Universal Bufferless Routing

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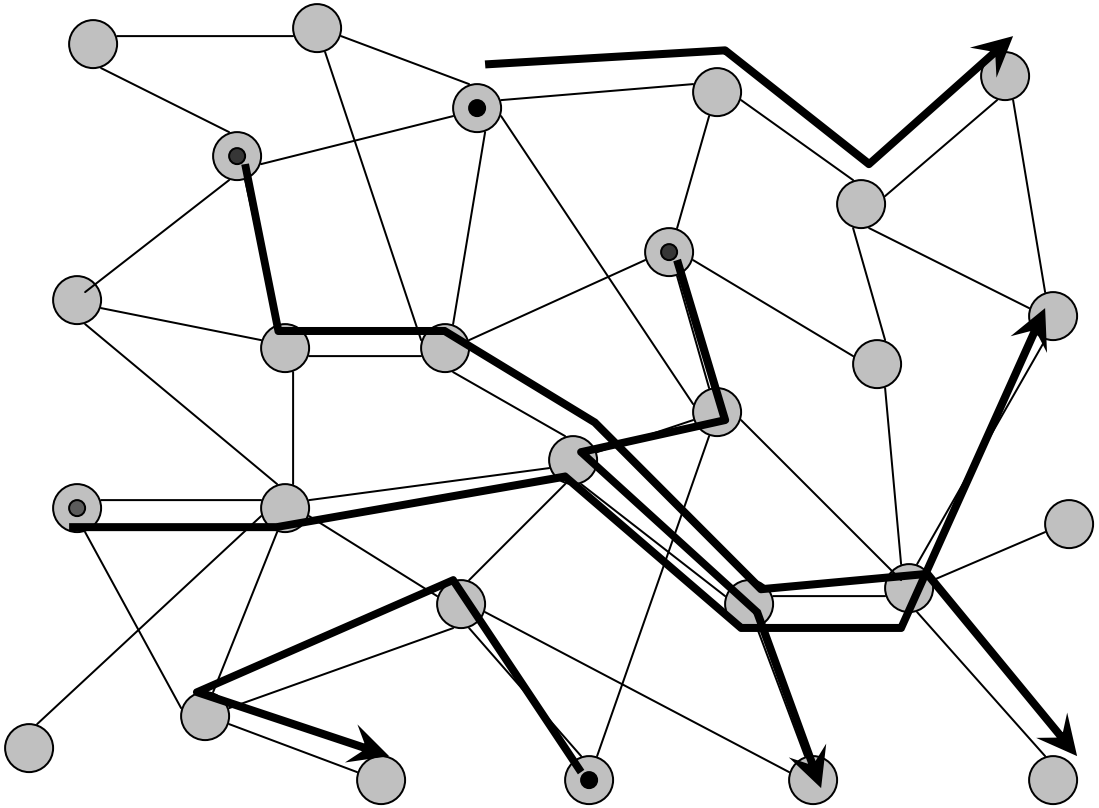
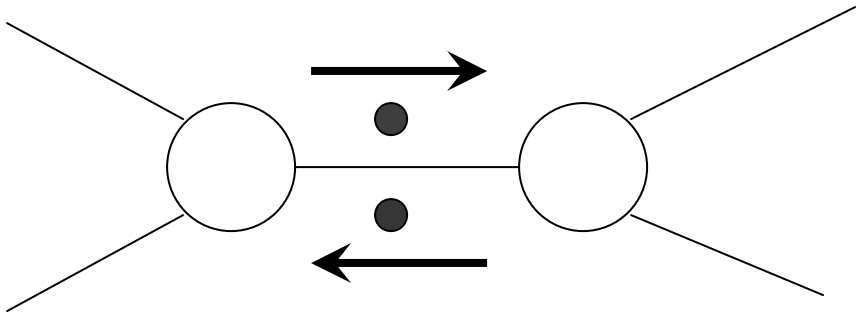
Routing Problem



- Arbitrary network
- Arbitrary set of packets

Network Model

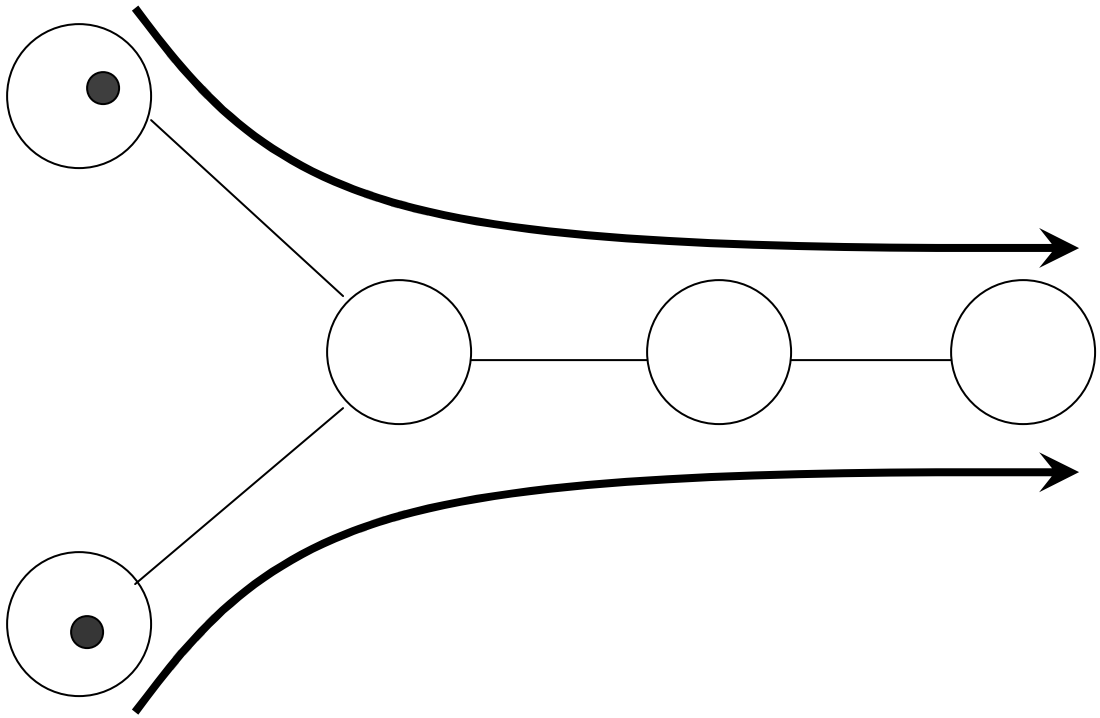
- Synchronous network (time steps)
- One packet per time step
- Bi-directional links



Network Model

Buffer-less nodes

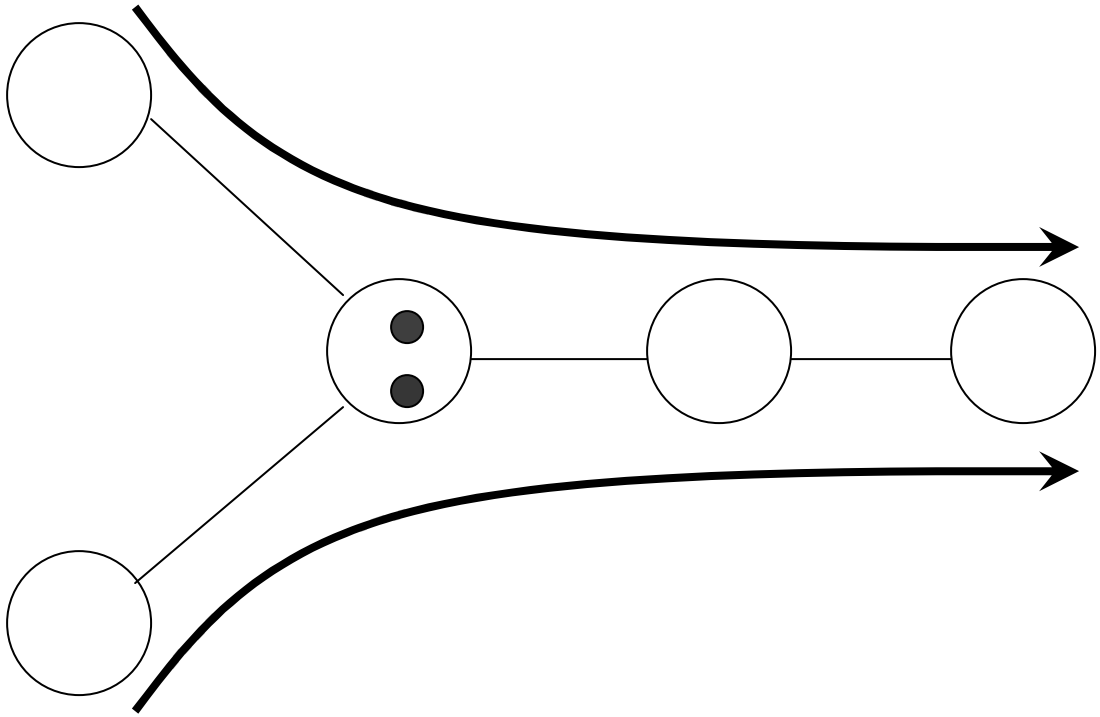
Time 0



Packets are always moving

Buffer-less nodes

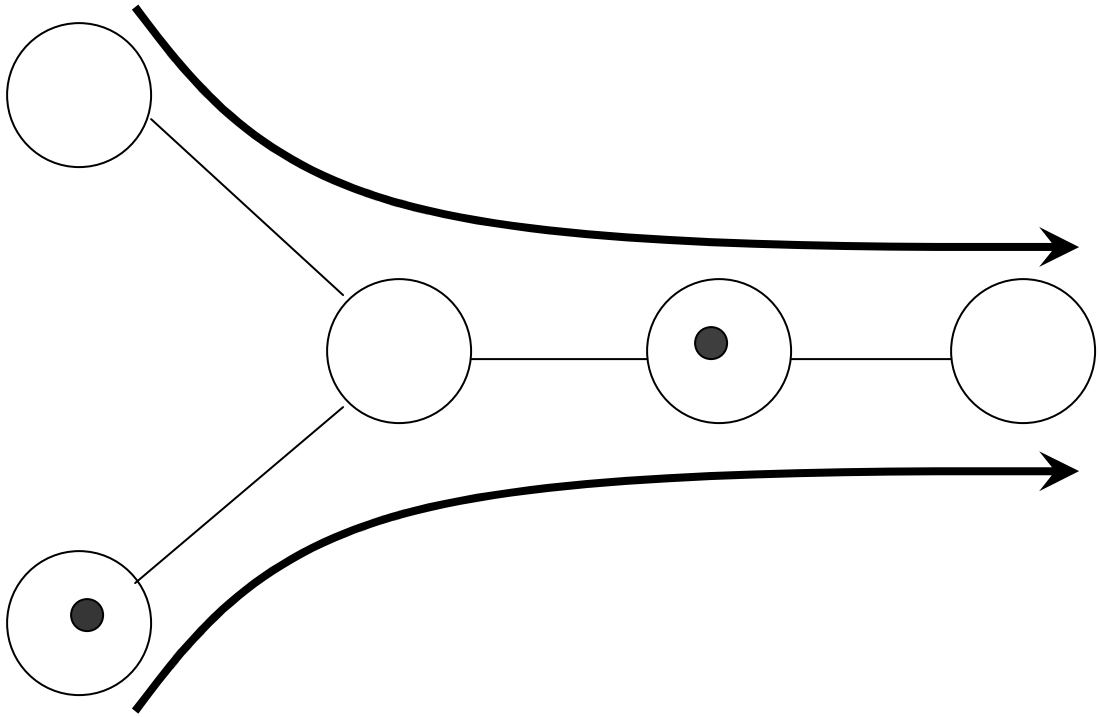
Time 1



Packets are always moving

Buffer-less nodes

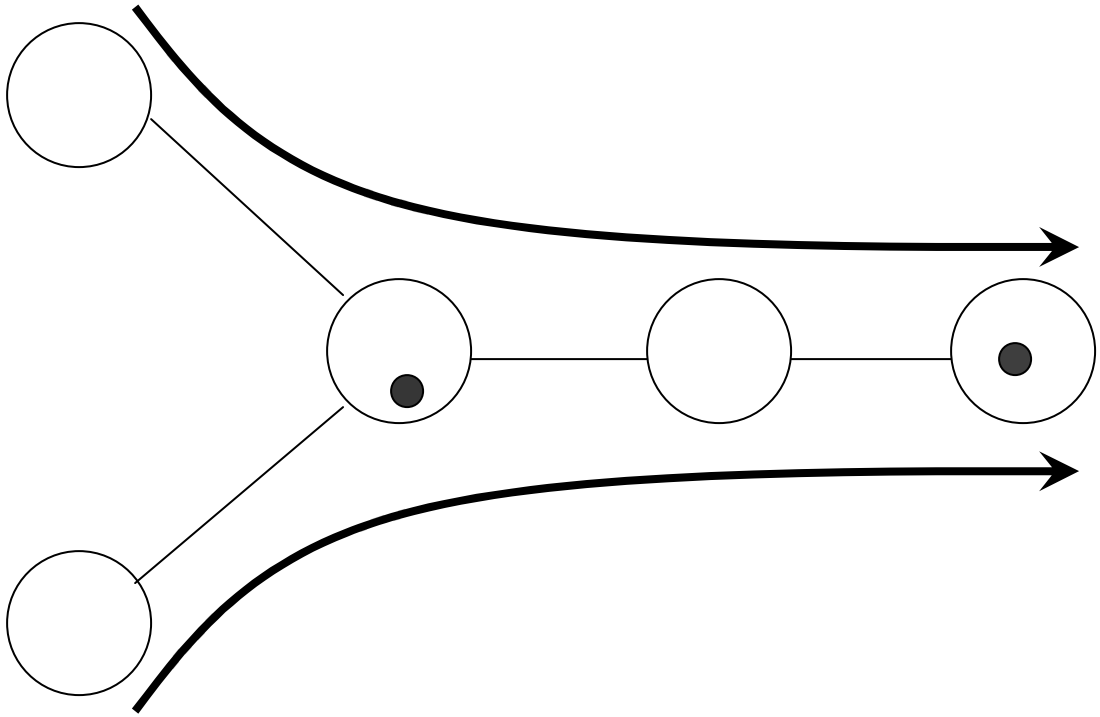
Time 2



Packets are always moving

Buffer-less nodes

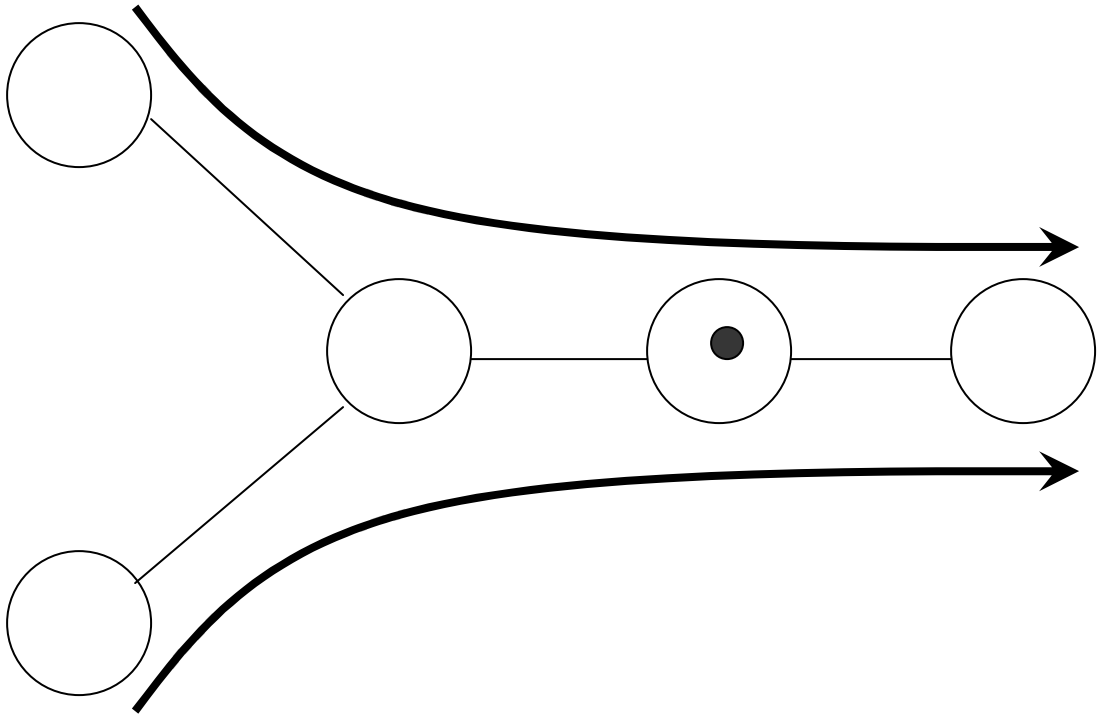
Time 3



Packets are always moving

Buffer-less nodes

Time 4



Packets are always moving

Bufferless routing is interesting:

- Optical networks
- Simple hardware implementations
- Works well in practice:

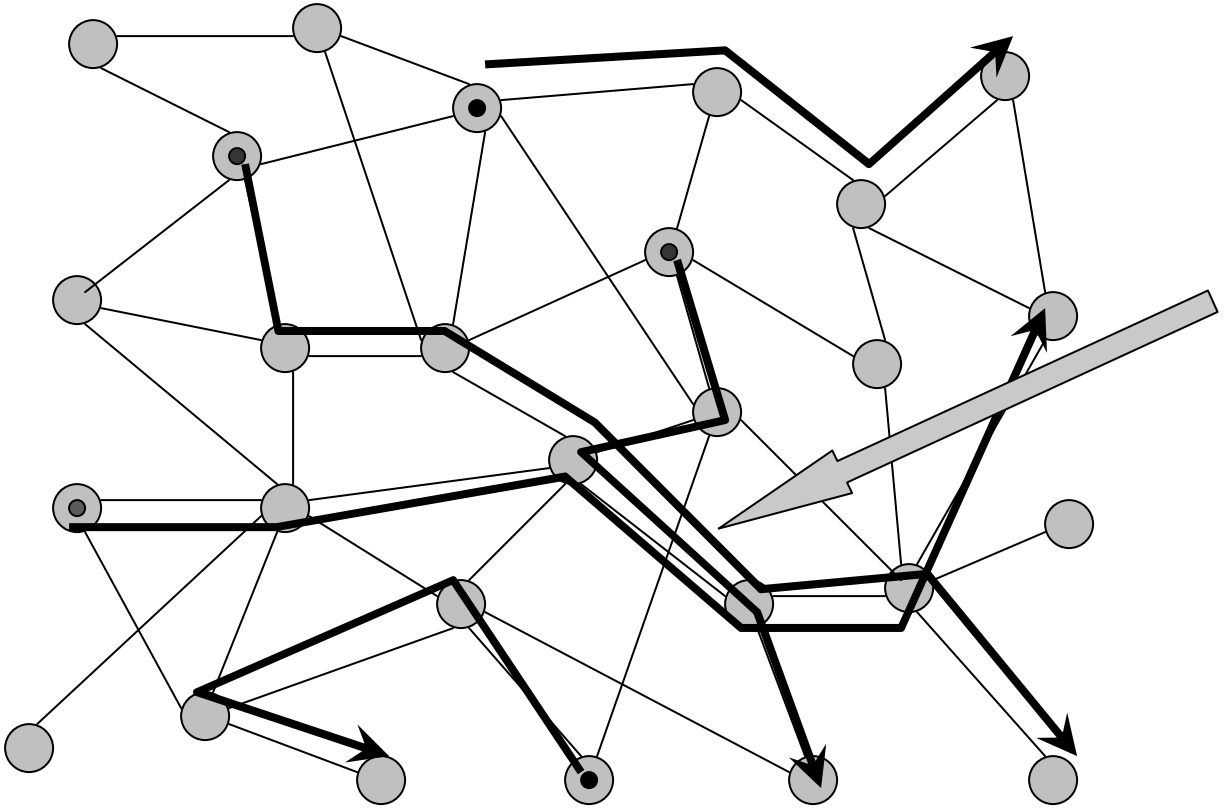
  Bartzis *et al.*: *EUROPAR 2000*

  Maxemchuck: *INFOCOM 1989*

Routing Time: the time until the last Packet is absorbed

Objective: Minimize Routing Time

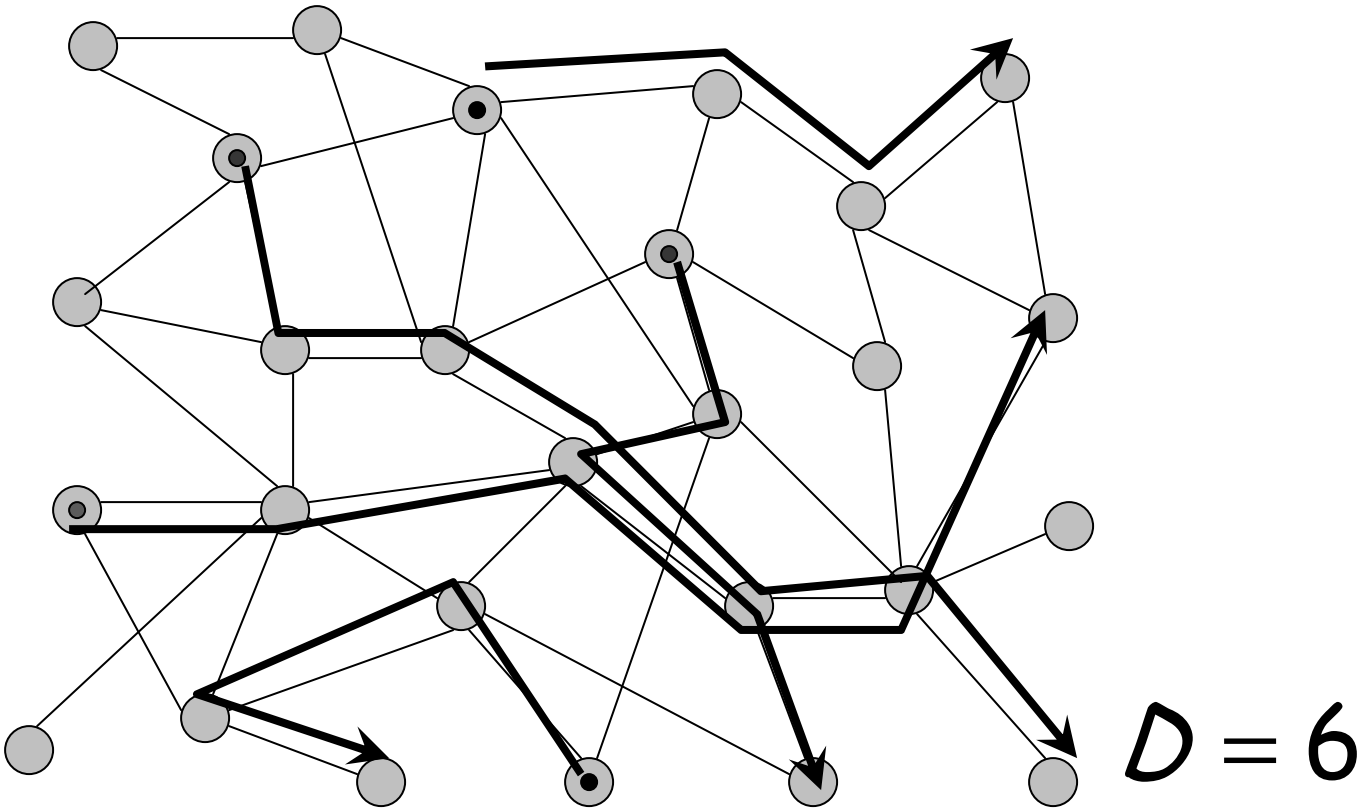
Congestion



$c = 3$

Maximum number of packets that cross an edge

Dilation



Maximum path length

Lower bound on Routing Time:

$$\Omega(C + D)$$

Congestion

Dilation

We want to find algorithms with routing time close to lower bound

Our contribution:

A buffer-less algorithm with routing time

$$O((C + D) \cdot \log^3(n + N))$$

nodes

packets

Related Work

For routing with buffers
there exist optimal $O(C + D)$ algorithms:

[Leighton - Maggs - Rao, Combinatorica 94]

[BS99, LMR99, MV99, OR97, RT96]

What about buffer-less routing?

Related Work

Buffer-less Routing has been studied for specific topologies

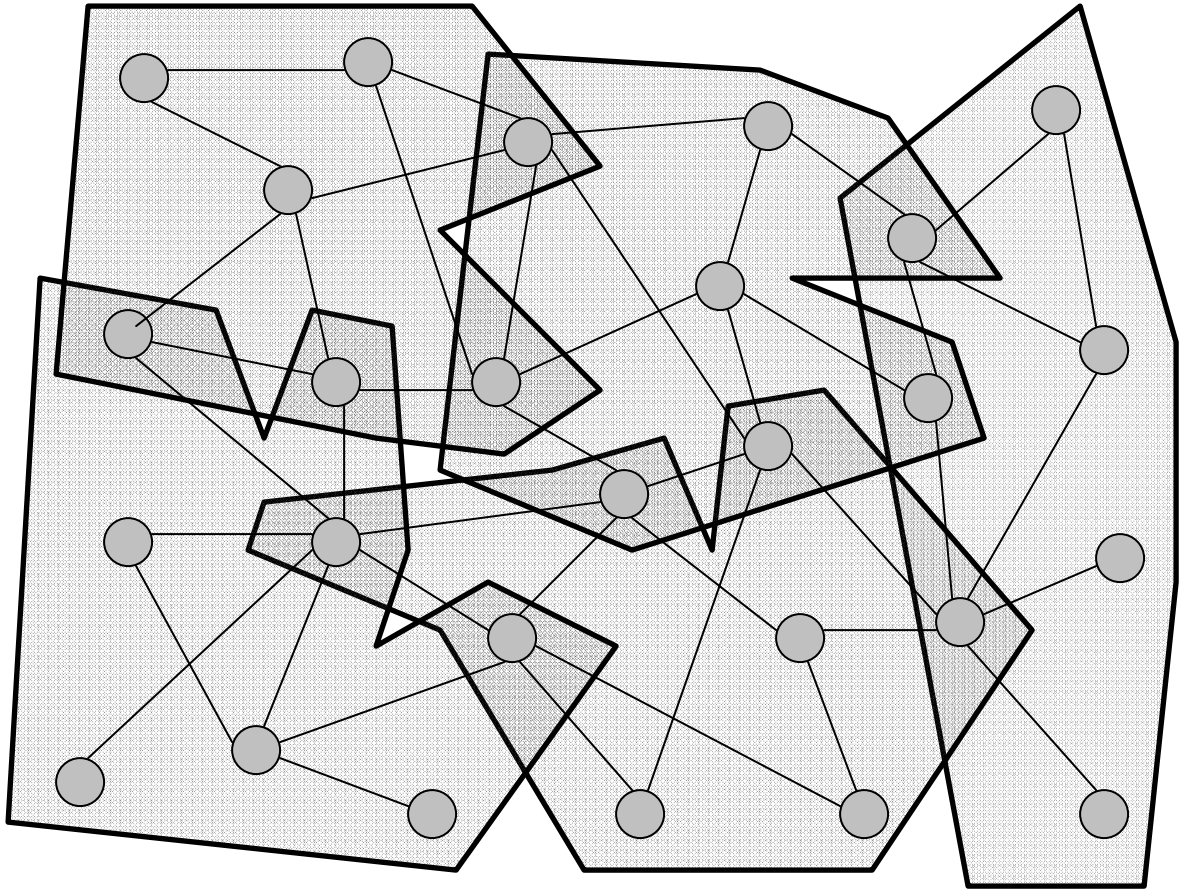
Mesh	[BRST93, BES97, BHS98, BU96, BHW00]
Hypercube	[BH85, BC95, FR92, H91]
Trees	[BMMW04, RSW00]
Leveled	[BBPRRS96, B02]
Vertex-symmetric	[MS95]

No previous work for $O(C + D)$ time on arbitrary networks

Presentation Outiline

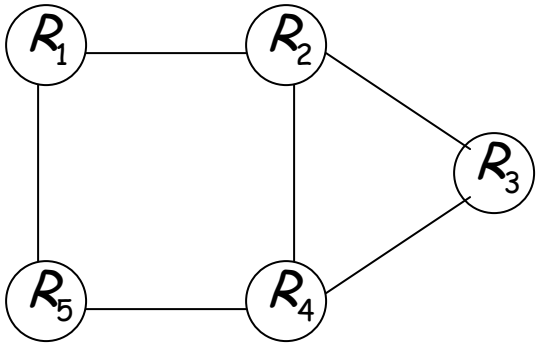
- Algorithm
- Analysis
- Conclusion

Partition the Network into Regions

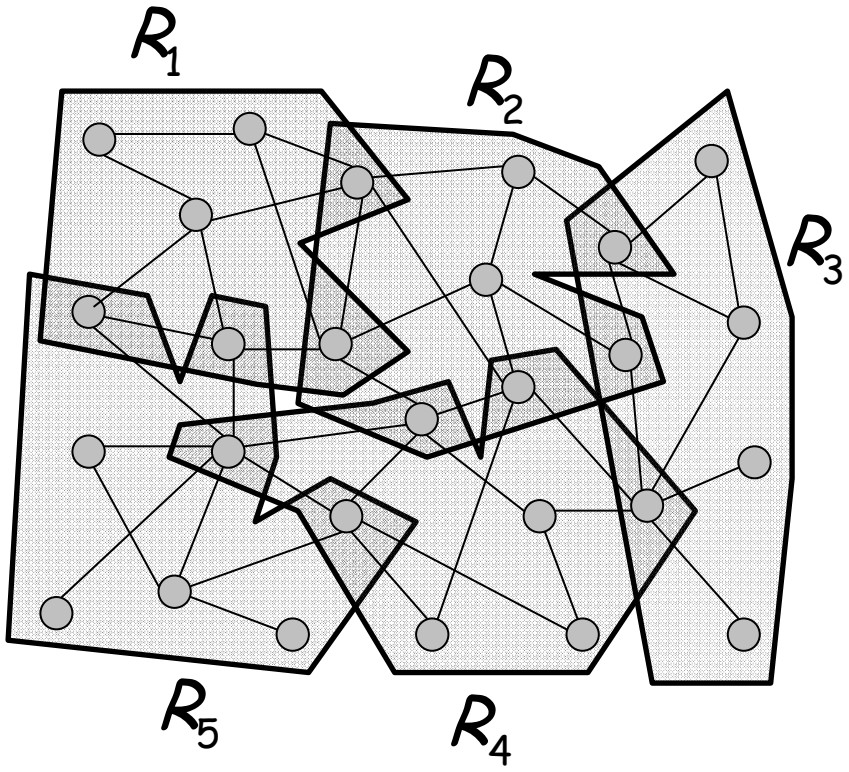


- Each edge belongs to only one region
- Regions share nodes

Construct Region Graph

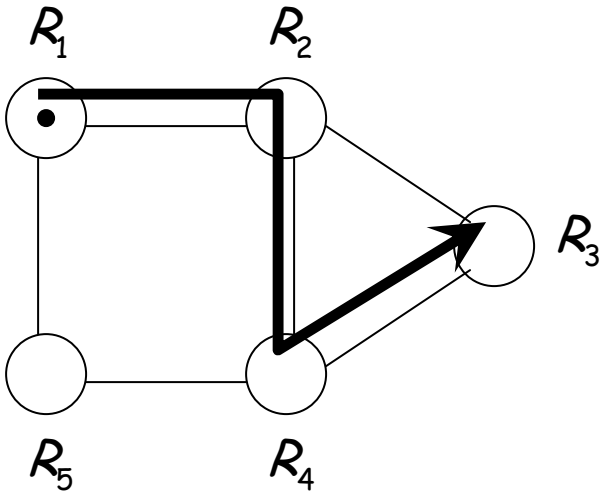


Region Graph

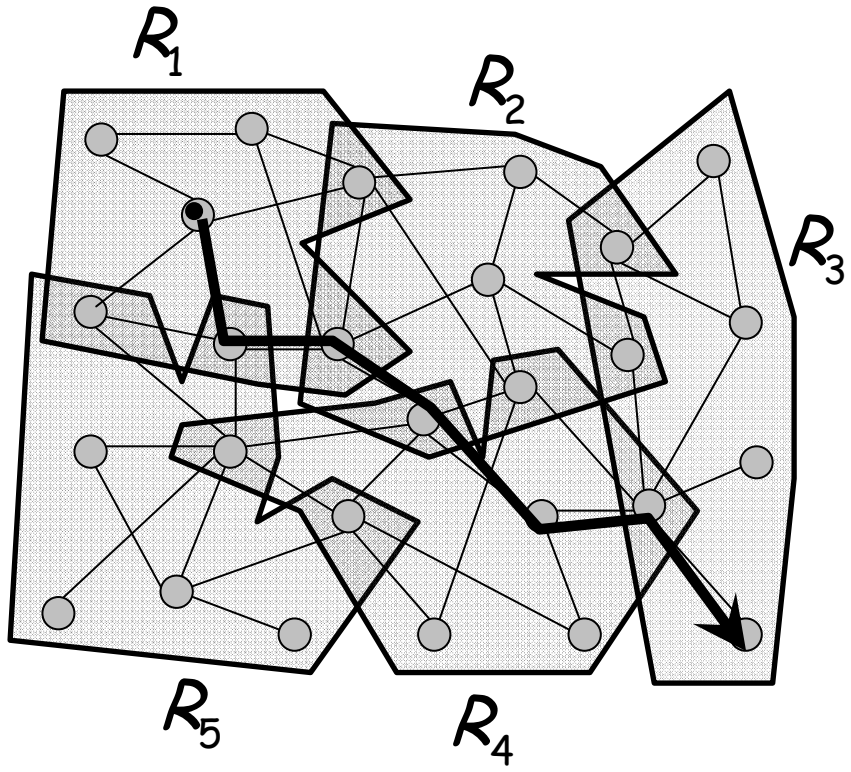


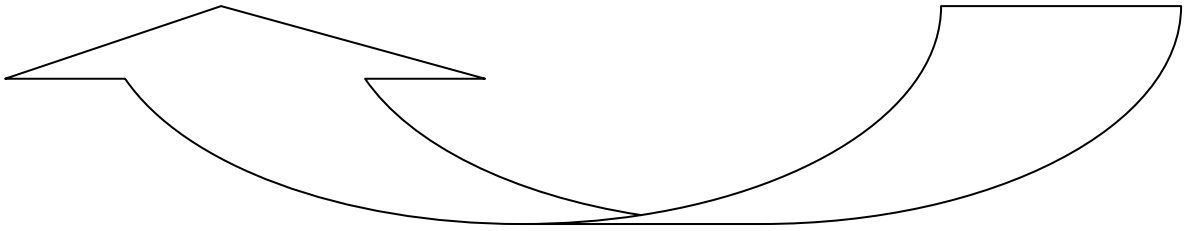
Regions are connected if they share a node in the original graph

Translate Packet Paths

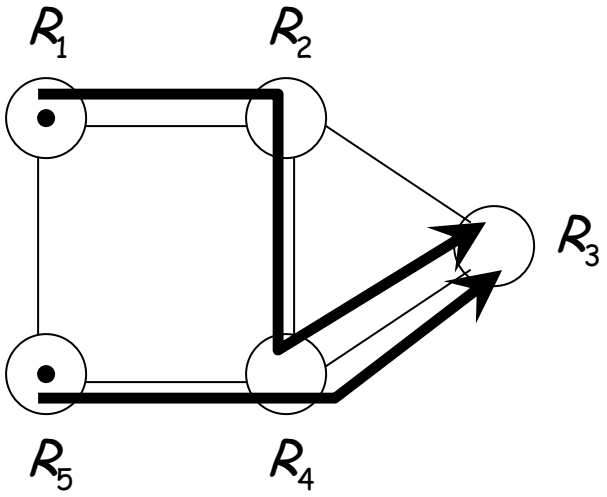


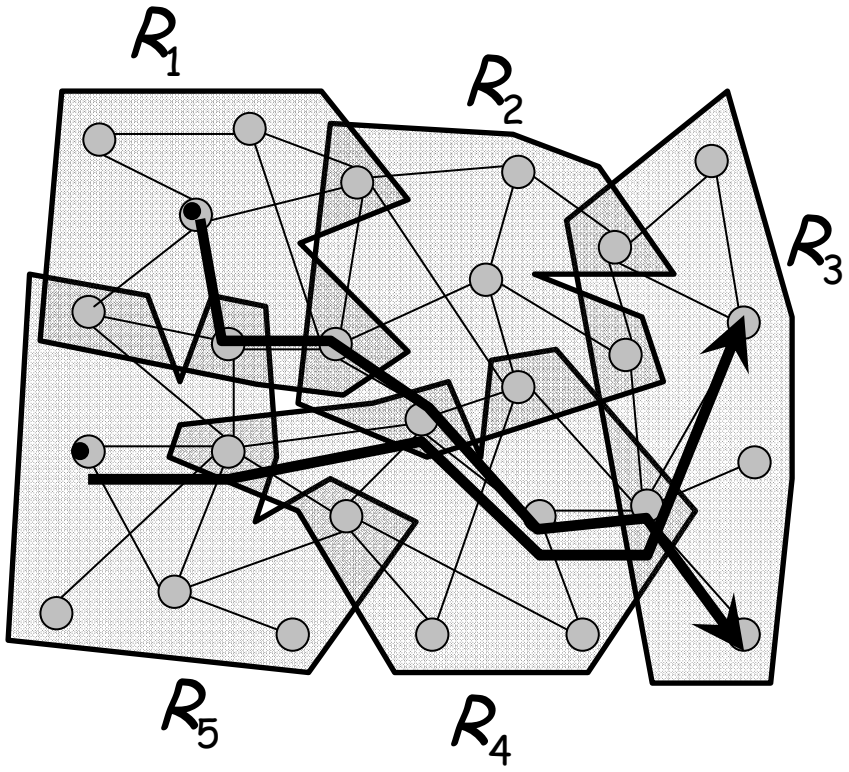
Region Graph



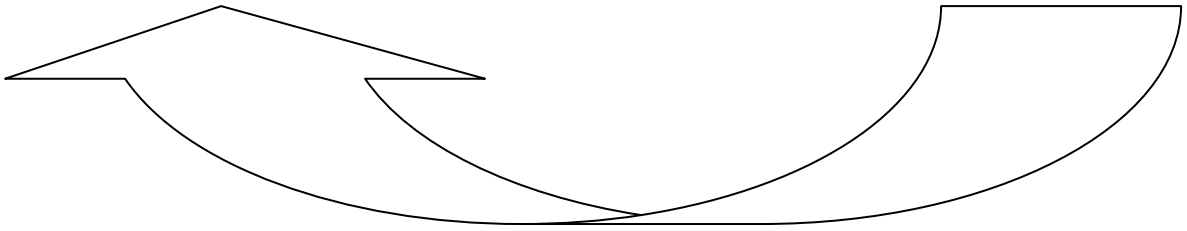


Translate Packet Paths

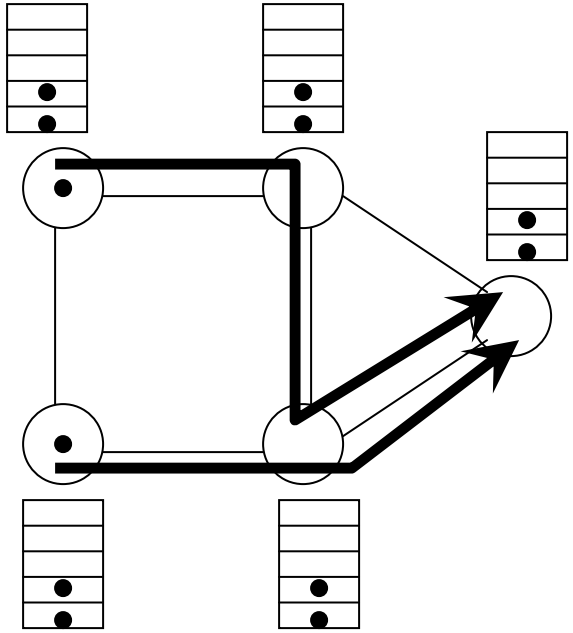




Region Graph

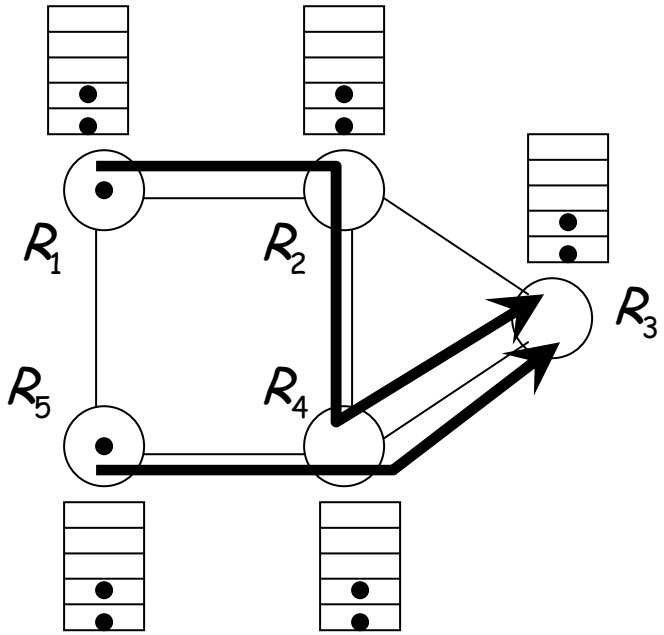


Execute a Buffered Algorithm on Region Graph



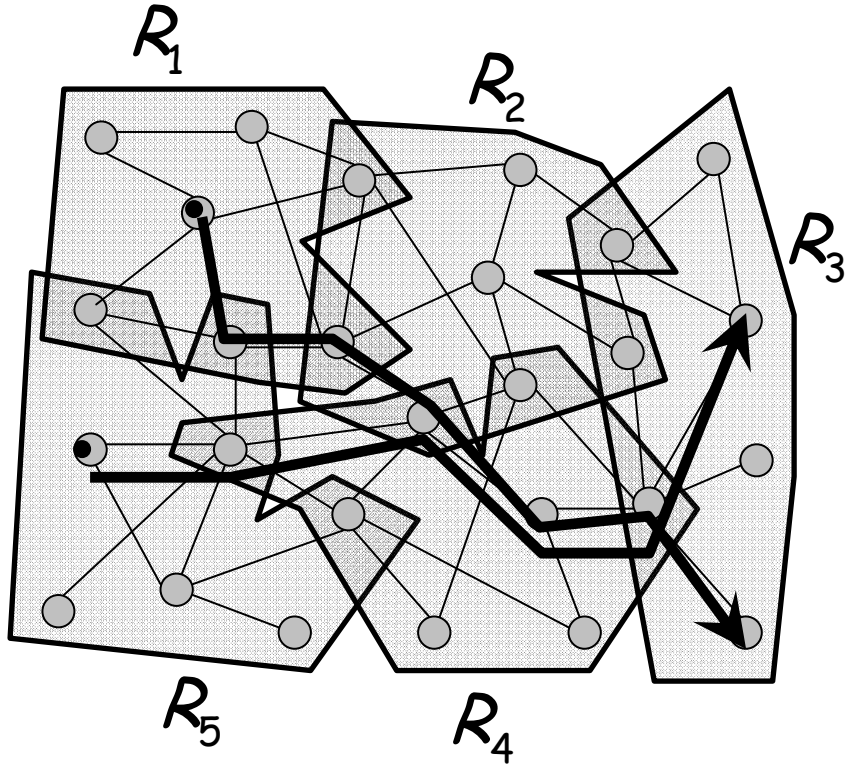
Packet Buffer

Buffered Algorithm



Buffers

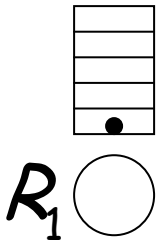
Simulate Buffered Algorithm



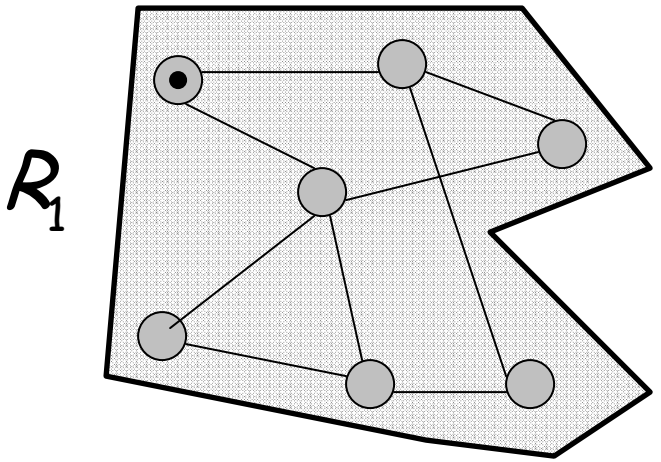
No Buffers

Region Graph Buffered Algorithm Action for a Packet	Original Graph Bufferless Algorithm Simulation Action
1 time step	1 phase
• Buffer in node R_i • Move from node R_i to R_j • Inject/Absorb at node R_i	• Circulare in region R_i • Move from region R_i to R_j • Inject/Absorb at source/destination in region R_i

Packet Buffer

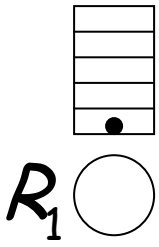


Simulation-Circulation

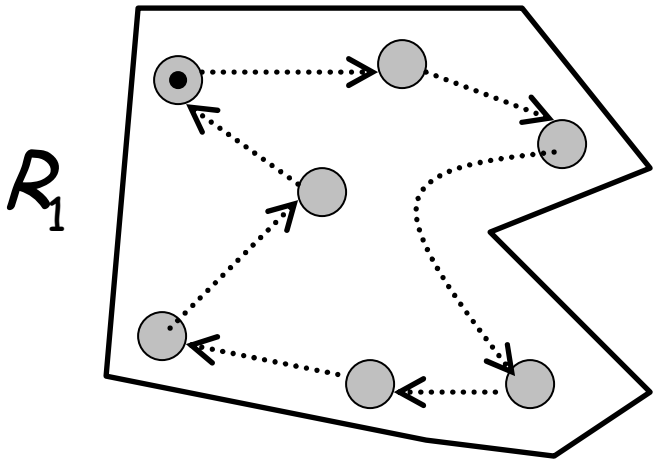


Follow repeatetly an Euler tour

Packet Buffer

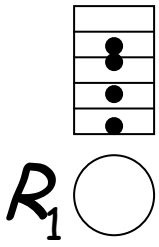


Simulation-Circulation



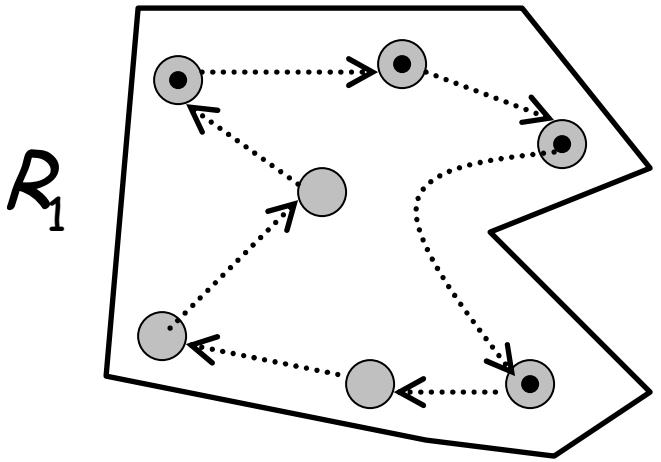
Euler tour (abstract)

Packet Buffer



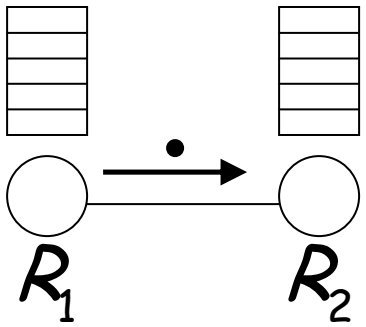
Multiple packets

Simulation-Circulation

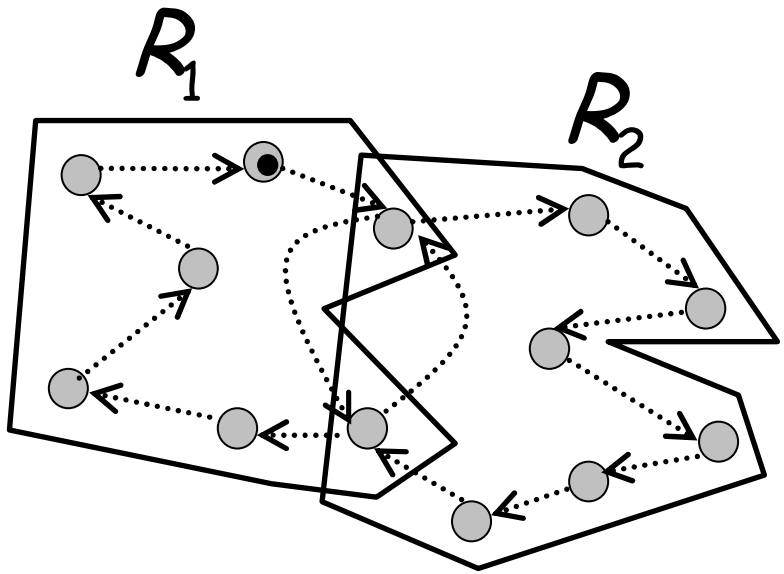


Pipeline packets on Euler Tour

Packet Move

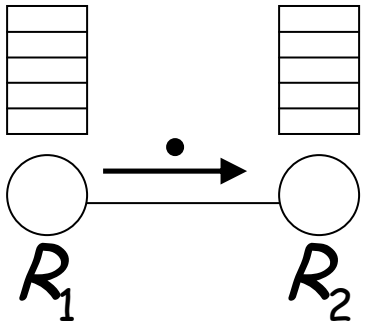


Simulation of move

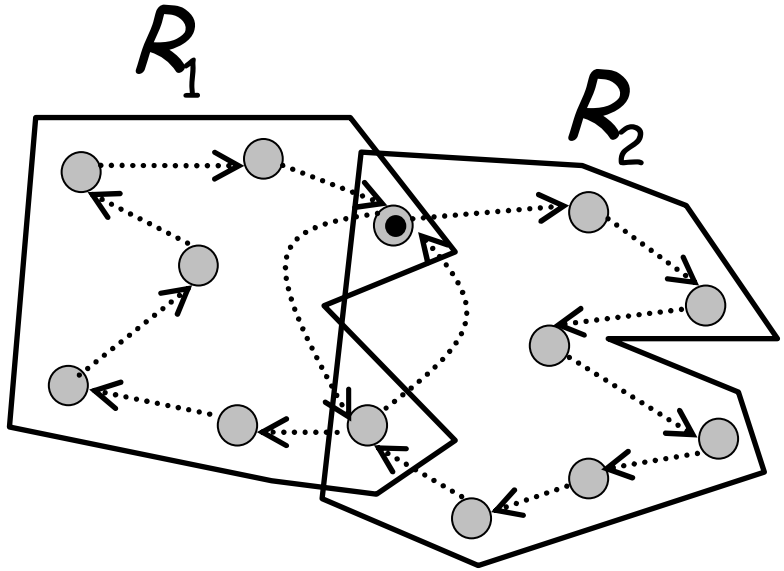


Packet moves to new region when it reaches common node

Packet Move



Simulation of move



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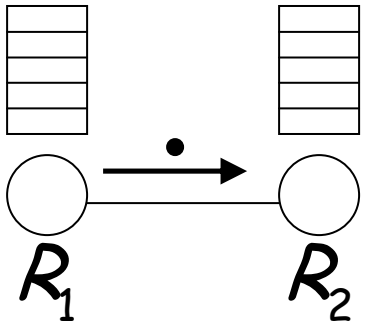
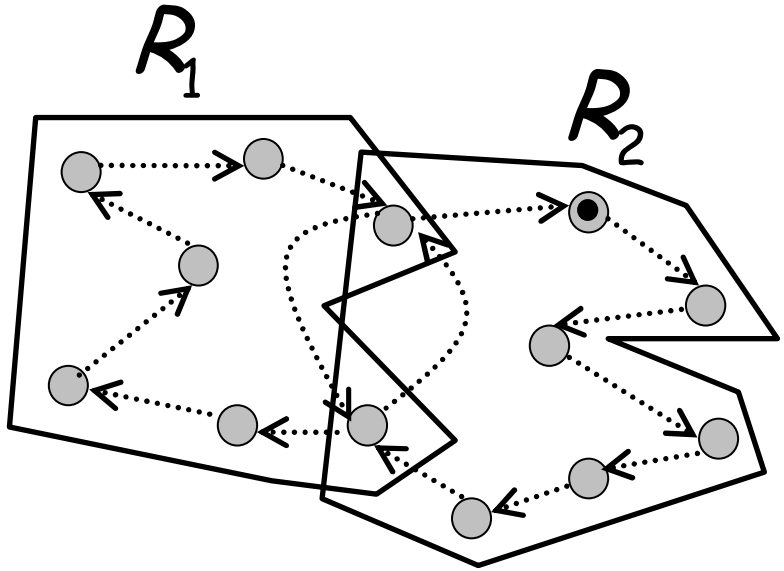
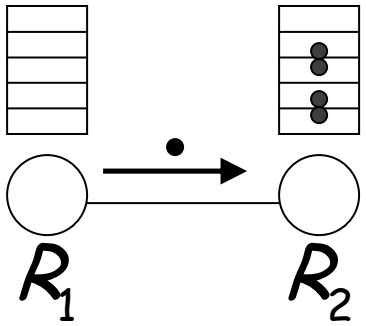


Simulation of move



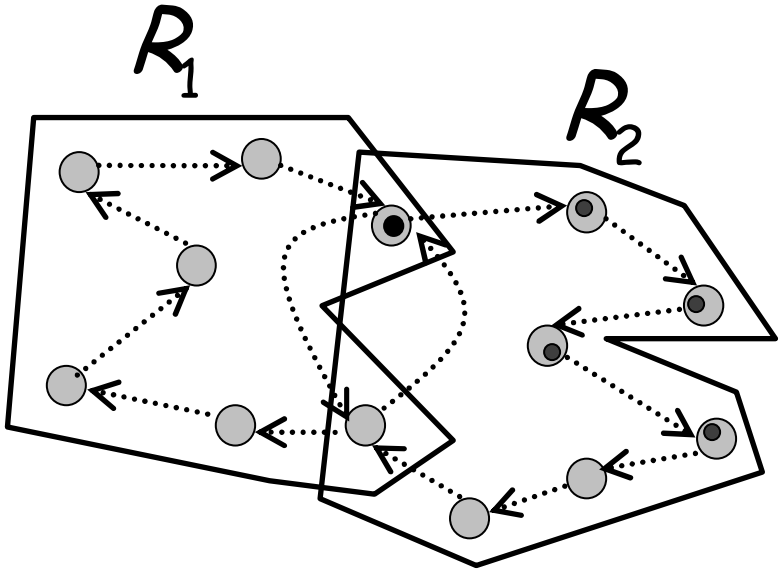
Packet moves to new region when it reaches common node

Packet Move



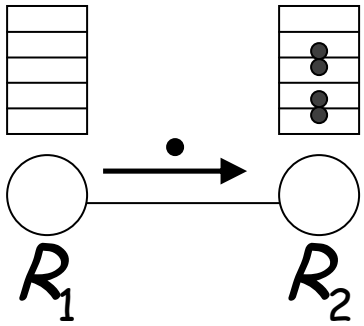
multiple packets in R_2

Simulation of move

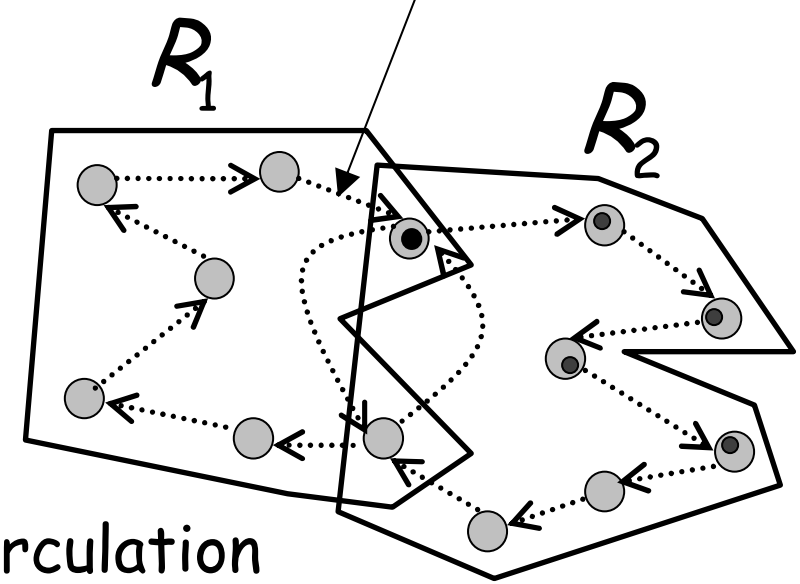


Insert packet when there is an empty spot in R_2

Packet Move



Oscillate here until empty slot is found



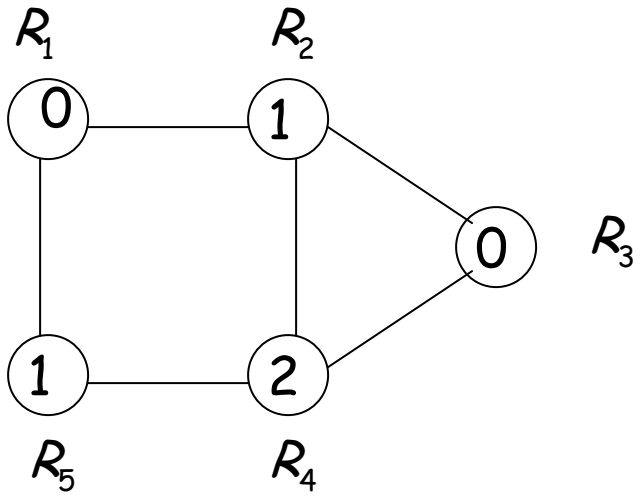
Slow Circulation

Sending Mode

Fast Circulation

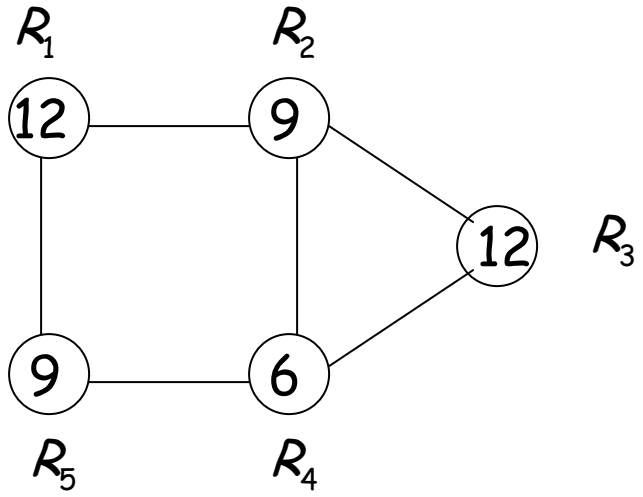
Receiving Mode

Color the Region Graph



Let k be the maximum color

Change Colors



Replace each color with $2\log k$ bits

$$\overline{b_{\log k} \ldots b_2 b_1}\; b_{\log k} \ldots b_2 b_1$$

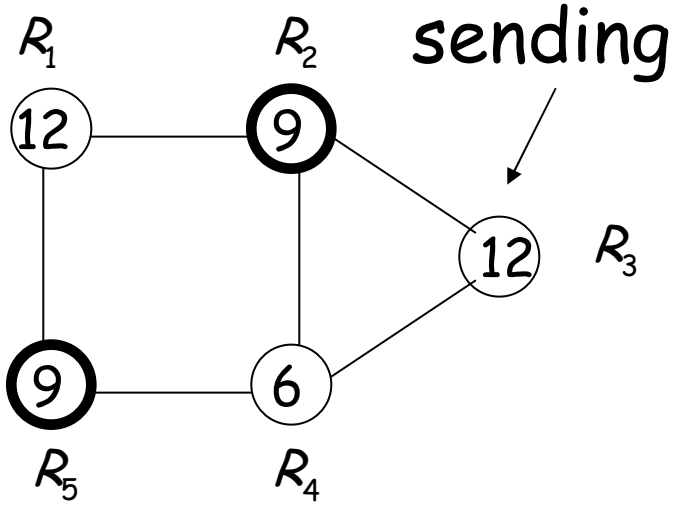
complement

original color

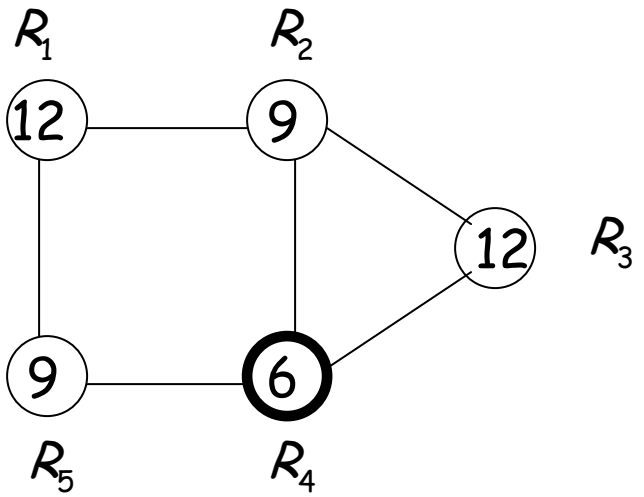
Each phase consists of $2\log k$ rounds

At round i a node is in sending mode
if its i color bit is equal to 1

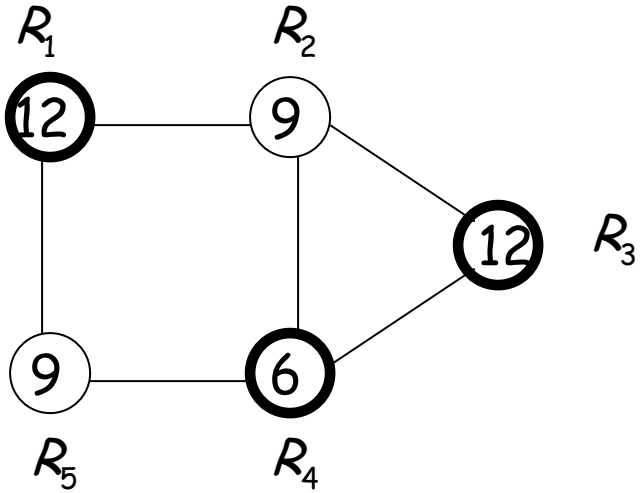
Round 0



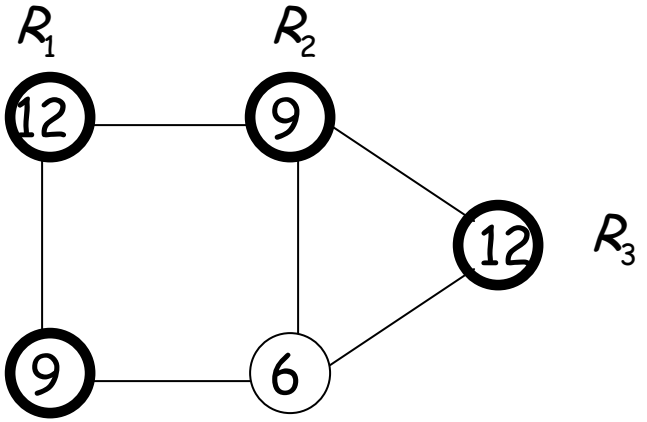
Round 1

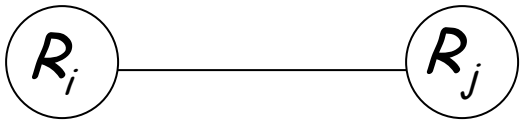


Round 2



Round 3





There is a round that

R_i is receiving

R_j is sending

And vice-versa

Presentation Outiline

- Algorithm
- Analysis
- Conclusion

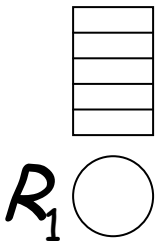
We give a buffered algorithm with

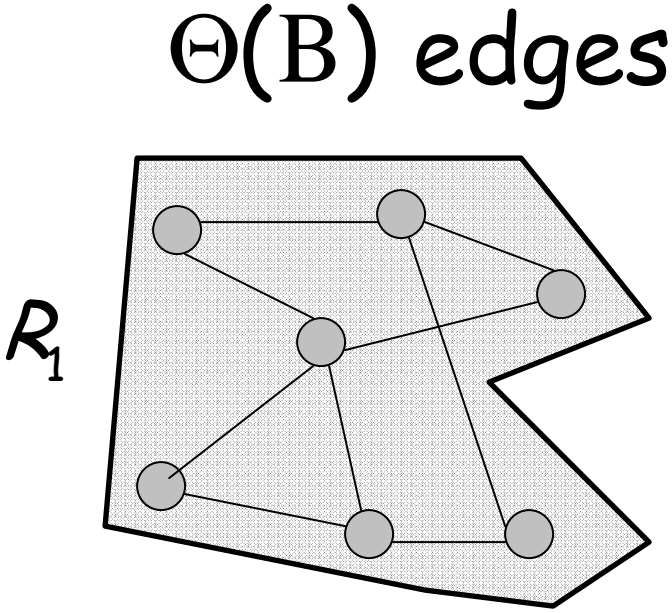
Buffer size: $B = 6\log(n' + N)$

Routing time: $O\left(\frac{C'}{B} + D'\right)$ "Time Steps"

n', C', D' : #nodes, node congestion, dilation in region graph

We partition original graph so that each region has $\Theta(B)$ edges





It holds:

$$n' \leq n$$

$$C' \leq \mathrm{B}\, C$$

Edges in region

Edge congestion

$$D' \leq D$$

Each simulation phase has duration

$$\mathrm{B}^2 \cdot 2\log k = O(\log^3(n+N))$$

duration of round

number of rounds

Total Routing Time

$$O\left(\left(\frac{C'}{B}+D'\right)\cdot\log^3(n+N)\right)=O((C+D)\cdot\log^3(n+N))$$

Number of phases

Phase Duration

Presentation Outiline

- Algorithm
- Analysis
- Conclusion

We presented the first bufferless algorithm for arbitrary networks within logarithmic factors from optimal

Future research:

- Improve logarithmic factors
- Maintain original paths