Laboratory 10

1. Write a Maple function that produces an **array** containing the following variant of Pascal's triangle:

$$\begin{array}{r|ccccccc} n=0: & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ n=1: & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ n=2: & 1 & 2 & 3 & 2 & 1 & 0 & 0 \\ n=3: & 1 & 3 & 6 & 7 & 6 & 3 & 1 \\ \hline i= & 0 & 1 & 2 & 3 & 4 & 5 & 6 \end{array}$$

   Here the construction rule is

$$\left\langle {n+1 \atop i} \right\rangle = \left\langle {n \atop i-2} \right\rangle + \left\langle {n \atop i-1} \right\rangle + \left\langle {n \atop i} \right\rangle,$$
$$\left\langle {n \atop 0} \right\rangle = \left\langle {n \atop 2n} \right\rangle = 1,$$
$$\left\langle {n \atop j} \right\rangle = 0 \text{ for } j < 0 \text{ or } j > 2n.$$

   Your procedure should take one argument, n, and return an $(n+1)\times(2n+1)$ array containing the first n rows of the triangle (the case $n = 3$ is depicted above).

   Can you prove that the numbers in each row add up to 3^n?

2. In this problem you will gain insight into using Maple to design a special square called the **magic square**. The purpose is to give you some practice in using Maple as a tool to develop programs. Magic squares have some interesting properties which are listed below:

   - The **order** of a magic square (say n) is the number of rows (or columns)
   - The number of rows and columns is the same
   - Each number in the magic square is unique
   - The numbers in the magic square are in the range $\{1..n^2\}$
   - The sum of all the numbers in each row = the sum of all numbers in each column = the sum of all numbers in both diagonals = N
   - There is a specific relationship between the sum N and the order n

   In this problem you will write a Maple program to print magic squares of odd order. An example 3×3 (order 3) and 5×5 (order 5) magic square is shown below.

8	1	6
3	5	7
4	9	2

3 × 3 magic square

17	24	1	8	15
23	5	7	14	16
4	6	13	20	22
10	12	19	21	3
11	18	25	2	9

5 × 5 magic square

It is quite straightforward to see how the numbers are filled. Before you come to the lab, study the magic squares of order 3 and 5 and try to think of a way in which a program could be written for any odd order. *In the beginning of the lab the algorithm will be explained to you. Therefore do not panic if you are finding it difficult.*

Deliverables At the end of the lab you should have accomplished the following:

- Understood how the magic squares of odd order are created
- Implemented and tested your program to print magic squares
- Demo your program to one of the assistants
- Find the relationship between N and n

After you have finished the above program, try to think of a way to construct magic squares of order $\{4, 8, 16, 32, 64, ...\}$. Is it possible to construct a magic square of order 6?