Immunization

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Modeling Epidemics

- Model spread of a phenomenon through a contact network
  - Nodes represent an individual that could become infected
  - Edges indicate the possibility that one individual could infect the other
  - Usually scale-free
- Possible models: SI, SIS, SIR
  - S: Susceptible
  - I: Infected
  - R: Recovered

Types of Epidemics

- Disease
- Malware
- Information

Contact networks can be social networks, email networks, etc.

Usually, it's negative

Immunization

- Effectively remove a node from a contact network
- Dual effect
  - Immunized node is protected from the epidemic
    - Why not just immunize everyone?
      - Logistically unrealistic
      - Failure
  - Immunized node will not spread the epidemic to others

Notation

- <k>: mean of degree distribution
- $<k^2>$: variance of degree distribution

- β: chance of infection
- μ: chance of recovery

- $\lambda = \beta / \mu$
  - spread rate
  - epidemic threshold (λ_c)
- $\tau = <k> / (\beta<k^2> * \mu<k>)$
  - characteristic time (1/e of population infected)

Random Graph

Reduce λ to λ_c

- $\lambda = \beta / \mu$
- $\tau = \langle k \rangle / (\beta \langle k^2 \rangle * \mu \langle k \rangle)$
  - $\langle k^2 \rangle = \langle k \rangle * (\langle k \rangle + 1)$
- $\tau = (\beta (\langle k \rangle + 1) - \mu)^{-1}$
- $\lambda = (\langle k \rangle + 1)^{-1}$
- $\lambda_c = (\langle k \rangle + 1)^{-1}$

Immunization

- g = fraction of individuals immunized
- $\lambda = \lambda (1 - g)$

Random Graph

Immunization

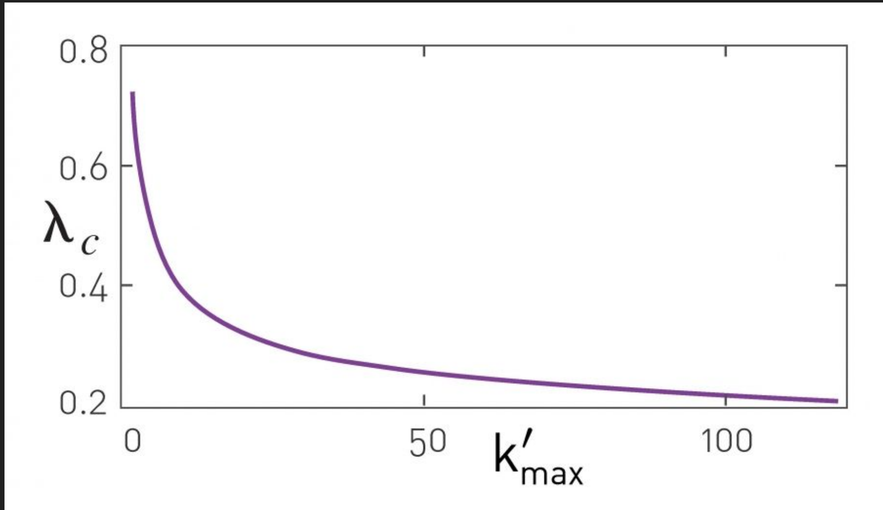
- g = fraction of individuals immunized
- ƛ = ƛ (1 - g)
- $((1 - g_c) \beta) / \mu = (<k> + 1)^{-1}$
- $g_c = 1 - (\beta/\mu)(<k> + 1)^{-1}$

Scale-free Graphs

- Now, $\lambda_c = \langle k \rangle / \langle k^2 \rangle$
  - Now a dependence on $\langle k^2 \rangle$
  - Limit of $\langle k^2 \rangle$ as $n \to \infty$ is infinite
  - Epidemic threshold will eventually vanish
  - Weak pathogens will still end up spreading in a sufficiently large network
- This result surfaces in any heterogeneous network ($\langle k^2 \rangle$ more than $\langle k \rangle(\langle k \rangle+1)$)

Random Immunization Is Ineffective

- A very large *g* is required to randomly immunize a scale-free network
  - Measles: 95%
  - Email virus: 99.7%
- Why?
  - Hubs are the most potent vectors as well as the first to become infected
  - Schools, stores, airports
- Need to immunize hubs
  - Requires explicit knowledge of the network
    - Difficult, maybe even not well-defined



Immunization Strategy Using Global Knowledge

Choose a threshold k'_{max}, and immunize all nodes with higher degree

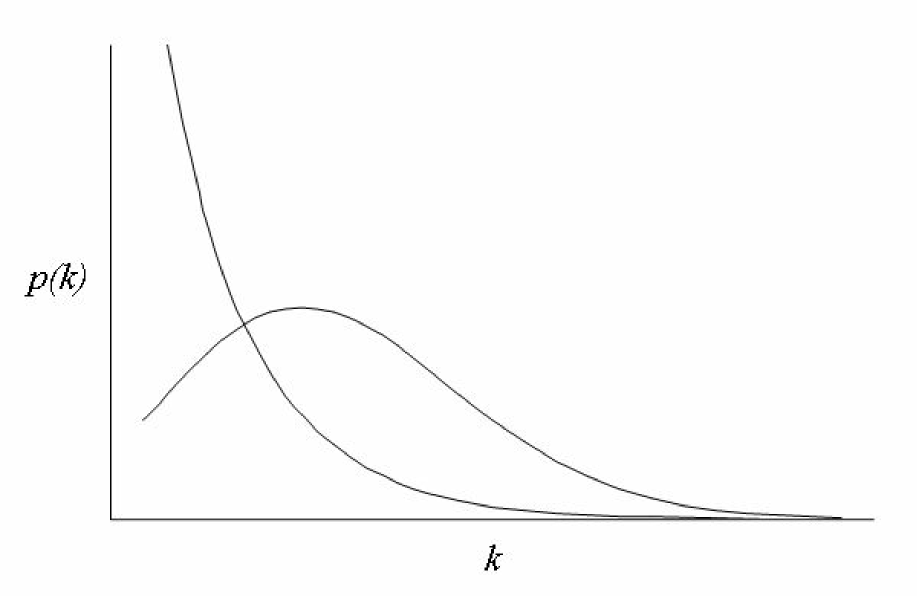
New approximation for λ_c:

$$((\gamma - 2) * k_{min}^{2-\gamma}) / ((3 - \gamma) * (k'_{max})^{\gamma-3})$$

As long as $\gamma < 3$, decreasing k'_{max} will increase the epidemic threshold

Acquaintance Strategy

- Choose a fraction of nodes
  - Rather than immunizing those nodes, immunize their neighbors randomly
  - Probability of selecting node with degree *k*:
    - *k(P(k)) / (N<k>)*
    - Randomly selected links have higher degree than randomly selected nodes



The fraction of nodes that must be immunized can be obtained analytically

Find p_c using:

$$\sum_k \frac{P(k)k(k-1)}{\langle k \rangle} \nu_{p_c}^{k-2} e^{-2p_c/k} = 1 \, .$$

Then f_c is:

$$f_c = 1 - \sum_k P(k) p(s_k|k) = 1 - \sum_k P(k) \nu_{p_c}^k \, ,$$

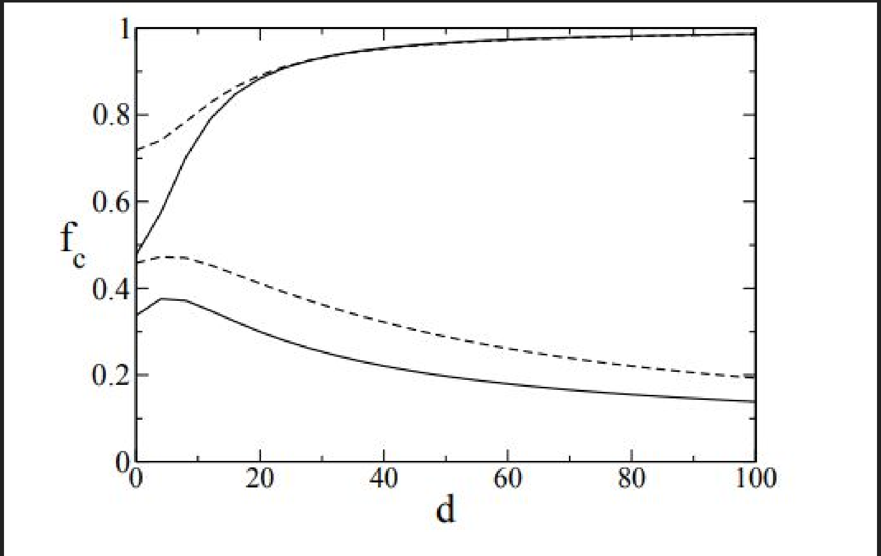
Performance on bimodal distribution

$F_c = g_c$

d = distance between modes

Solid lines has variance 2

Dashed lines has variance 8



Robustness of Scale-free Graphs

- The basic insight pertains to breaking up a scale-free graph
- Suggested use: terrorist networks
  - Focus on obtaining information from identified terrorists rather than eliminating them directly
  - Names acquired tend to be more well-connected, more important individuals