2.7 FFT

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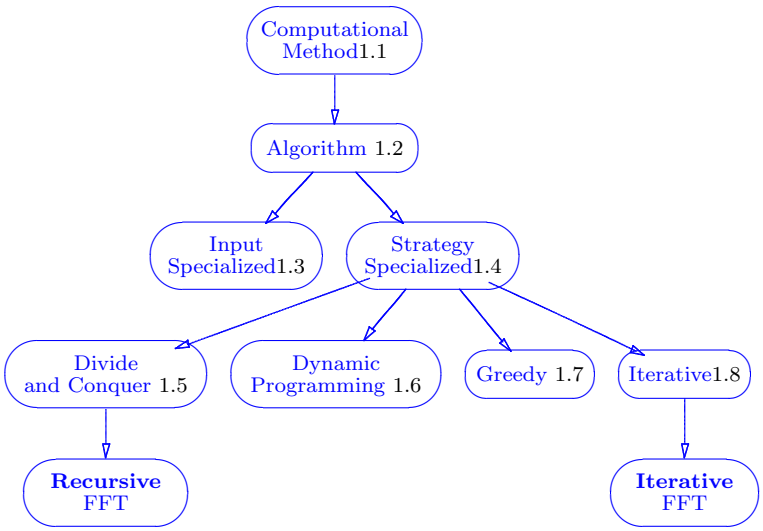


Refinement of: To some extent, the 7 FFT algorithms below are refined step by step.

Prototype:

- Case 1: Radix2 recursive cooley-Tukey FFT with decimation in time

```
vector< complex<double> >
r2_DIT_R_FFT(vector< complex<double> > x,
bool forward )
```

- Case 2: Radix2 interative cooley-Tukey FFT with decimation in time

```
template<class RandomAccessIterator>
void r2_DIT_I_FFT
(const RandomAccessIterator x,
const int n,const bool forward )
```

- Case 3: Enhanced radix2 interative cooley-Tukey FFT with decimation in time

```
template<class RandomAccessIterator>
void r2_DIT_I_FFTx
(const RandomAccessIterator x,
const int n,const bool forward )
```

- Case 4: RADIX-2 FFT ALGORITHM with Decimation in Frequency

```
template<class RandomAccessIterator>
void r2_DIF_FFT
( RandomAccessIterator data,
const int n,const bool forward )
```

- Case 5: Enhanced RADIX-2 FFT ALGORITHM with Decimation in Frequency

```
template<class RandomAccessIterator>
void r2_DIF_FFTx
( RandomAccessIterator data,
const int n,const bool forward )
```

- Case 6: RADIX-4 FFT ALGORITHM with Decimation in Frequency

```
template<class RandomAccessIterator>
void r4_DIF_FFT
( RandomAccessIterator data,
const int n,const bool forward )
```

- Case 7: Enhanced RADIX-4 FFT ALGORITHM with Decimation in Frequency

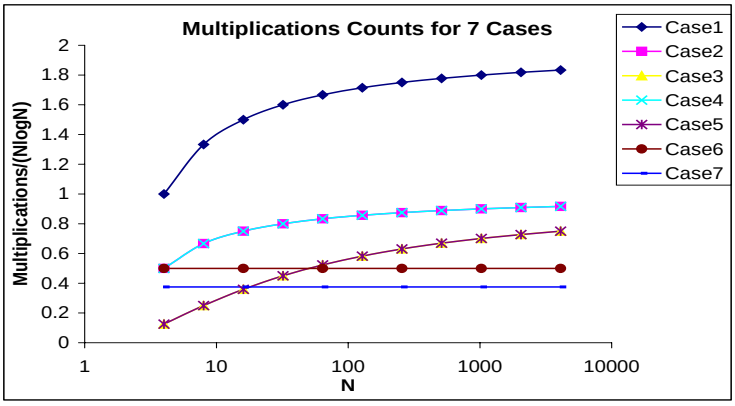
```
template<class RandomAccessIterator>
void r4_DIF_FFTx
( RandomAccessIterator data,
const int n, const bool forward )
```

Effects: Computing a point-value representation for a polynomial given in coefficient form. Asymptotic complexity: Let $N=$ degree-bound

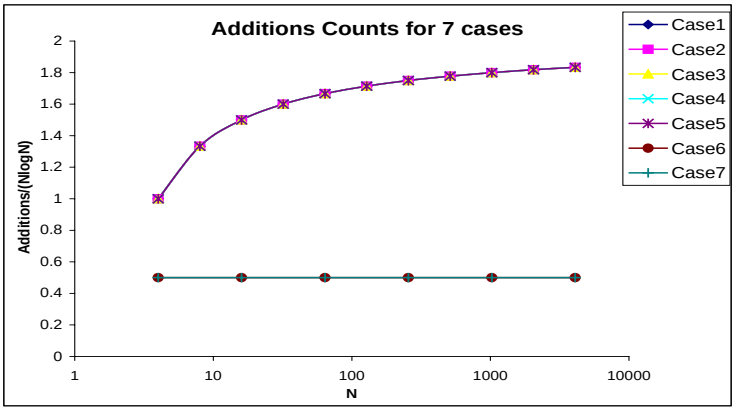
- Average case: $O(N \log N)$
- Worst case: $O(N \log N)$

Complexity in terms of operation counts:

	Multiplications						
N	Case1	Case2	Case3	Case4	Case5	Case6	Case7
4	8	4	1	4	1	4	3
8	32	16	6	16	6		
16	96	48	23	48	23	32	24
32	256	128	72	128	72		
64	640	320	201	320	201	192	144
128	1536	768	522	768	522		
256	3584	1792	1291	1792	1291	1024	768
512	8192	4096	3084	4096	3084		
1024	18432	9216	7181	9216	7181	5120	3840
2048	40960	20480	16398	20480	16380		
4096	90112	45056	36879	45056	36879	24576	18432



	Additions						
N	Case1	Case2	Case3	Case4	Case5	Case6	Case7
4	8	8	8	8	8	4	4
8	32	32	32	32	32		
16	96	96	96	96	96	32	32
32	256	256	256	256	256		
64	640	640	640	640	640	192	192
128	1536	1536	1536	1536	1536		
256	3584	3584	3584	3584	3584	1024	1024
512	8192	8192	8192	8192	8192		
1024	18432	18432	18432	18432	18432	5120	5120
2048	40960	40960	40960	40960	40960		
4096	90112	90112	90112	90112	90112	24576	24576



	Multiplications + Additions						
N	Case1	Case2	Case3	Case4	Case5	Case6	Case7
4	16	12	9	12	9	8	7
8	64	48	38	48	38		
16	192	144	119	144	119	64	56
32	512	384	328	384	328		
64	1280	960	841	960	841	384	336
128	3072	2304	2058	2304	2058		
256	7168	5376	4875	5376	4875	2048	1792
512	16384	12288	11276	12288	11276		
1024	36864	27648	25613	27648	25613	10240	8960
2048	81920	61440	57358	61440	57340		
4096	180224	135168	126991	135168	126991	49152	43008