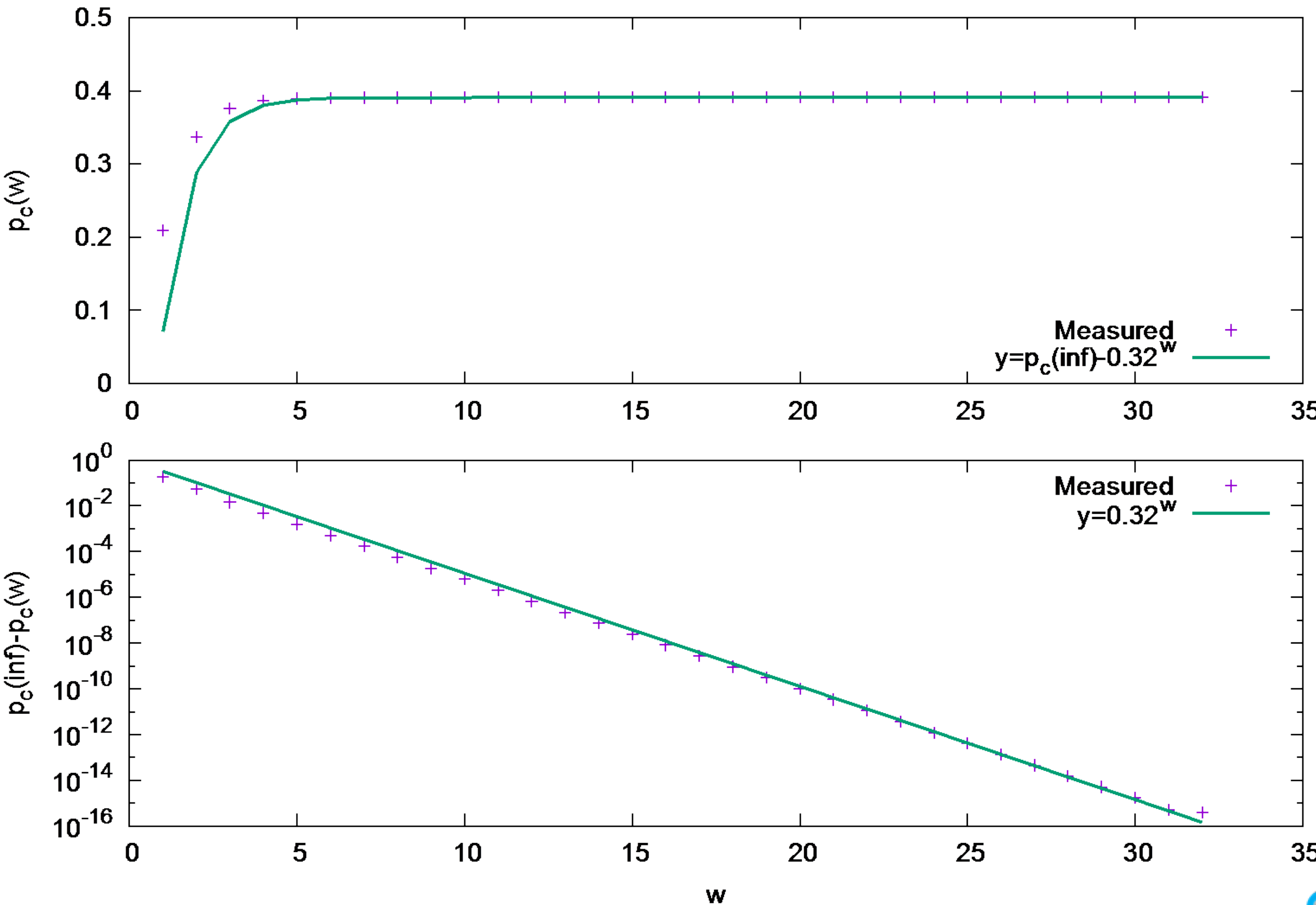
# $a_{0<a>}(1-r)^w=0$

- w : the number of consecutive interactions to become committed
- $a_{0<a>}$ : the fraction of potential committed agents at state A.
- $1-r = a+c/2$ : the probability of a listener receiving opinion A
  - a : the fraction of all agents at state A
  - c : the fraction of all agents at state AB
- $(1-r)^w$ : the probability of an listener consecutively receiving A w times
- $a_{0<a>}(1-r)^w=0$ : no more uncommitted agent becoming committed

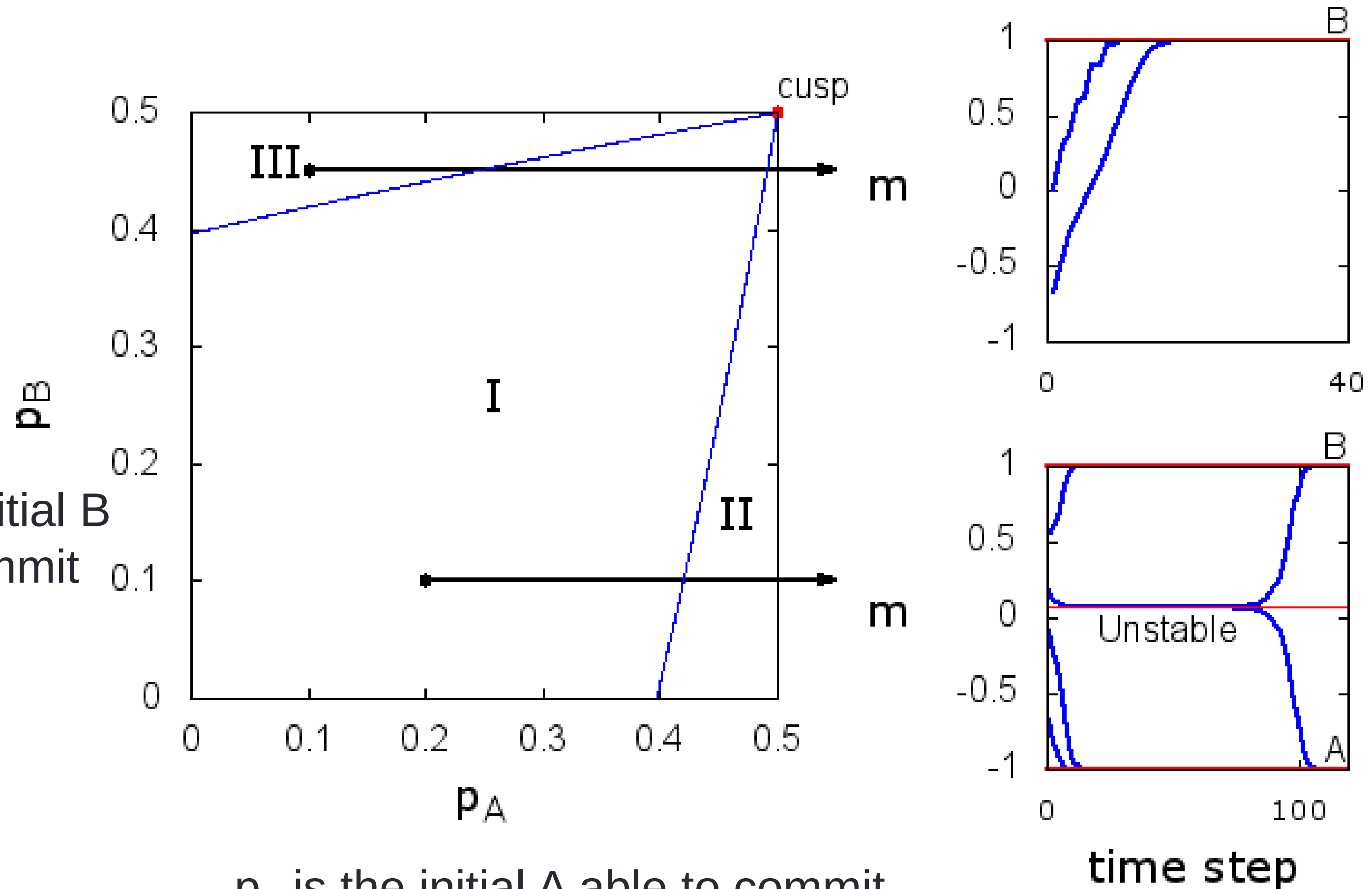
# $a_{0<a>}(1-r)^w=0$

- When w is finite:
  - r=0 (a=1, b=0, c=0): consensus state A. All agents reach a consensus on opinion A.
  - r=1 (a=0, b=1, c=0): consensus state B. All agents reach a consensus on opinion B.
  - 0<r<1: unstable state or active steady state

# Critical Value Function



# Two competing committed groups




$p_B$ is the initial B able to commit

$p_A$ is the initial A able to commit

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# Conclusion

- Critical Value Function: $p_c(w) \approx kq^w + p_c(\infty)$
  - Waning commitment: k>0, q~0.815, $P_c$(inf)~0.1
  - Increasing commitment: k<0, q~0.32, $P_c$(inf)~0.4

- Phase Diagram:
  - Waning commitment: **beak** shape
  - Increasing commitment: the beak boundaries -> **straight lines**

Reference:
The impact of variable commitment in the Naming Game on consensus formation
Xiang Niu, Casey Doyle, Gyorgy Korniss & Boleslaw K. Szymanski
Scientific Reports 7, Article number: 41750 (2017)

# THANK YOU!

Questions?