# Near-Optimal Hot-Potato Routing on Trees


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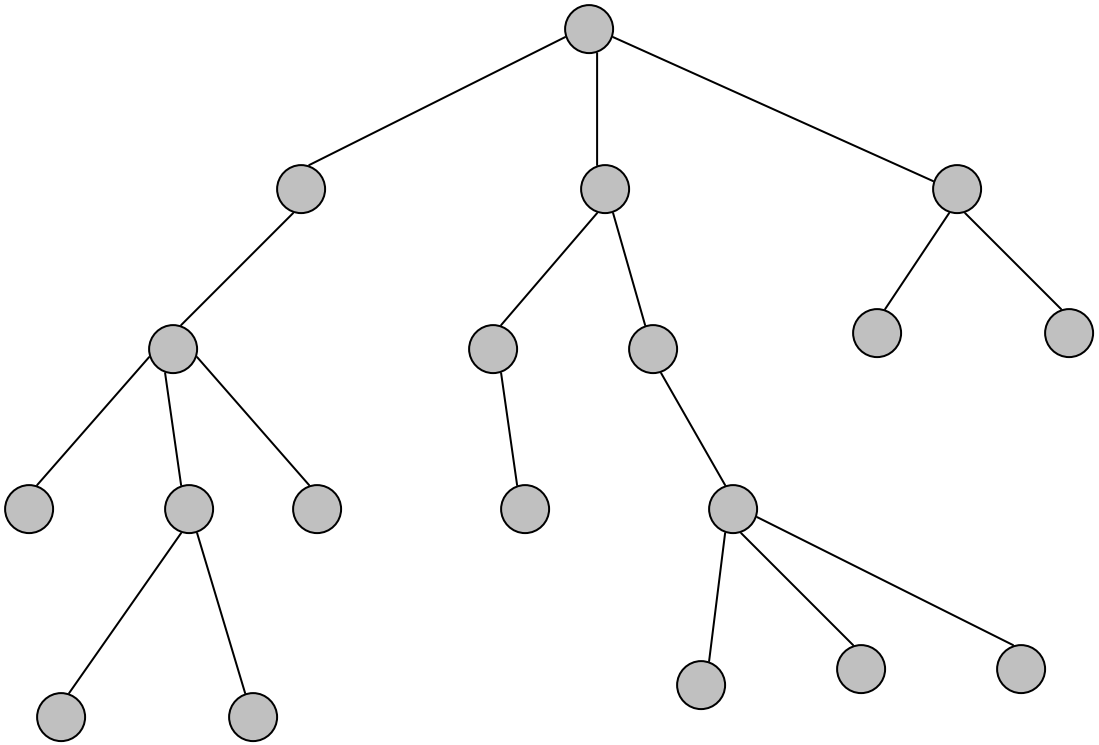
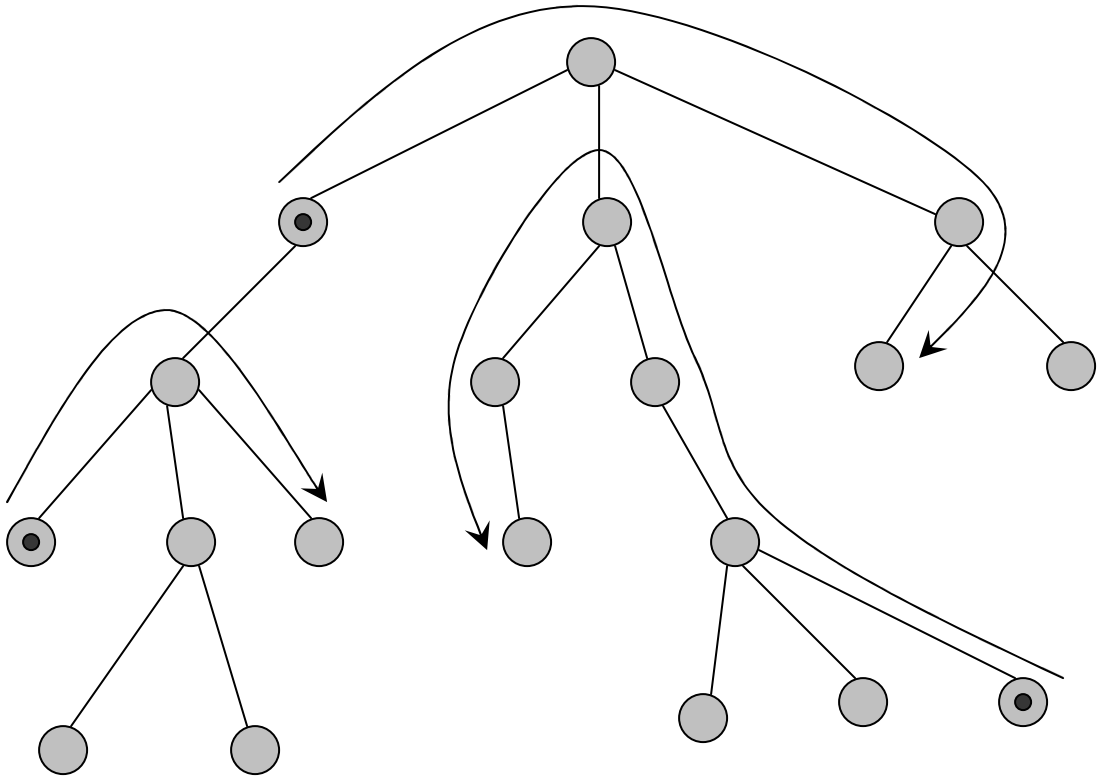
| **Costas Busch** | Rensselaer Polytechnic Inst. |
| Malik Magdon Ismail | Rensselaer Polytechnic Inst. |
| Marios Mavronicolas | University of Cyprus |
| Roger wattenhofer | ETH Zurich |

# Trees



Trees are important in many networks
(i.e. spanning trees)

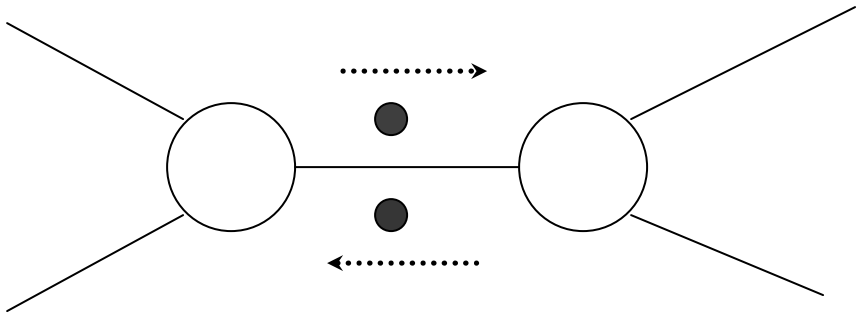
# Routing on Trees



- Every node generates at most one packet
- Packets follow shortest paths

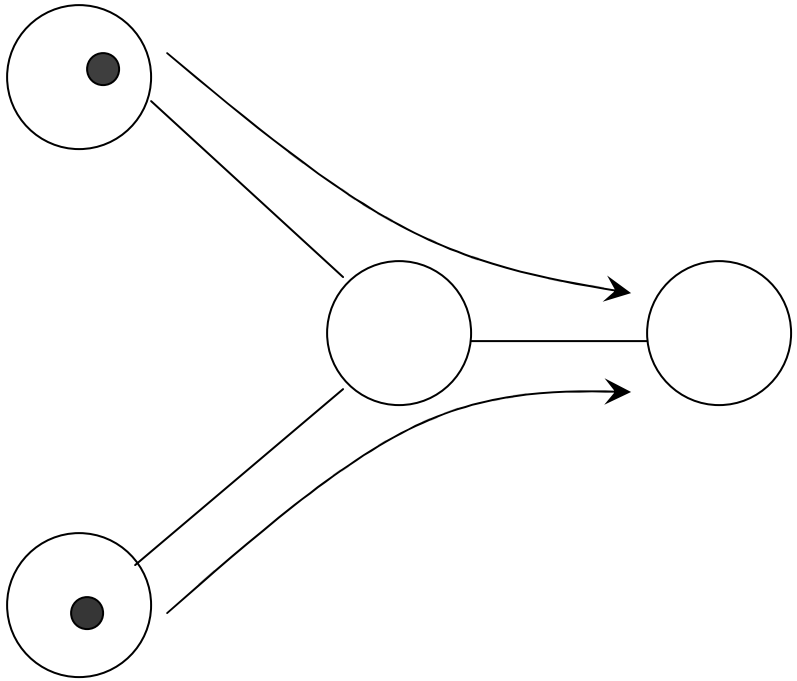
# Network Model

- Synchronous network
- Bi-directional links
- One packet per time step



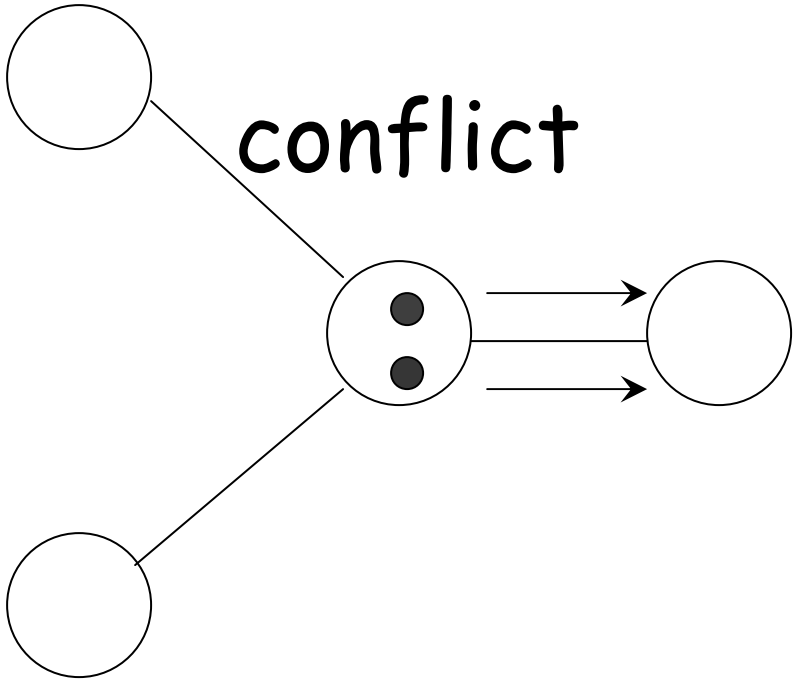
# Hot-Potato Routing

Time 0

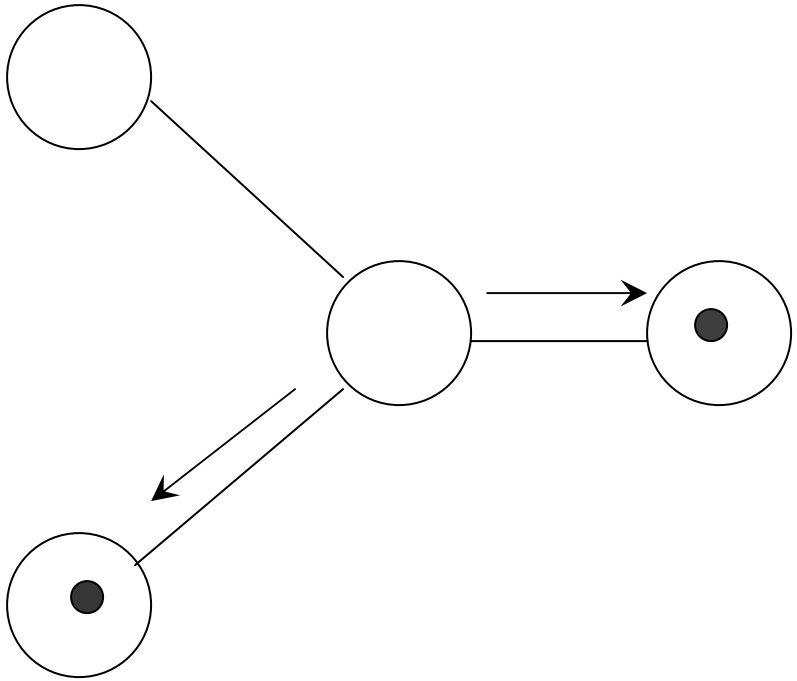


Buffer-less nodes

Time 1

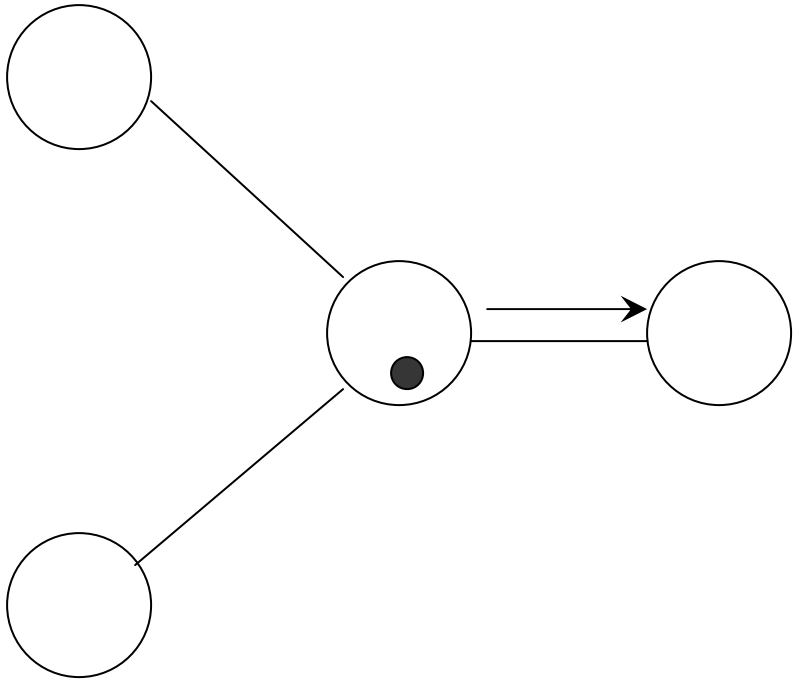


Time 2

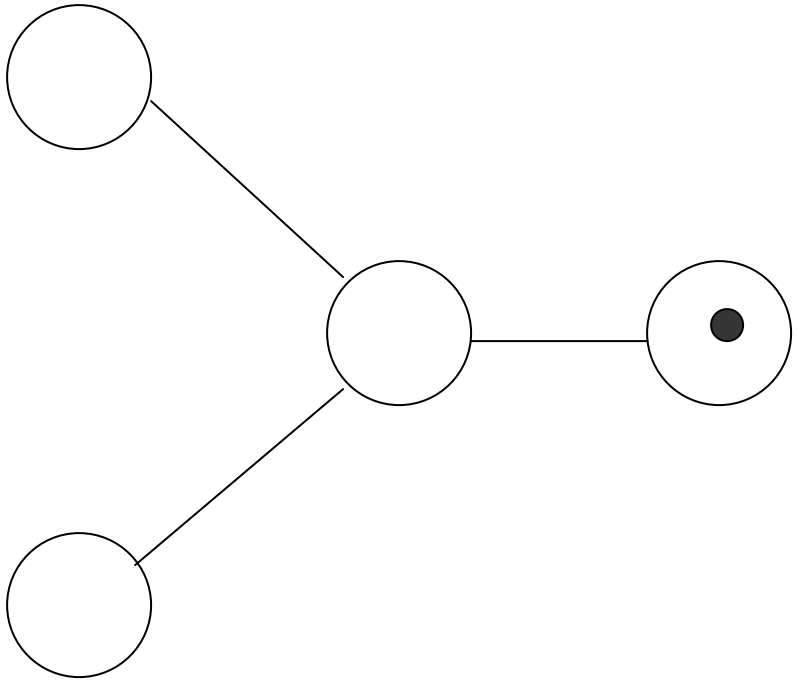


deflected

Time 3



Time 4



Hot-potato routing is interesting:

- Optical networks
- Simple hardware implementations
- Works well in practice:

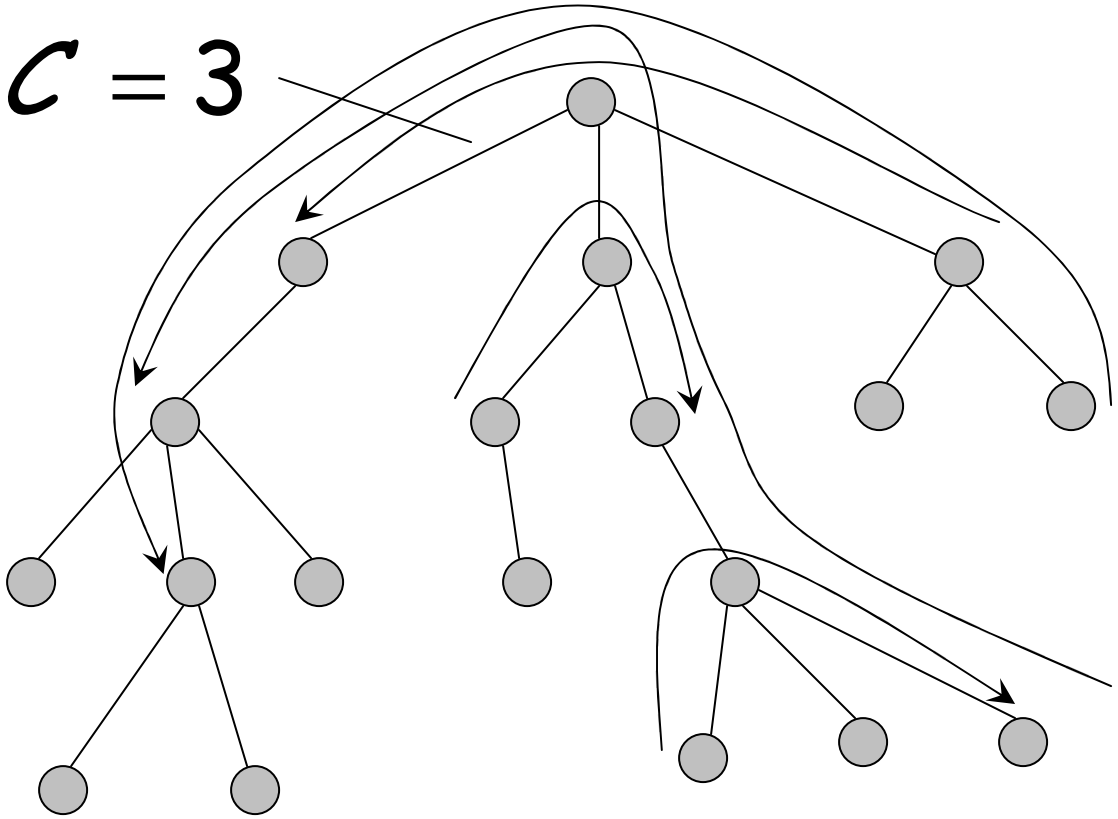
  Bartzis *et al.*: *EUROPAR 2000*

  Maxemchuck: *INFOCOM 1989*

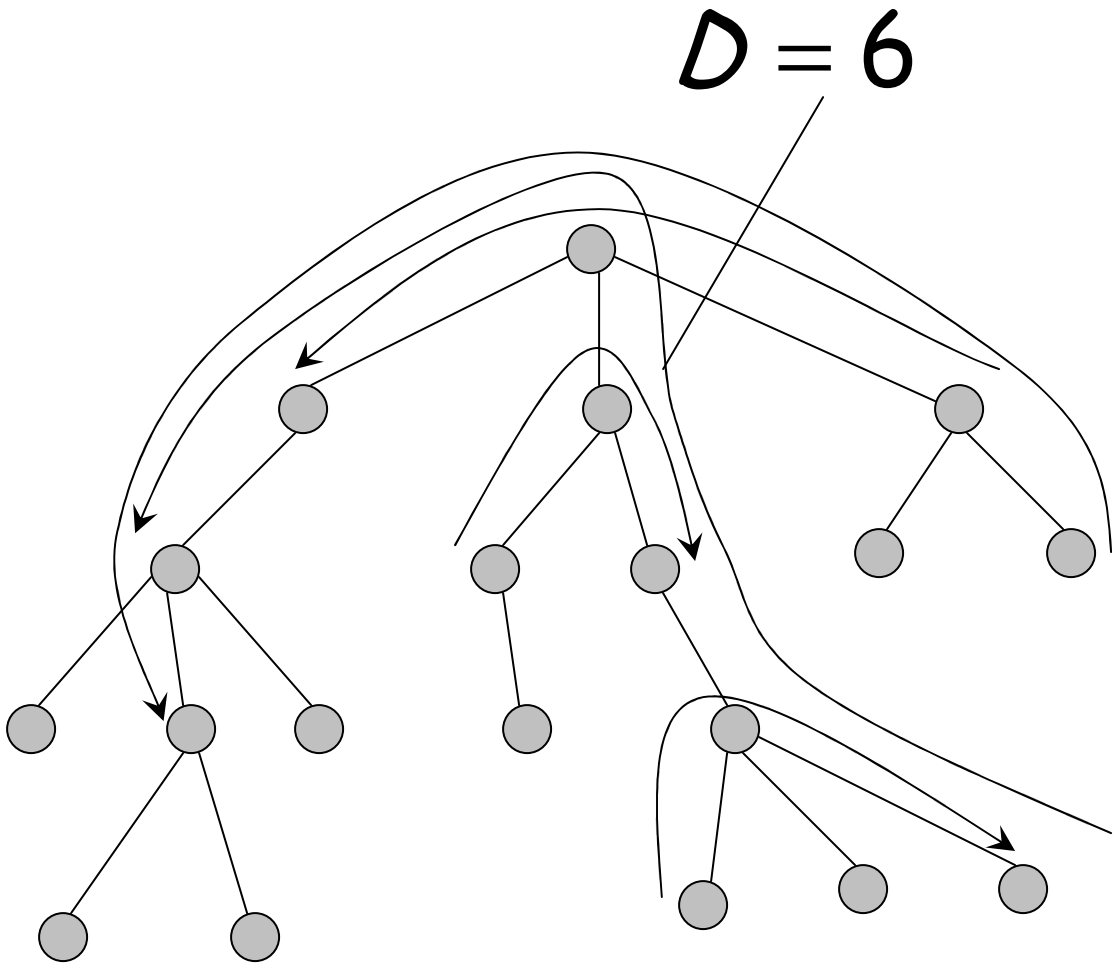
Objective: Find hot-potato algorithm which minimizes routing time

The time until the last packet is delivered to its destination

Congestion: Maximum numbers of packets that share an edge



# Dilation: Maximum path length



A lower bound on Routing Time:

Congestion+Dilation

$$\Omega(C + D)$$

We want to find an algorithm close to this lower bound

# Our contributions:

- Deterministic Algorithm: $O((\delta C + D)\log n)$

node degree

network size

- Randomized Algorithm: $O((C + D)\log^2 n)$

degree independent

# Related Work for Trees

- Matching Routing [ACG94] [PRS97] [Z97]
- Direct Routing [AHLT98][BMMS04]
- Hot-Potato routing [RSW00]

Most results have routing time $O(n)$

(worst case bound for $O(C+D)$)

# Presentation Outiline

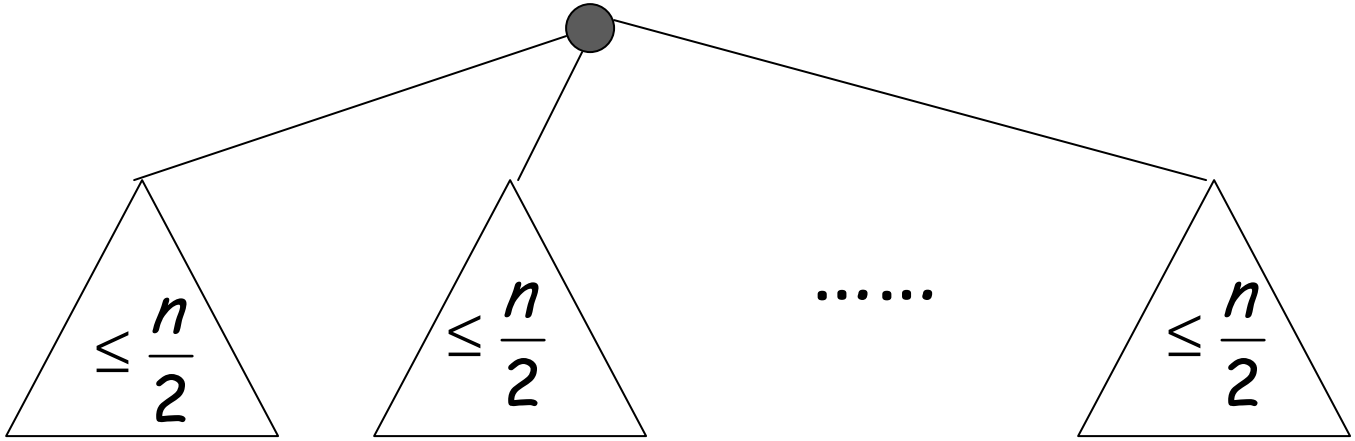
- Deterministic Algorithm
- Randomized Algorithm

# Deterministic Algorithm

1. Divide time into phases according to short nodes

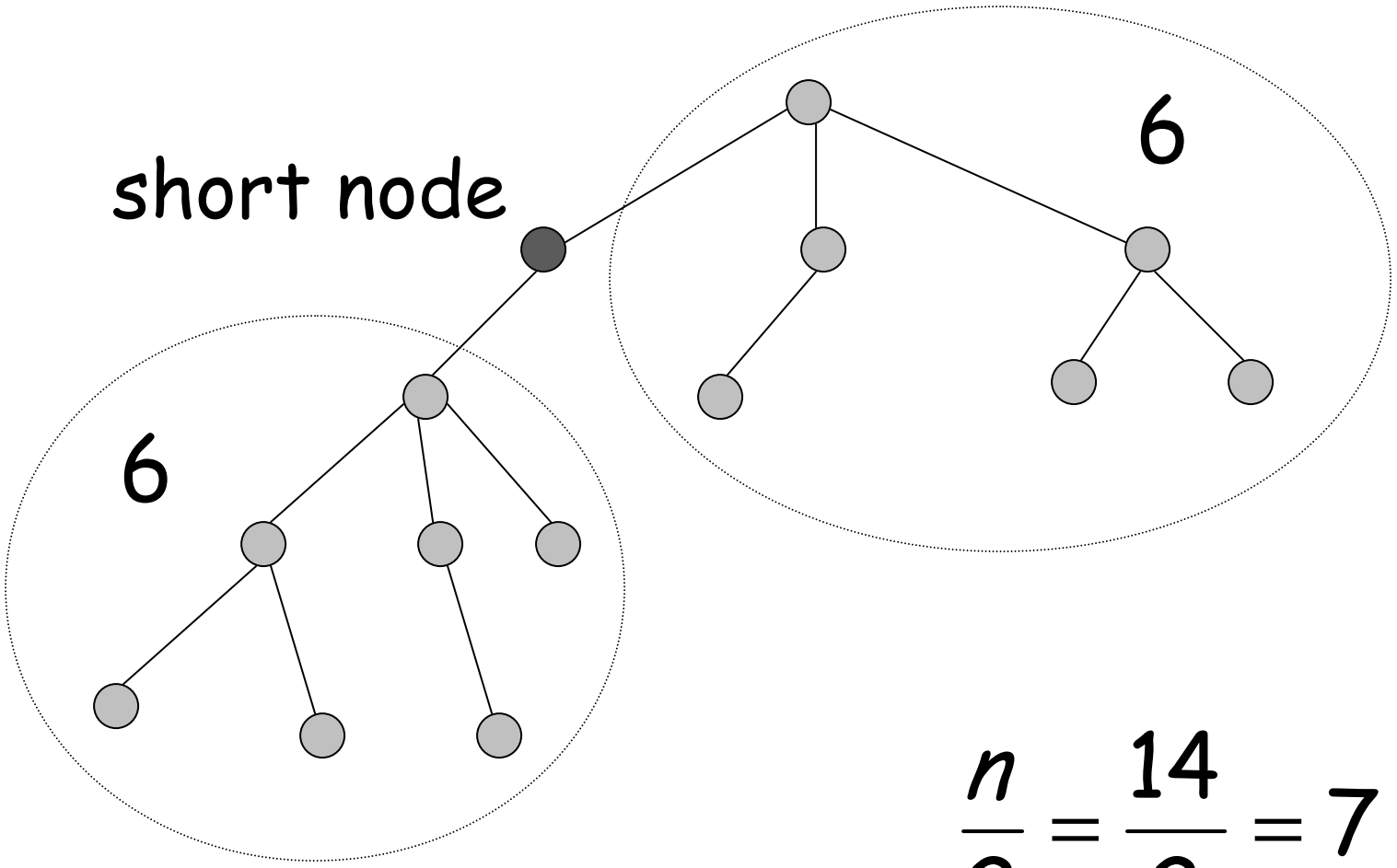
2. At each phase send packets to their destinations greedily

# Short Node:




every subtree has at most $\frac{n}{2}$ nodes

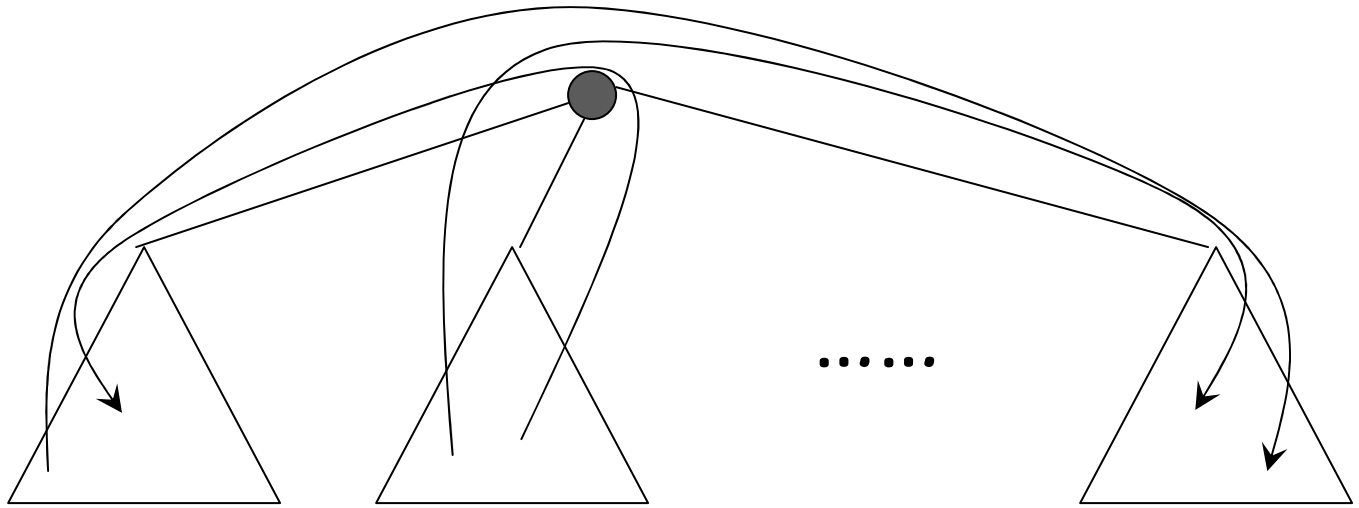
# Example




$$\frac{n}{2} = \frac{14}{2} = 7$$

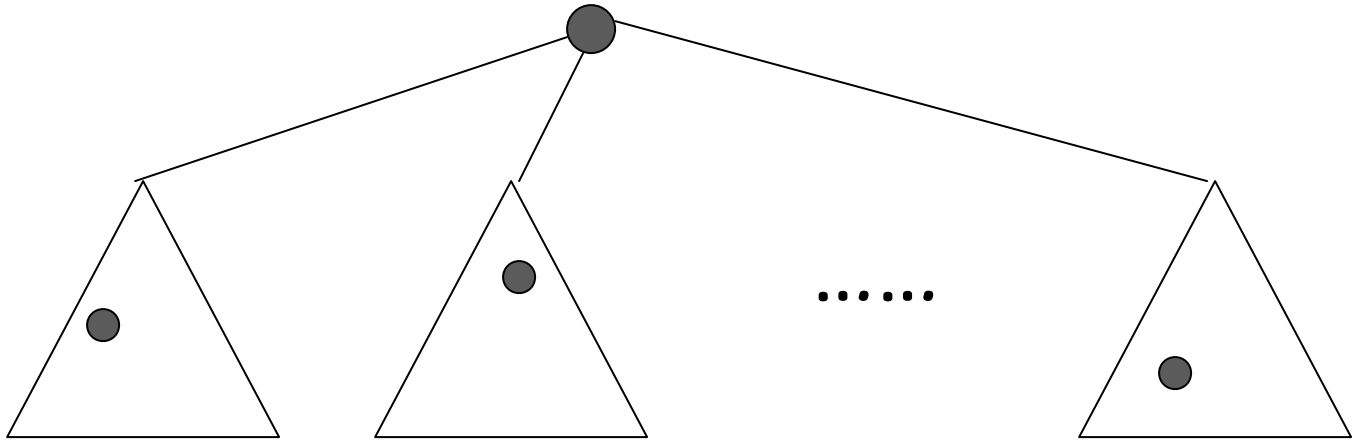
# Phase 1:

## Route packets that cross the short node



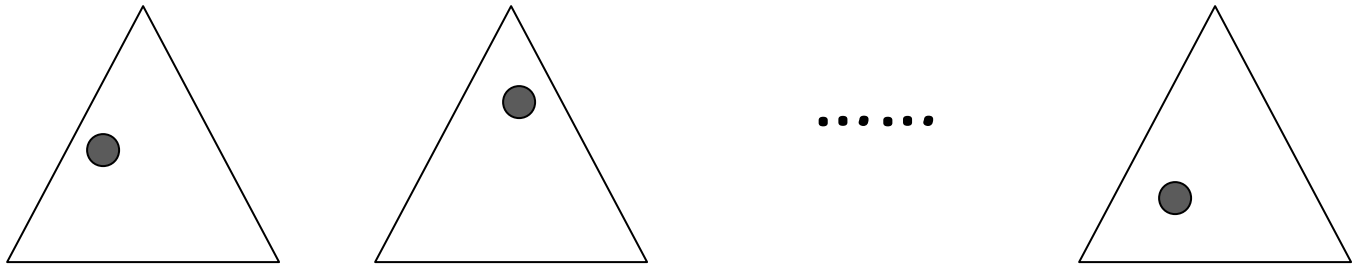
# Phase 2:

## In each subtree get the short nodes



# Phase 2:

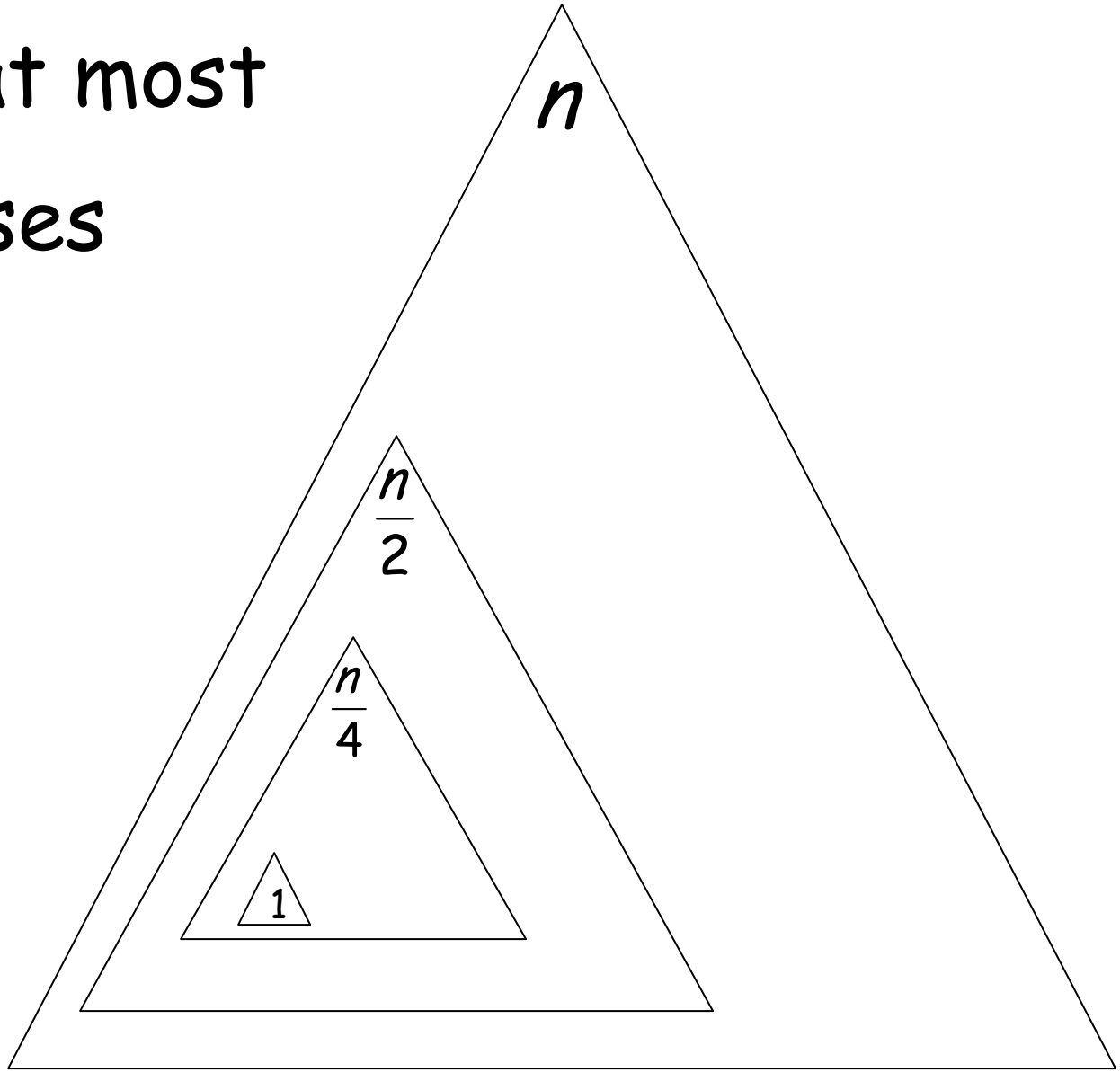
## In each subtree get the short nodes



# Phase 2:
# Route packets that cross the short nodes

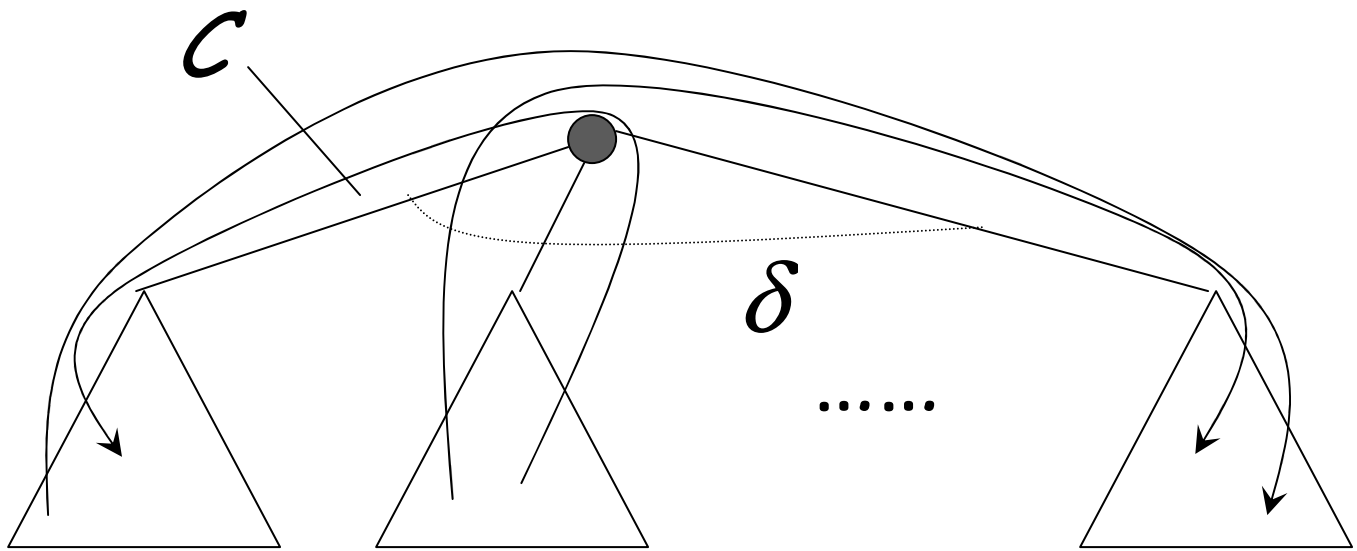
There are at most

$\log n$ phases



# Phase k:

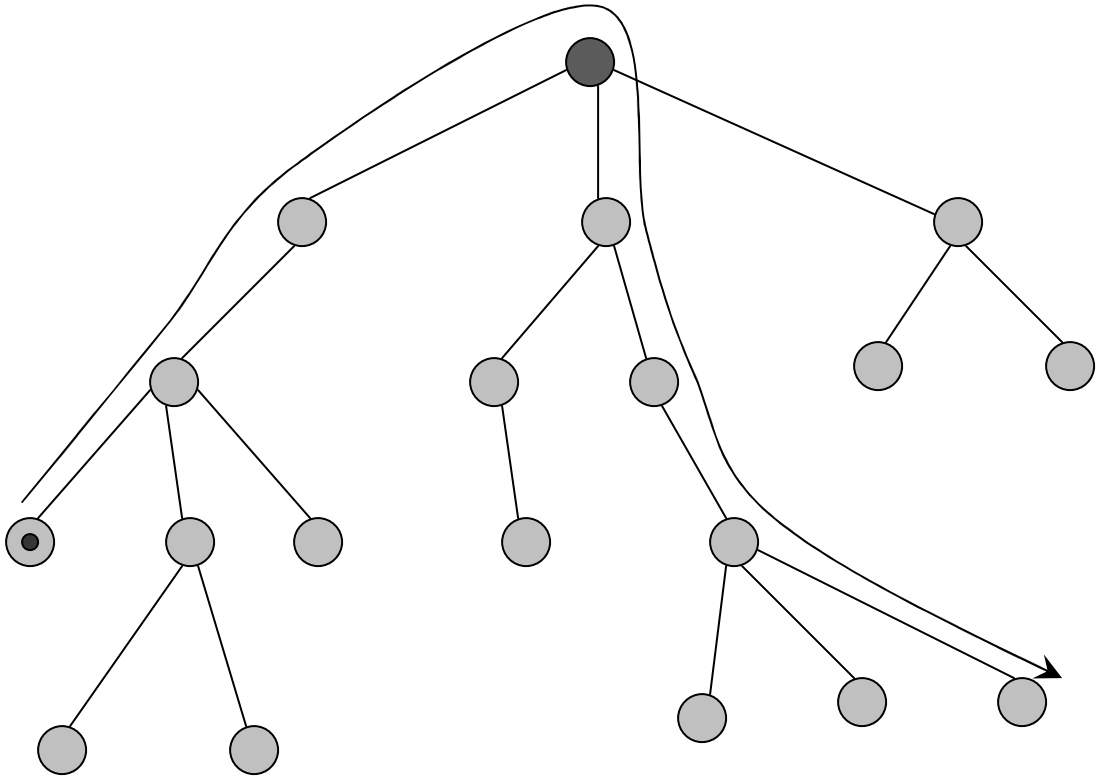
Route packets that cross the short node




Bound on number of packets: $\delta C$

# Phase k:

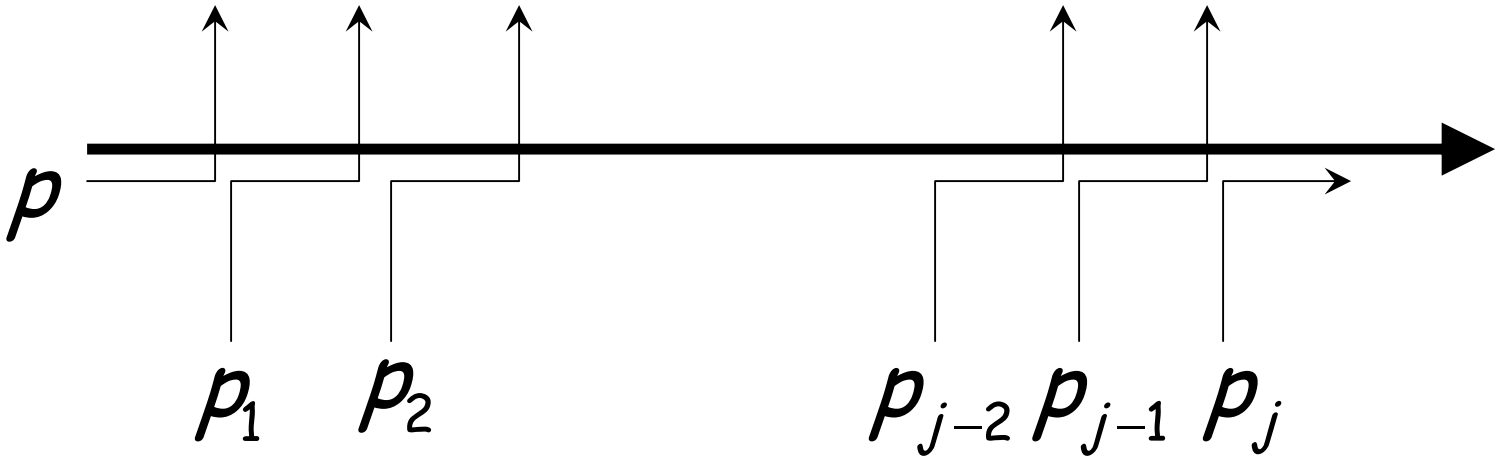
## A packet follows its path greedily



# However, packets can conflict and get deflected

# Deflection Sequence

[Borodin, Rabani, Schieber 1997]




If a packet $p$ is deflected then some other packet $p_j$ reaches its destination

Since there are at most $\delta C$ packets, there are at most $\delta C$ deflections

Worst Routing Time for a packet:

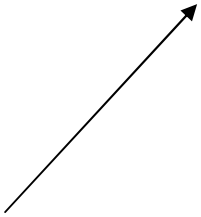
$$2\delta C + D$$

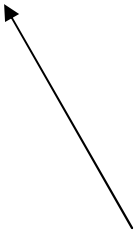
deflections

Initial distance

# Total Routing Time

$$(2\delta C + D) \cdot \log n$$





Packet time
In a phase

Number
of Phases

# Presentation Outiline

- Deterministic Algorithm
- Randomized Algorithm

# Randomized Algorithm

Same with deterministic algorithm,
with only difference:

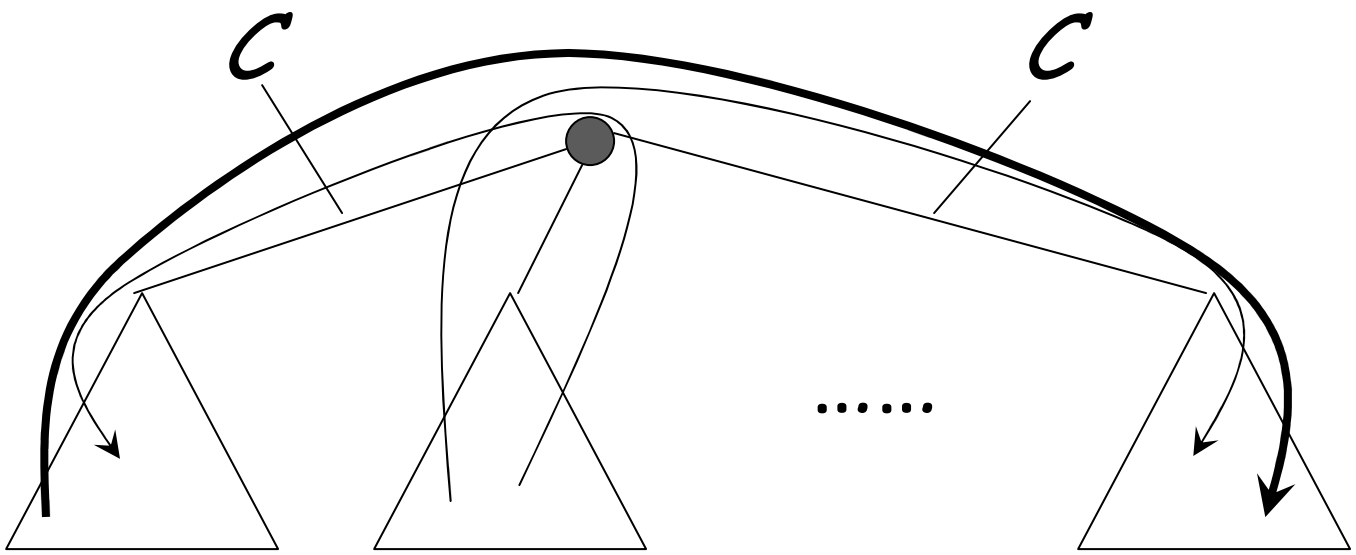
Packet conflicts are resolved
according to random packet priorities

# Packet Priorities:

Low: each packet starts with a low priority

High: when a packet is deflected it increases its priority with probability $\frac{1}{4(C+D)}$

A high priority packet can conflict with at most $2C$ packets



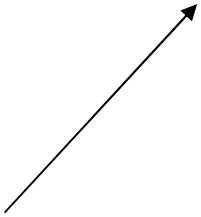

From those packets, $O(1)$ are expected to be in high priority

A high priority packet successfully reaches its destination with probability $\frac{1}{2}$

Thus $O(\log n)$ attempts to become high priority are enough.

# Total Routing Time

$$\kappa(C + D)\log n \cdot \log n$$



Packet time
In a phase

Number
of Phases

# Discussion

We presented two near-optimal hot-potato algorithms for trees

(within logarithmic factors from optimal)

Open problem:

Remove the logarithmic factors