3.4.2 Floyd-Warshall

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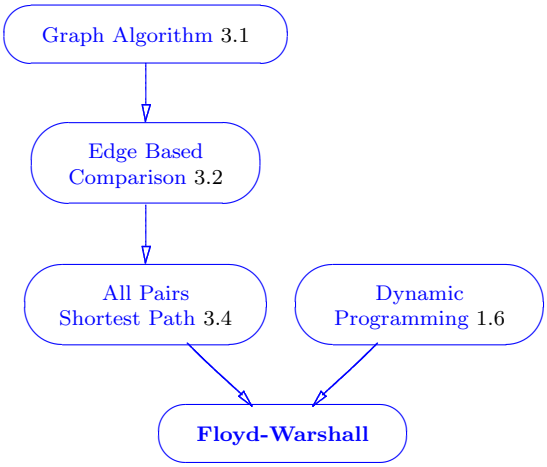


Prototype: bool Floyd_Warshall_all_pairs_shortest_paths(
const std::vector<std::vector<int> > & graph,
std::vector<std::vector<double> > & distance_matrix,
const std::vector<std::vector<double> >
& weight_map)

Refinement of: Graph Algorithm (§3.1),
therefore of Edge Based Comparison (§3.2)
and of All Pairs Shortest Path (§3.4).
Also a refinement of the Dynamic Programming (§1.6) design strategy.

Effects: Standard effects of an All Pairs Shortest Path (§3.4) algorithm. In brief:
The algorithm detects the presence of a negative weight cycle in a directed

or undirected graph. If such a cycle is found, the algorithm terminates returning false immediately. If no such cycle is found, the algorithm returns true and the shortest path between each pair of vertices, (u, v), is stored in $distance\_matrix[u][v]$.

Asymptotic complexity: For a graph $G = (V, E)$.

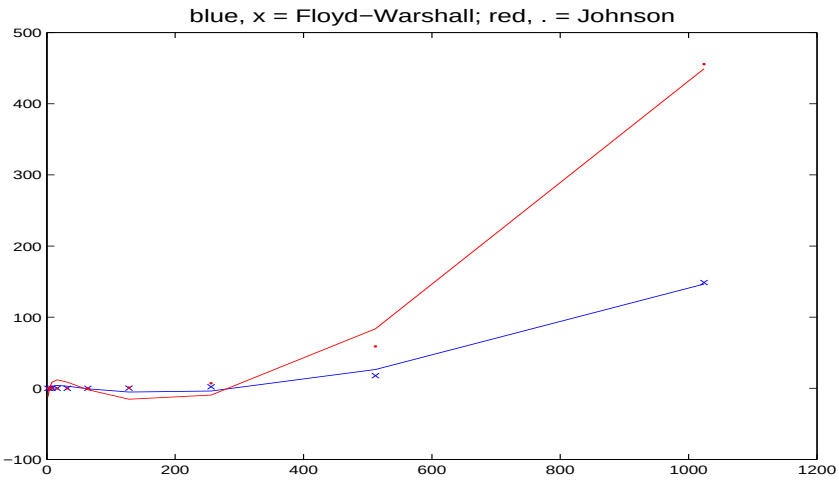
- Average case: $\Theta(V^3)$
- Worst case (fully connected graph): $\Theta(V^3)$

3.4.3 Floyd-Warshall Fully Connected Graph Timings

Comparison of timings on a fully connected graph against those for Johnson's Algorithm (§3.4.1).

Number Vertices	Floyd Results	Johnson Results
2	0.00	0.00
4	0.00	0.00
8	0.00	0.00
16	0.00	0.00
32	0.01	0.02
64	0.04	0.14
128	0.28	1.02
256	2.20	7.18
512	17.79	58.97
1024	148.74	455.57

3.4.4 Least Squares Fitting of Floyd-Warshall Versus Johnson Fully Connected Graph Timings

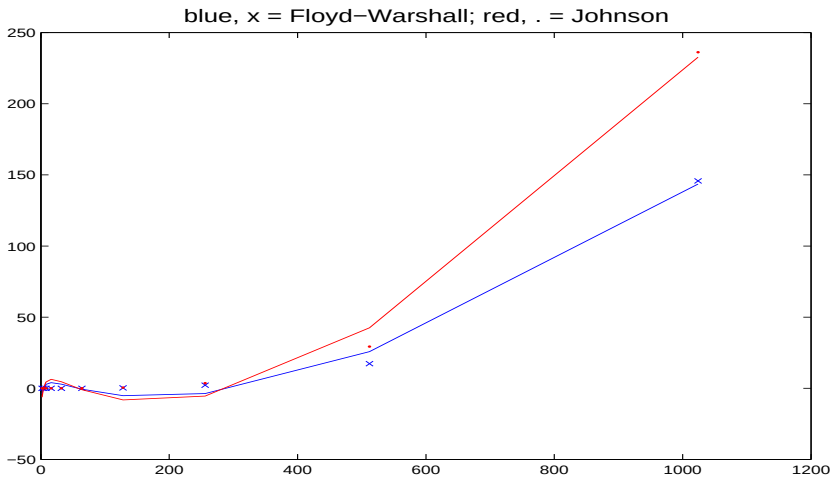


3.4.5 Floyd-Warshall Dense Graph Timings

Comparison of timings on a dense, but not fully connected graph against those for Johnson's Algorithm (§3.4.1).

Number Vertices	Floyd Results	Johnson Results
2	0.00	0.00
4	0.00	0.00
8	0.00	0.00
16	0.00	0.00
32	0.01	0.01
64	0.04	0.08
128	0.28	0.53
256	2.18	3.63
512	17.34	29.30
1024	145.76	236.21

3.4.6 Least Squares Fitting of Floyd-Warshall Versus Johnson Dense Graph Timings

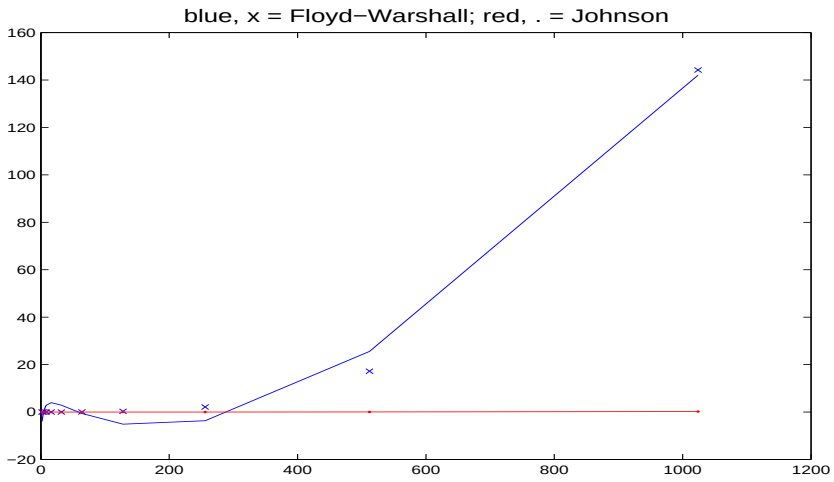


3.4.7 Floyd-Warshall Sparse Graph Timings

Comparison of timings on a sparse graph against those for Johnson's Algorithm (§3.4.1).

Number Vertices	Floyd Results	Johnson Results
2	0.00	0.00
4	0.00	0.00
8	0.00	0.00
16	0.00	0.00
32	0.00	0.00
64	0.03	0.00
128	0.30	0.00
256	2.13	0.01
512	17.20	0.05
1024	144.23	0.19

3.4.8 Least Squares Fitting of Floyd-Warshall Versus Johnson Sparse Graph Timings



3.4.9 Notes

The following conventions are used in the above analyses:

- A dense graph contains $((V^2)/2)$ edges.
- A sparse graph contains $(V/2)$ edges.
- All Graphs are represented by adjacency matrices.