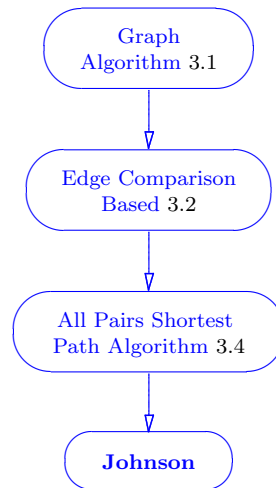


3.4.1 Johnson's algorithm

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Refinement of: All Pairs Shortest Path Algorithm (§3.4), therefore of Edge Comparison Based (§3.2), Graph Algorithm (§3.1).

Prototype:

```
//named parameter version
template <typename Graph, typename DistanceMatrix, typename P,
          typename T, typename R>
bool johnson_all_pairs_shortest_paths(Graph &g, DistanceMatrix
&D, const bglNamedParams<P,T,R> &params=all defaults)

// non-named parameter version
template <typename Graph, typename DistanceMatrix, typename
          VertexIndex, typename WeightMap, typename DT>
bool johnson_all_pairs_shortest_paths(VertexAndEdgeListGraph
&g1, DistanceMatrix &D, VertexIndex id1,
Weight w1, DT zero)
```

Input: A directed or undirected weighted graph

The input graph is randomly generated with parameters V and E . It has only positive-weight edges to prevent negative cycles. Negative-weight edges can make negative cycles and thus skip Dijkstra's algorithm. We want to separate graphs with negative cycles and those without negative cycles.

Output: The length of the shortest path between each pair of vertices u and v in the graph is stored in $D[u][v]$ where D is the distance matrix. It also returns the boolean value that indicates whether there exists a negative weight cycle.

Effects: Standard effects of an All Pairs Shortest Path Algorithm (§3.4). This algorithm finds the shortest distance between every pair of vertices in the graph. The algorithm returns false if there is a negative weight cycle in the graph and true otherwise. The distance between each pair of vertices is stored in the distance matrix D .

Asymptotic complexity: $O(|V||E| \lg |V|)$, where V = the number of vertices and E the number of edges.

Complexity in time: ¹ - Initialization: $0.002 \cdot (|E| + |V|)$

- Bellman-Ford: $2.444 \cdot 10^{-3}(|E| + |V|)$

- Reweighting: $8.79 \cdot 10^{-4}(|E| + |V|)$

- Dijkstra's: $9.4 \cdot 10^{-5}|V||E| \lg |V| + 0.0125(|E| + |V|) + 0.0082|V|^2$

- Setting vertex weights: $6.6 \cdot 10^{-4}|V|^2$

Total complexity: $9.4 \cdot 10^{-5}|V|(|E| + |V|) \lg |V| + 1.838 \cdot 10^{-2}(|E| + |V|) + 8.76 \cdot 10^{-3}|V|^2$

¹The test is done with P4 2GHz CPU, 256MB RAM under the operating system Windows XP.

3.4.2 Johnson's trace plot

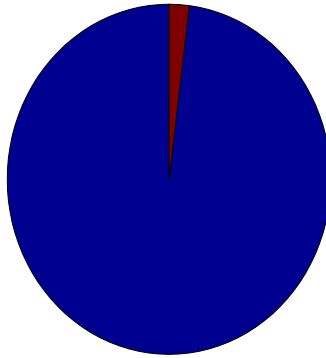
We use the following notation in the tables, and the time unit is 1 millisecond.

C: Construction
B: Bellman-Ford
R: Reweighting
S: Setting vertex weights
D: Dijkstra
T: Total time
E: Estimated time

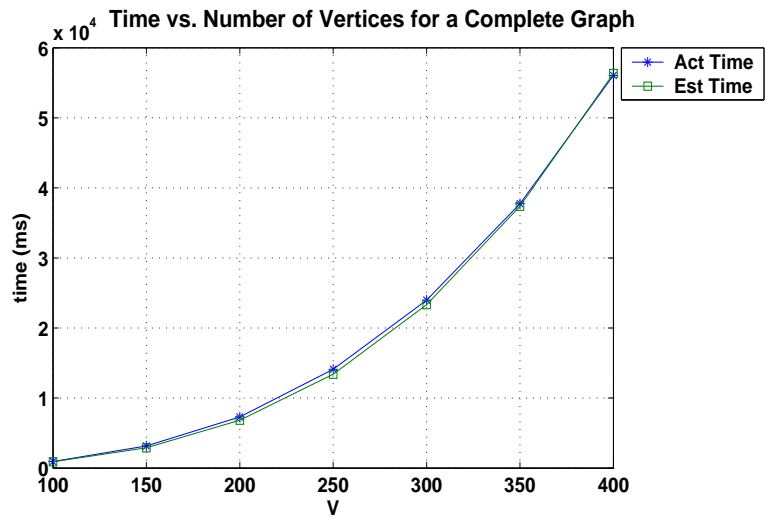
Complete graph:

V	E	C	B	R	S	D	T	E
100	9900	31	16	15	0	907	938	896
150	22350	47	62	16	0	3078	3156	2904
200	39800	94	93	32	30	7142	7297	6834
250	62250	124	156	63	31	13844	14094	13396
300	89700	188	219	78	112	23559	23968	23327
350	122150	265	282	109	108	37248	37747	37385
400	159600	343	390	141	108	55408	56047	56344

Complete Graph
Other(2%)



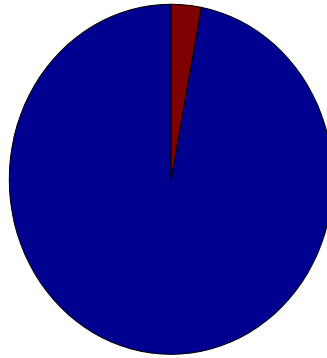
Dijkstra (98%)



Sparse graph:

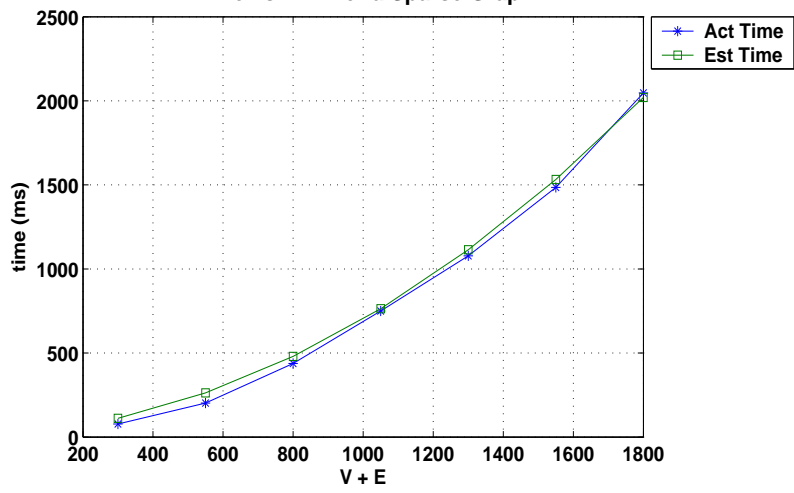
V	E	C	B	R	S	D	T	E
100	200	0	0	0	0	78	78	112
150	400	0	0	0	0	202	202	263
200	600	0	0	0	16	422	438	480
250	800	0	0	15	16	719	750	763
300	1000	0	0	0	31	1047	1078	1114
350	1200	0	0	0	95	1390	1485	1533
400	1400	15	0	0	125	1922	2047	2020

Sparse Graph
Other(3%)



Dijkstra (97%)

Time vs. V+E for a Sparse Graph



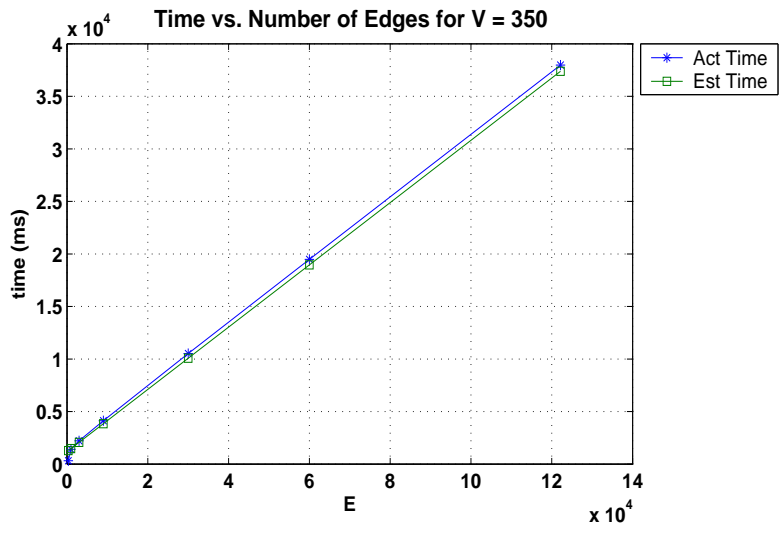
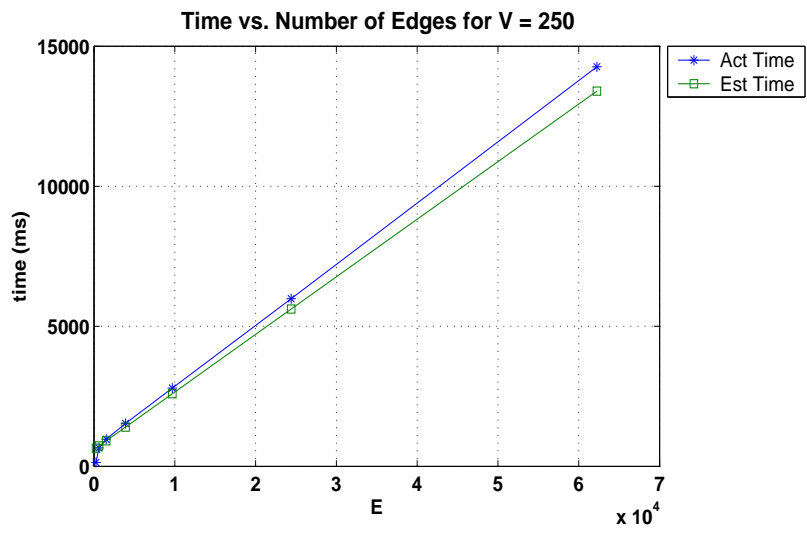
Fixed V:

Table 1 (V=350)

V	E	C	B	R	S	D	T	E
350	350	0	0	0	47	265	312	1281
350	1000	0	0	0	61	1345	1406	1473
350	3000	0	16	0	46	2157	2219	2066
350	9000	31	16	16	95	3967	4125	3845
350	30000	63	63	31	126	10234	10517	10070
350	60000	140	157	47	46	19094	19484	18962
350	122150	266	297	110	96	37201	37970	37385

Table 2 (V = 250)

V	E	C	B	R	S	D	T	E
250	250	0	0	0	16	125	141	650
250	625	0	0	0	15	642	657	727
250	1520	15	0	0	15	939	969	911
250	3900	0	0	0	15	1516	1531	1401
250	9700	16	31	16	48	2686	2797	2593
250	24400	62	63	15	62	5782	5984	5615
250	62250	141	172	47	62	13844	14266	13396



Fixed E:

Table 1 (E=10,000)

V	E	C	B	R	S	D	T	E
200	10000	15	15	16	31	2094	2171	2004
250	10000	15	32	15	16	2796	2874	2655
300	10000	16	31	0	16	3546	3609	3368
350	10000	16	31	16	124	4219	4406	4141
400	10000	16	31	16	123	5111	5297	4973

Table 2 (E=20,000)

V	E	C	B	R	S	D	T	E
200	20000	47	47	15	16	3828	3953	3625
250	20000	47	46	16	16	4922	5047	4710
300	20000	47	46	32	31	6062	6218	5872
350	20000	47	46	32	108	7235	7468	7105
400	20000	47	46	32	94	6578	8797	8407

