Efficient Bufferless Routing on Leveled Networks

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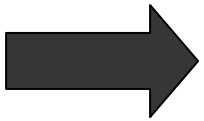
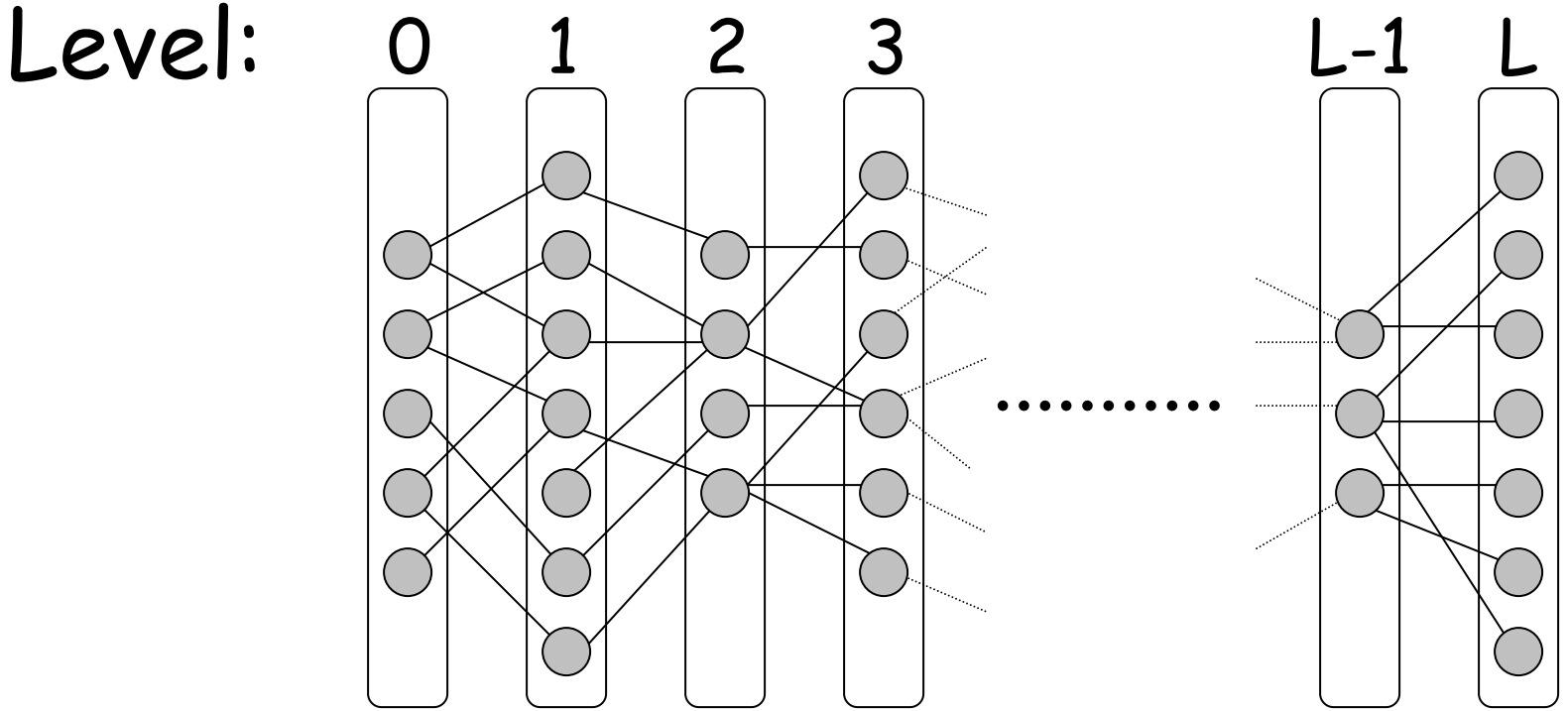
Talk Outline

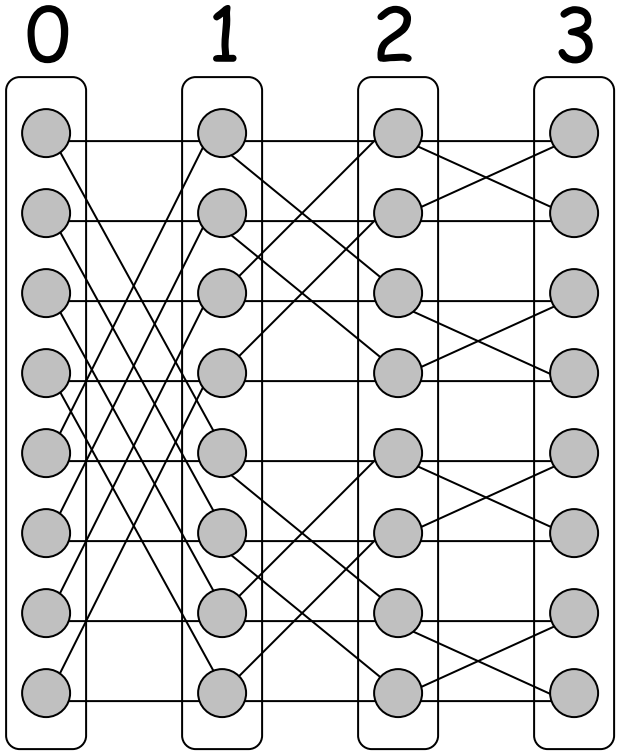


- Introduction
- Centralized Algorithm
- Distributed Algorithm
- Conclusion

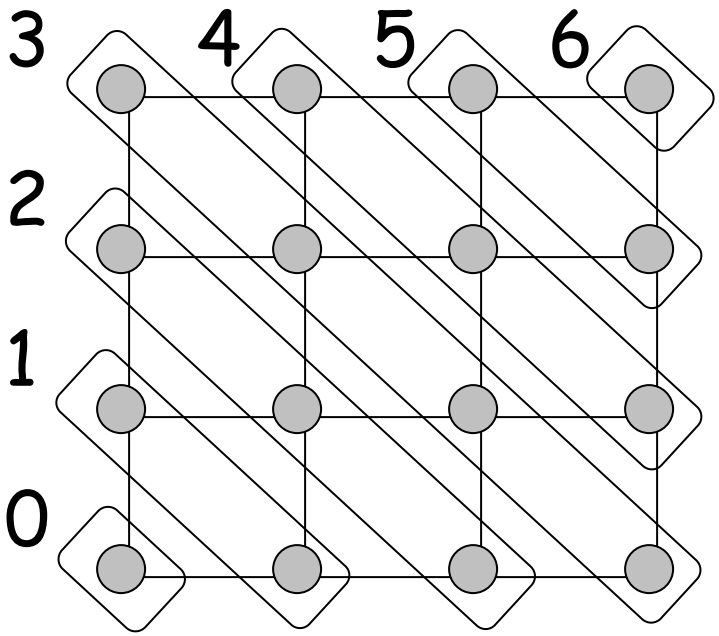
Leveled Networks



Examples of Leveled Networks



Butterfly

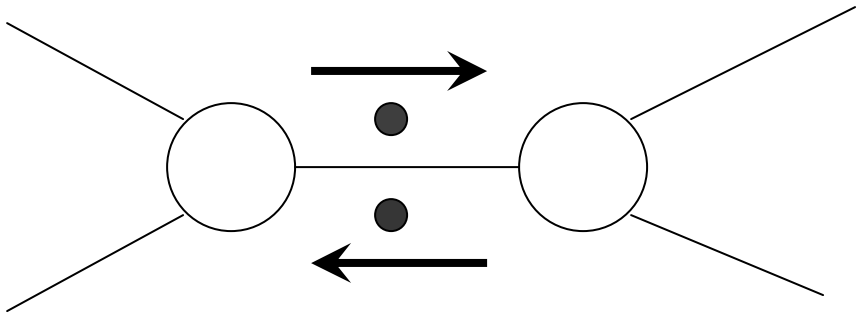


Mesh

Network Model

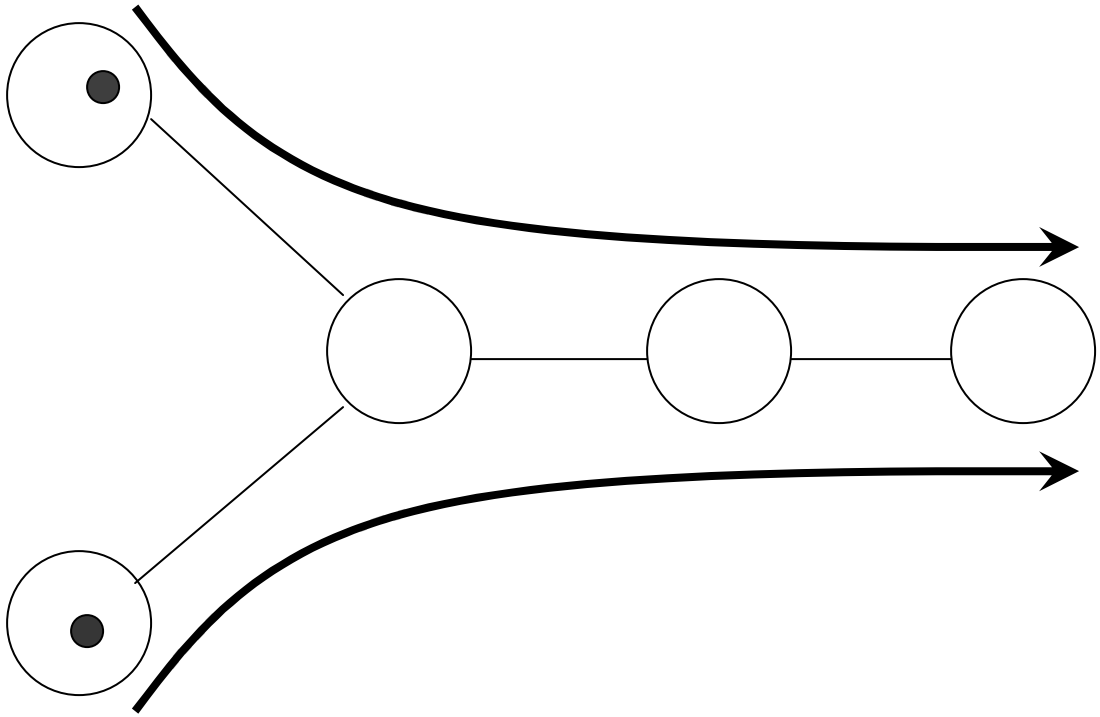
- Synchronous network (time steps)
- Bi-directional links

One packet per direction, per time step



Buffer-less nodes

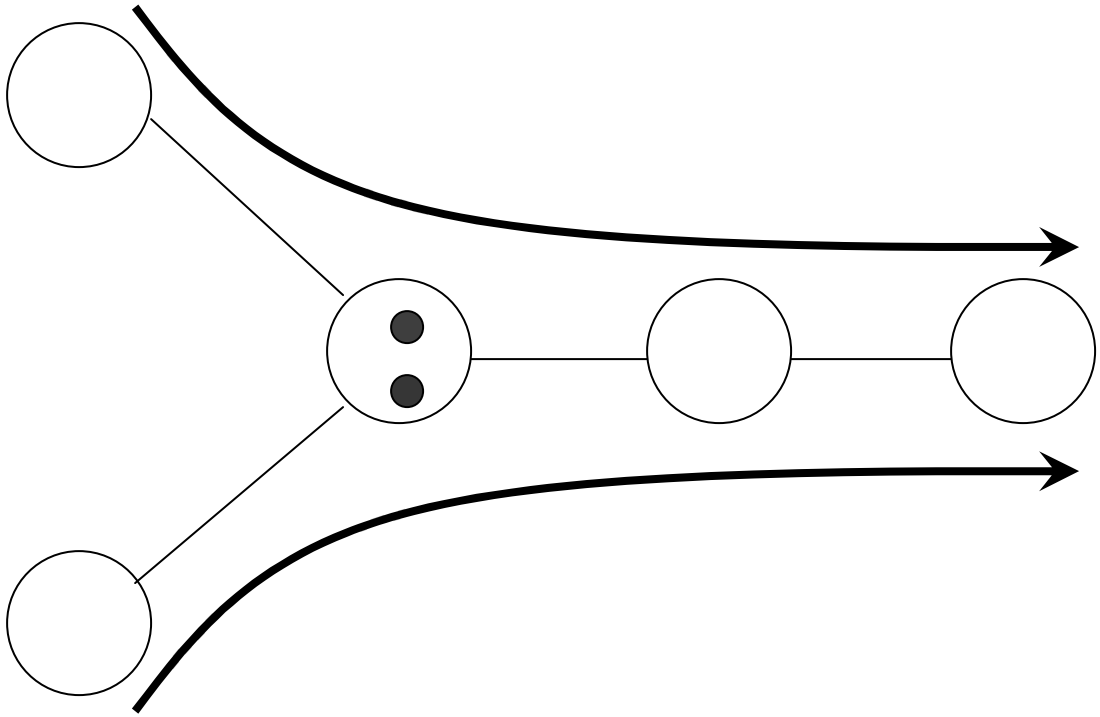
Time 0



Packets are always moving

Buffer-less nodes

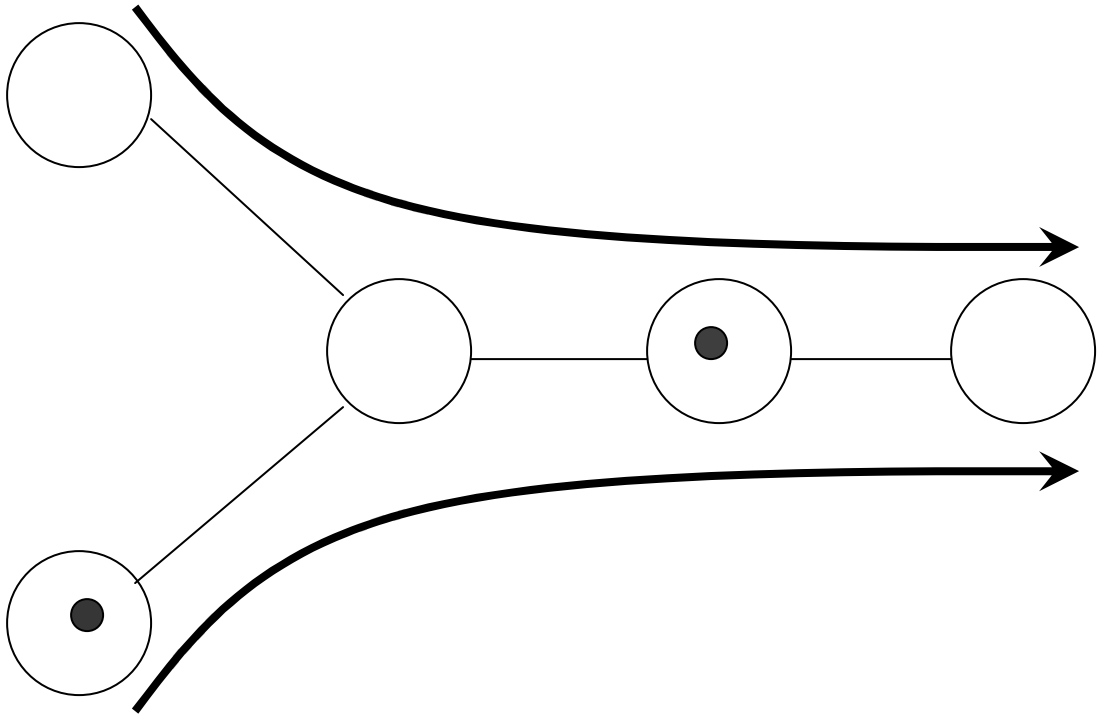
Time 1



Packets are always moving

Buffer-less nodes

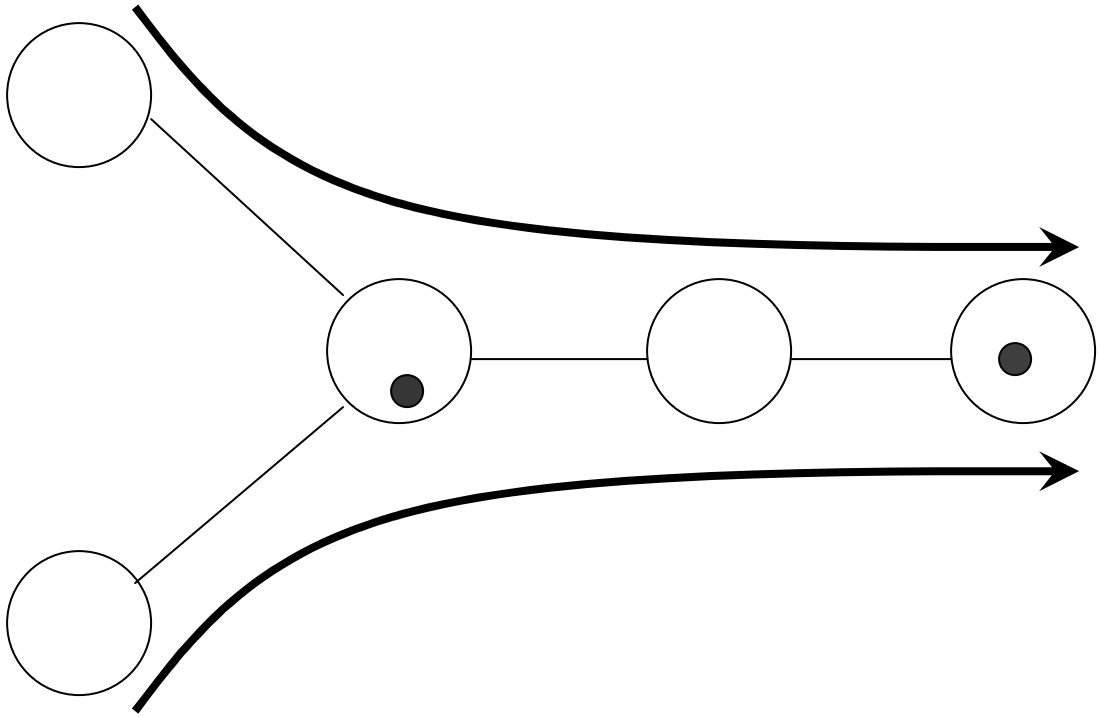
Time 2



Packets are always moving

Buffer-less nodes

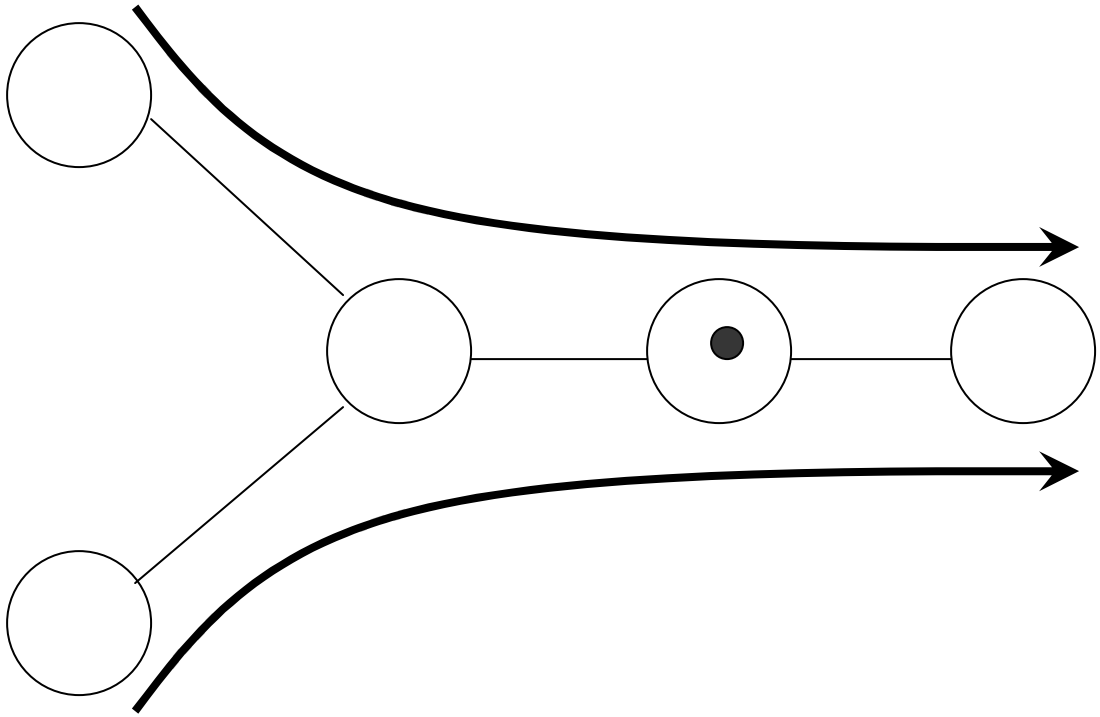
Time 3



Packets are always moving

Buffer-less nodes

Time 4



Packets are always moving

Bufferless routing is interesting:

- Optical networks
- Simple hardware implementations
- Works well in practice:

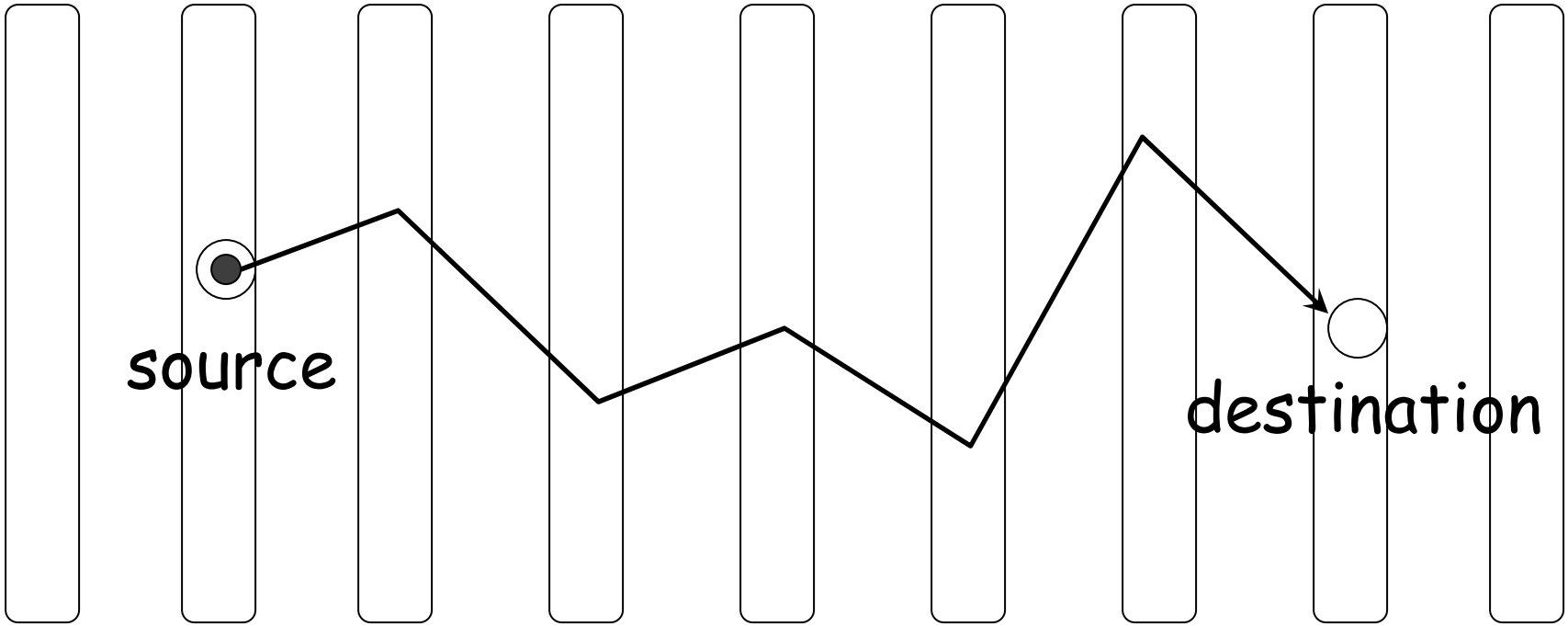
  Bartzis *et al.*: *EUROPAR 2000*

  Maxemchuck: *INFOCOM 1989*

Routing Time: the time until the last packet is absorbed

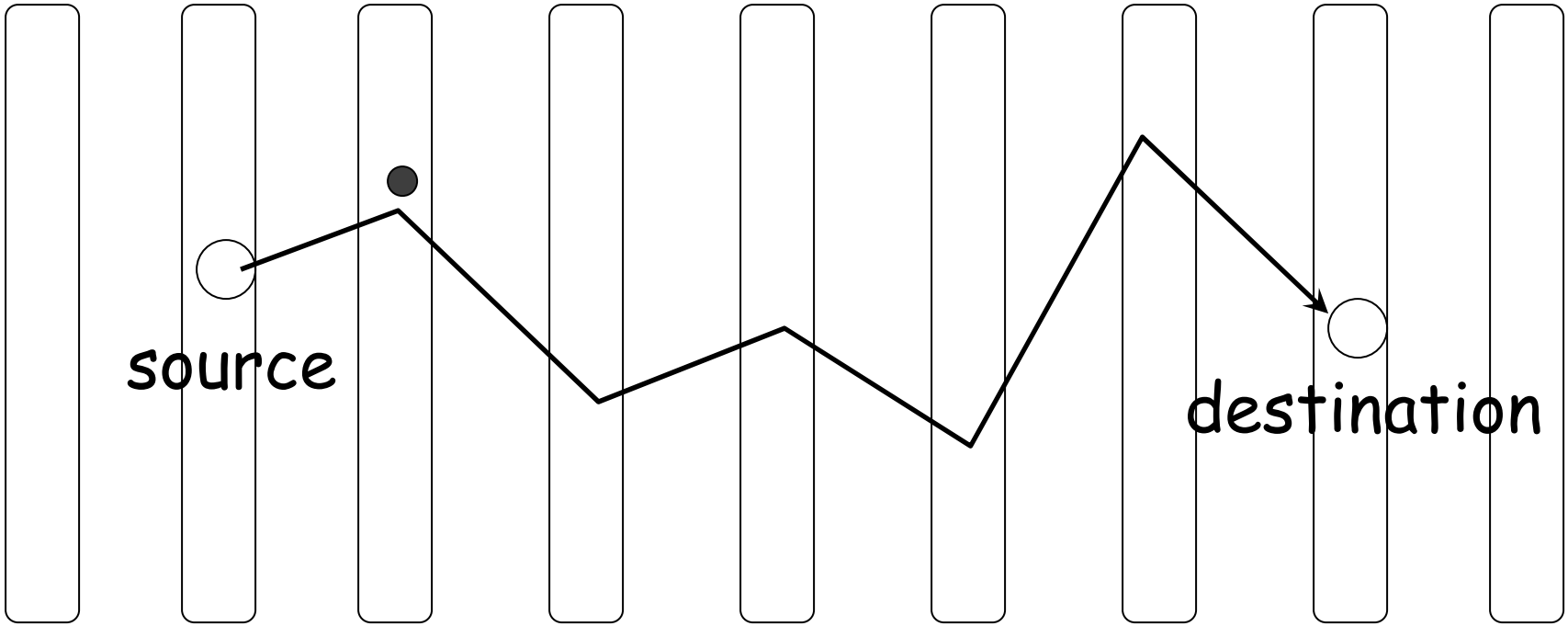
Objective: Minimize Routing Time

Each packet has a pre-selected path

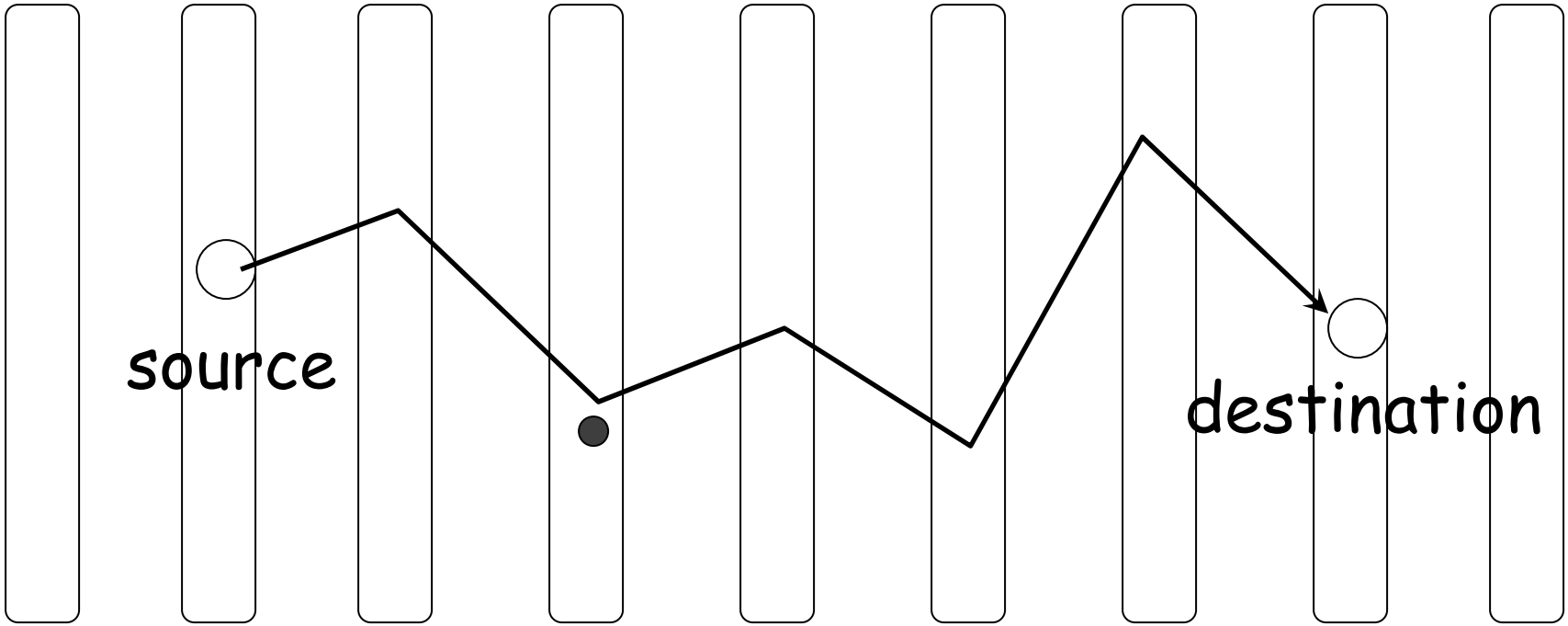


Packet path is from left to right

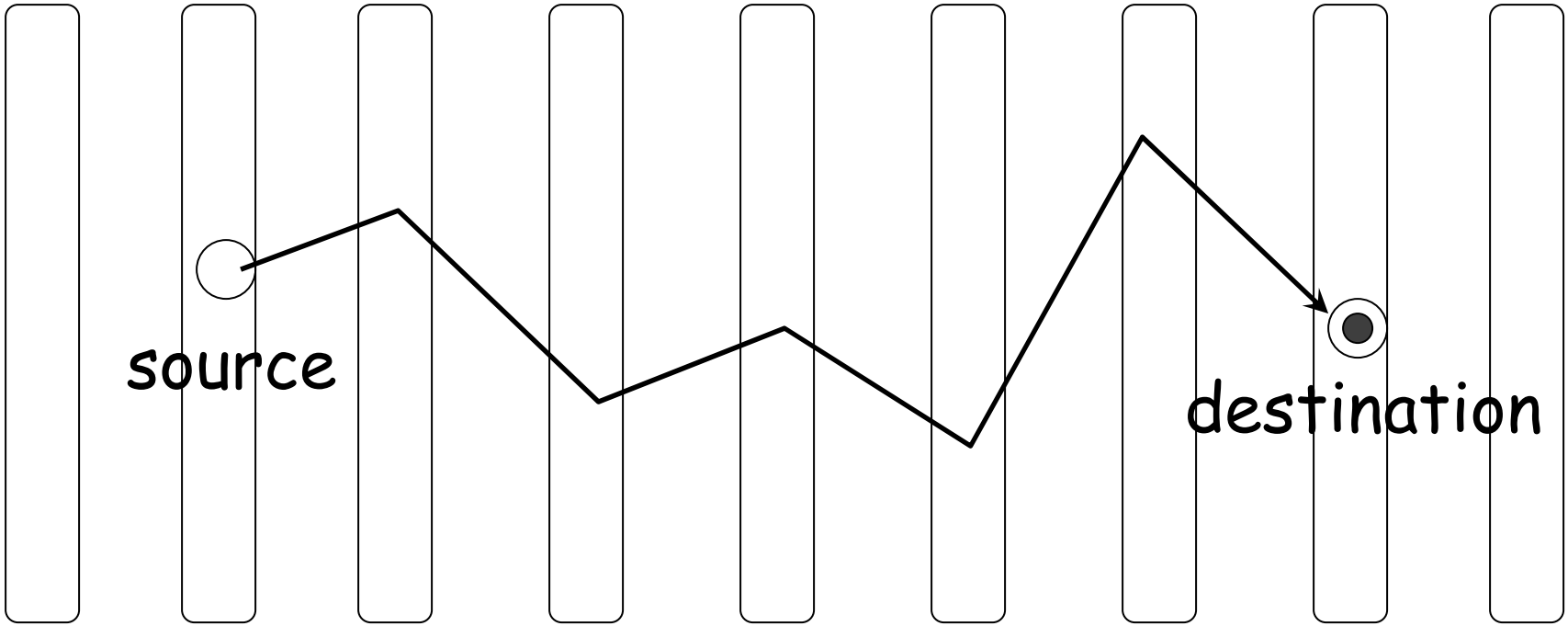
The packet follows the pre-selected path



The packet follows the pre-selected path

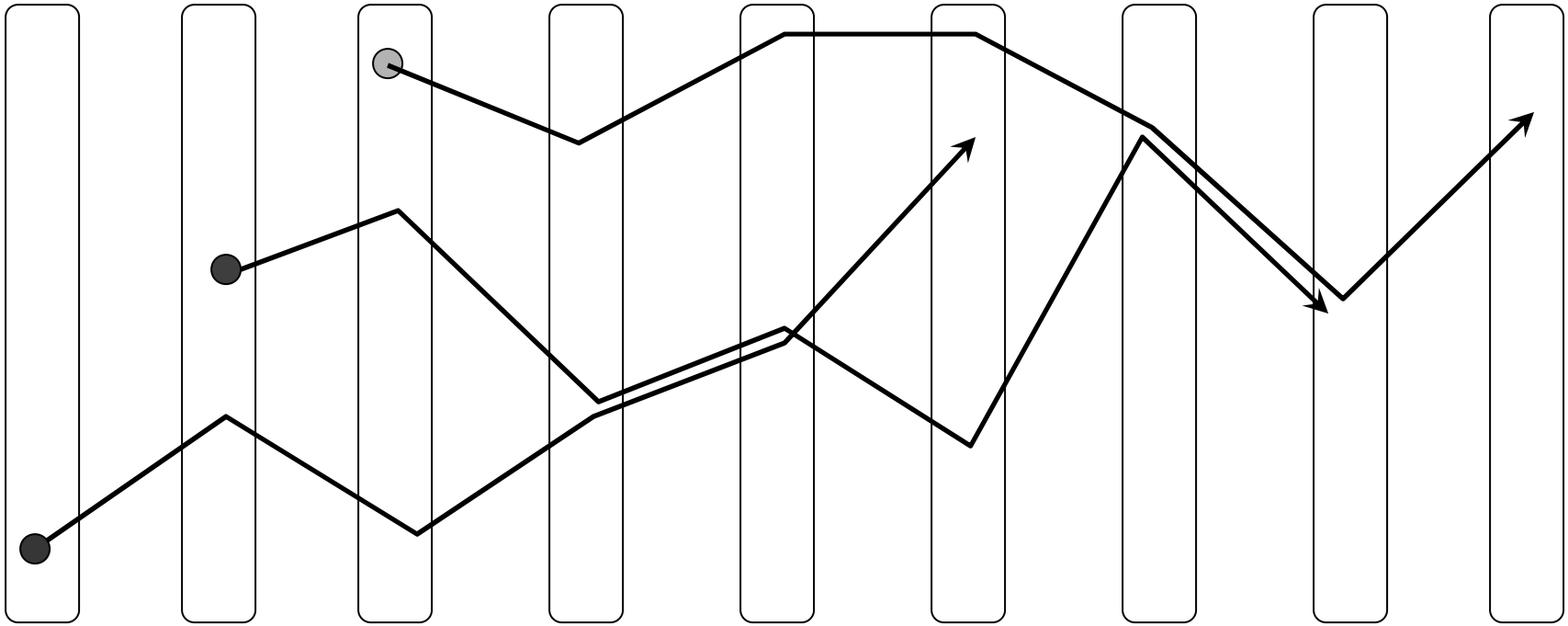


The packet follows the pre-selected path



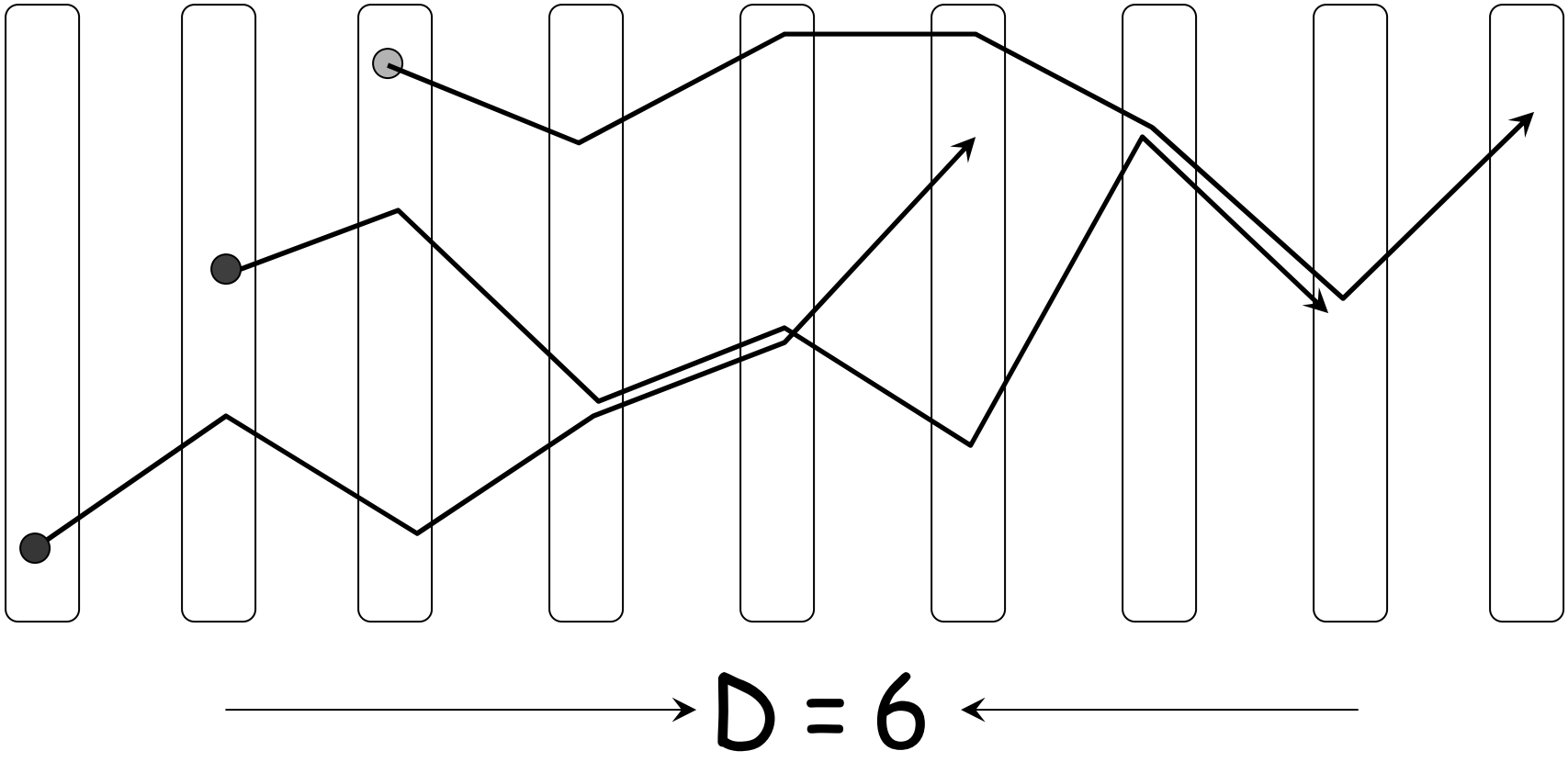
There are N packets

Each packet has its own path



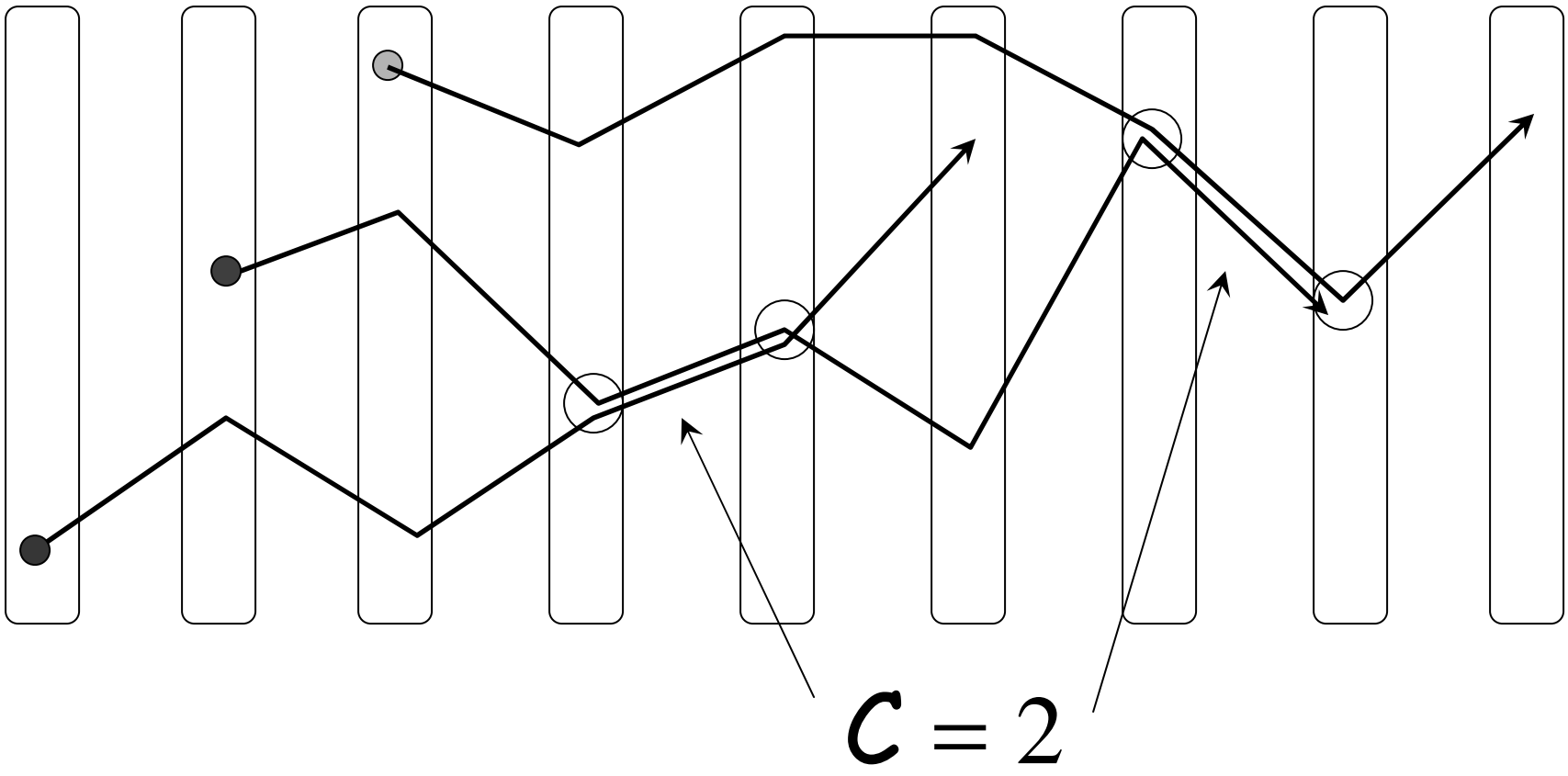
N = 3

Dilation D: The maximum length of any path



Routing time: $\Omega(D)$

Congestion C: The maximum number of packets traversing any edge



Routing time: $\Omega(C)$

Lower bound on Routing Time:

$$\Omega(C + D)$$

Congestion Dilation

We want algorithms with Routing Time close to:

$$C + D$$

Our Contributions

- Centralized Algorithm:

$$O(C \cdot \log(DN) + D)$$

- Distributed Algorithm:

$$O(C \cdot \log^2(DN) + D \cdot \log(DN))$$

Both algorithms are randomized

Results hold with high probability $\left(1 - \frac{1}{DN}\right)$

N : number of packets

Related Work

Networks with buffers

Leveled networks: $O(C + L + \log N)$

Leighton, Maggs, Ranade, Rao: *J. Algorithms 1992*

Arbitrary networks: $O(C + D)$

[Leighton - Maggs - Rao, Combinatorica 94]

[BS99, LMR99, MV99, OR97, RT96]

Bufferless networks

Mesh	[BRST93, BES97, BHS98, BU96, BHW00]
Hypercube	[BH85, BC95, FR92, H91]
Trees	[BMMW04, RSW00, BMMW]
Leveled	[BBPRRS96, B02]

Vertex-symmetric [MS95]

Arbitrary networks [BMM04]

Most related work

Arbitrary networks: $O((C+D)\cdot \log^3(n+N))$

Busch, Magdon-Ismail, Mavronicolas WAOA'04

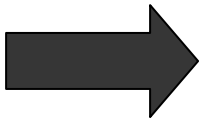
Leveled Networks: $O((C+D)\cdot \log^9(LN))$

Busch TCS'04

Leveled networks in different routing model:

Bhatt, Bilardi, Pucci, Ranade, Rosenberg, Scwabe TC'96

Talk Outline



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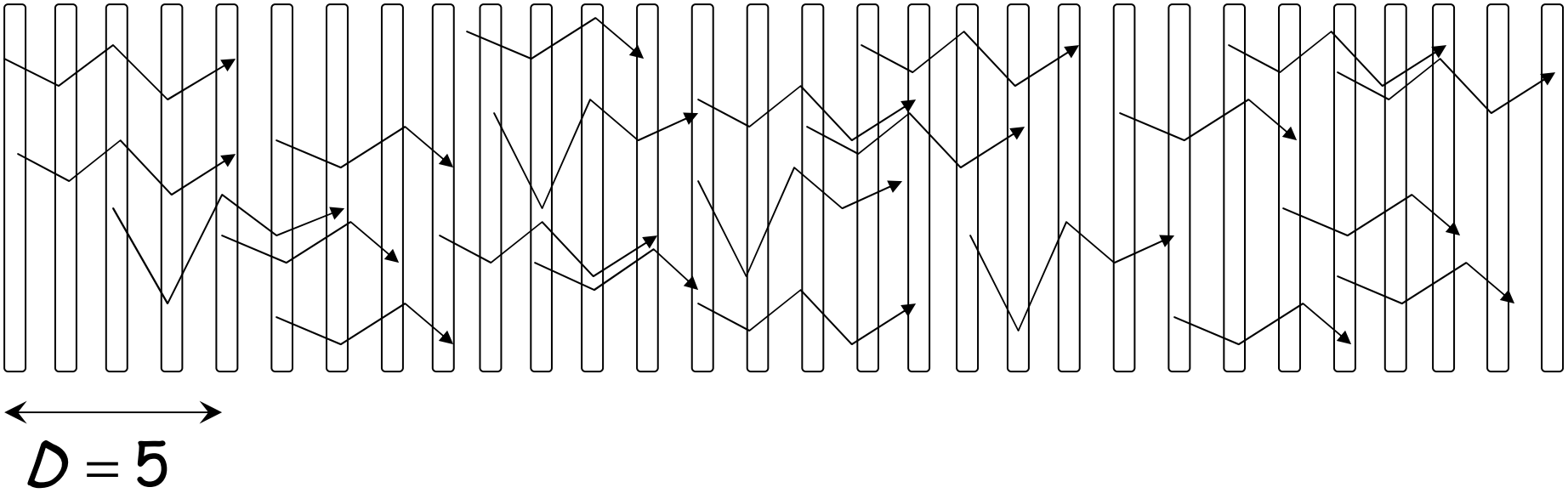
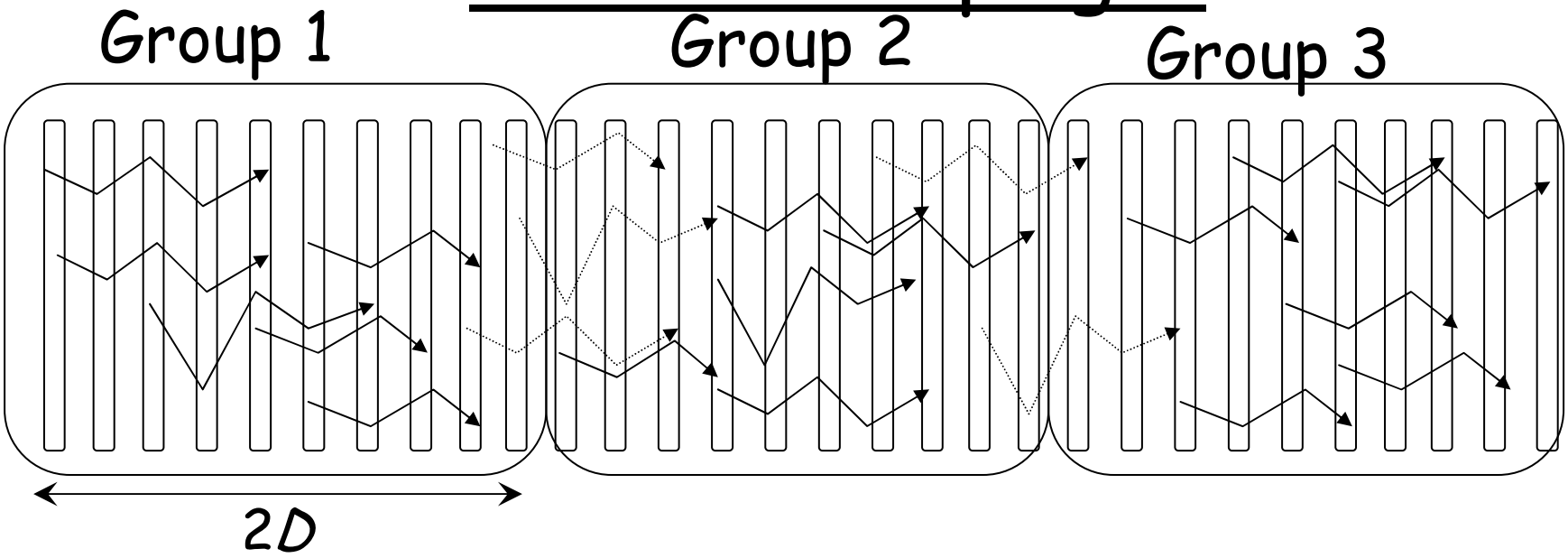
Centralized Algorithm

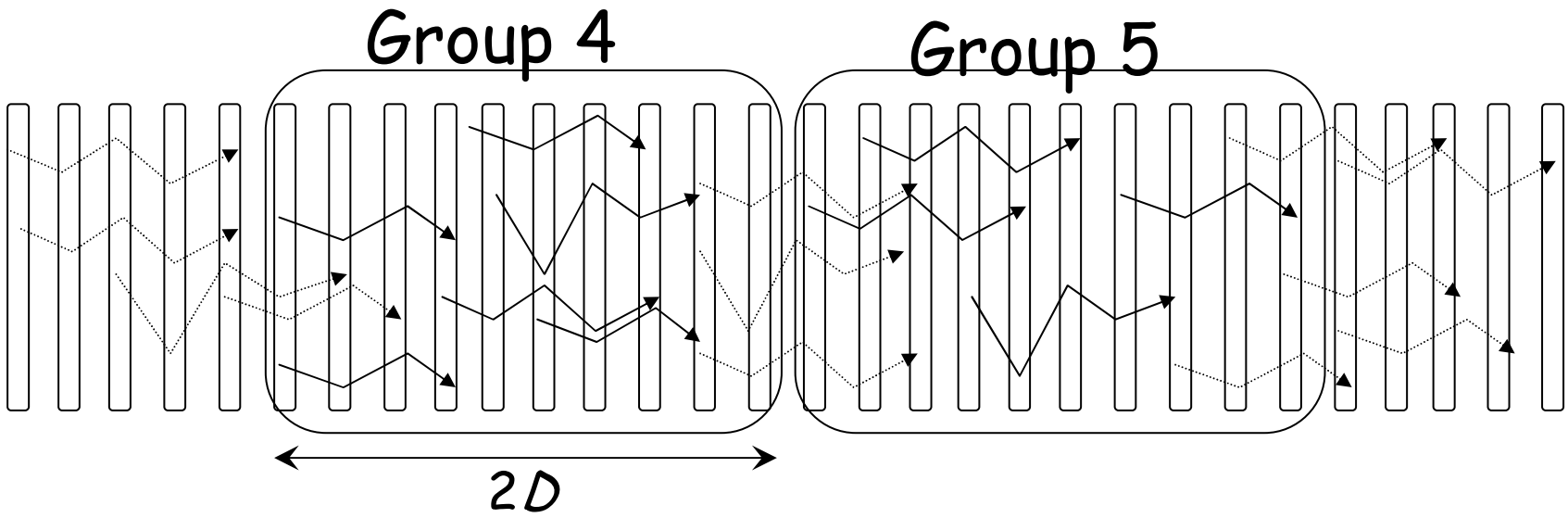
A central node knows all the parameters of the problem and computes a packet schedule


D = 5

Packet Grouping A



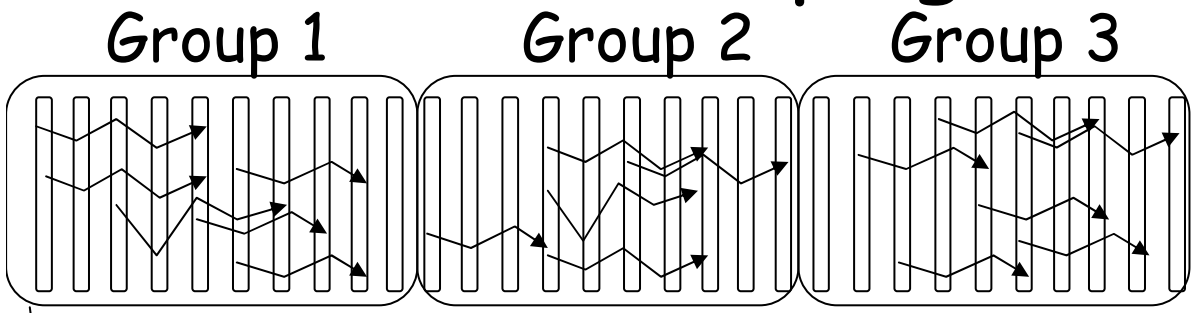
Packet Grouping B



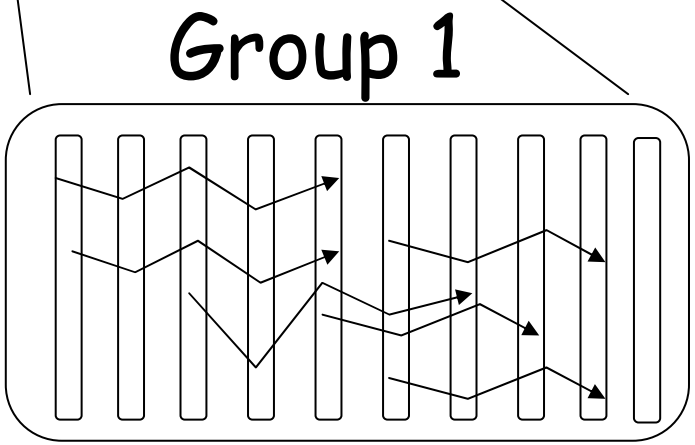
1. Send packets of Grouping A
2. Send packets of Grouping B

Increases routing time by only a factor of 2

Packet Grouping A

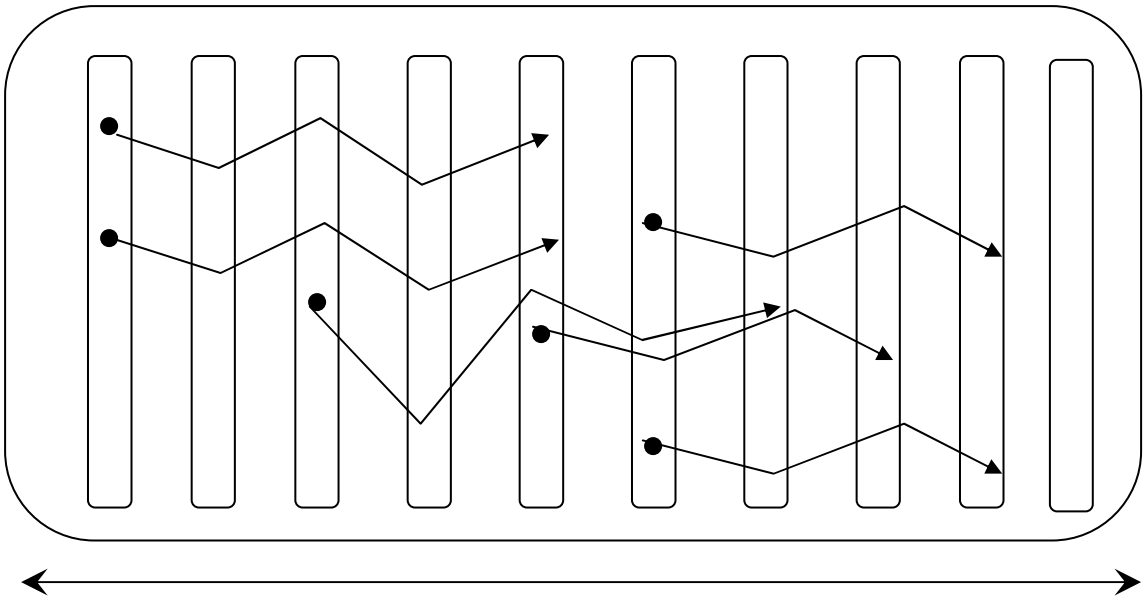


Packets in different groups can be sent simultaneously

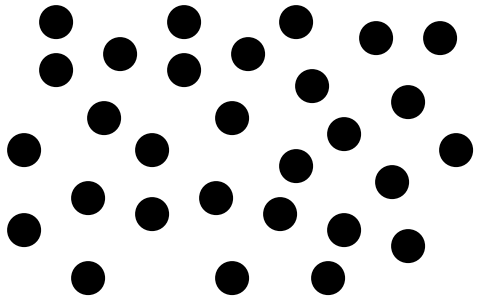


We focus only in one group

Group 1



Π set of packets

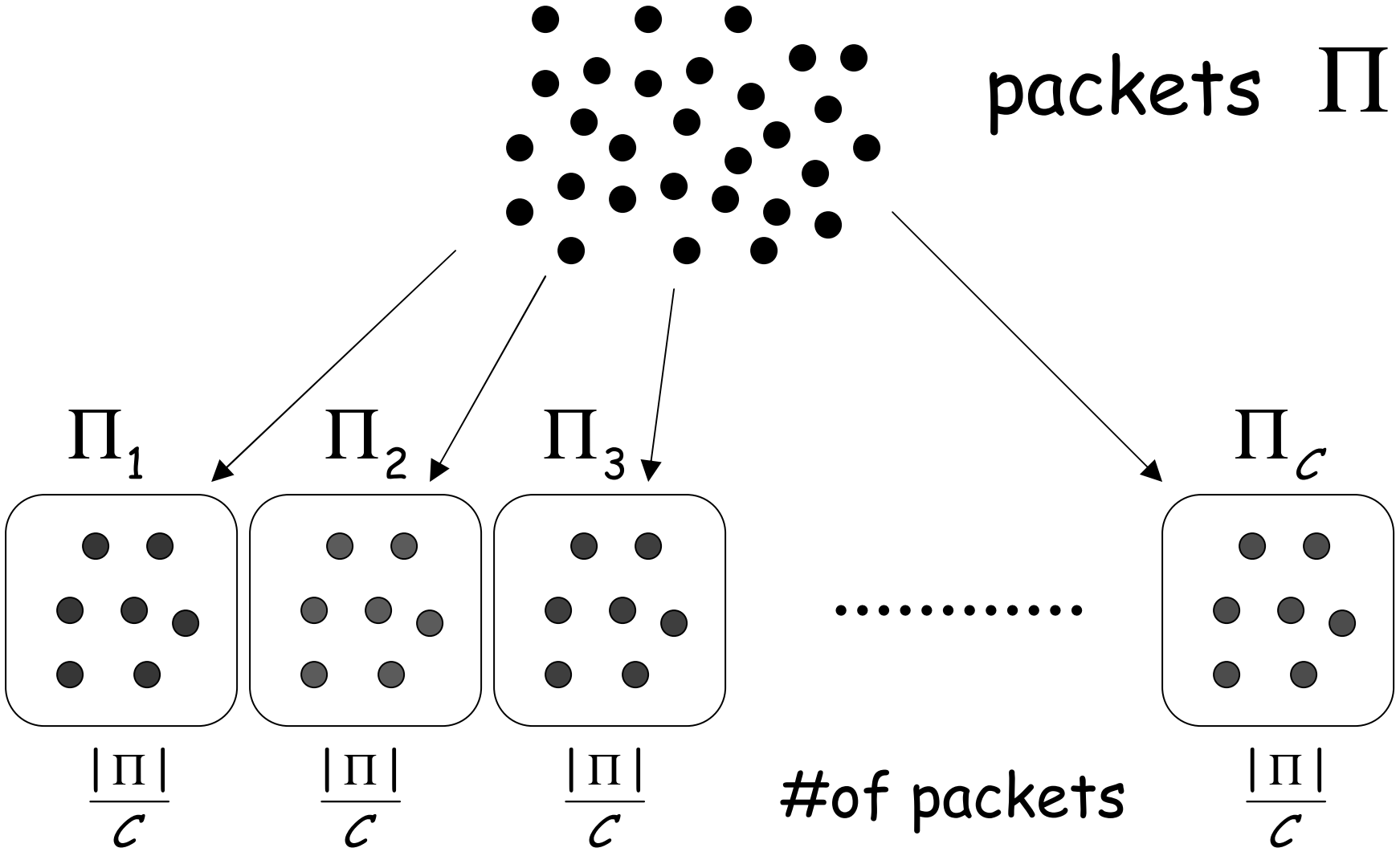


$|\Pi| \leq N$

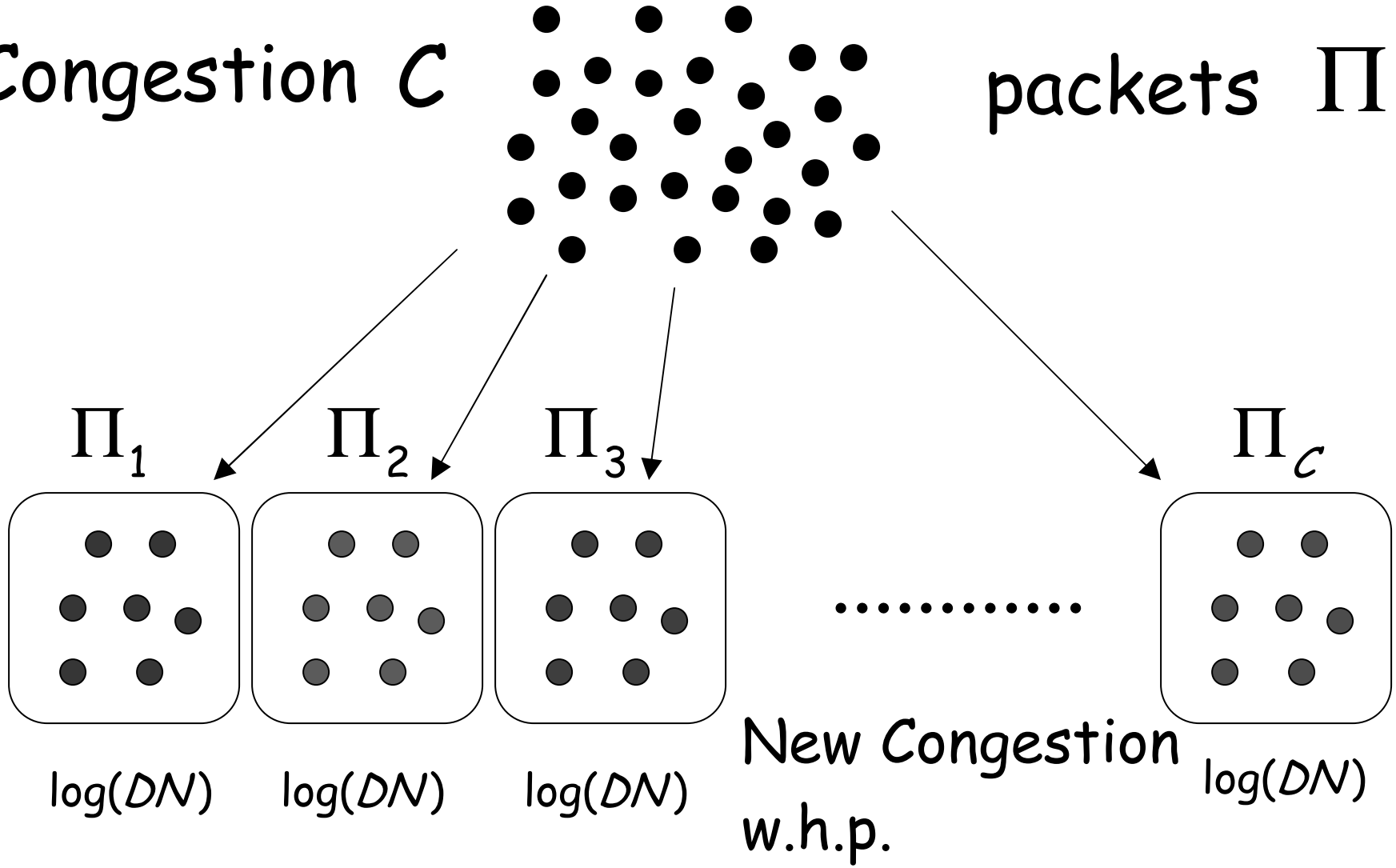
Congestion $\leq C$

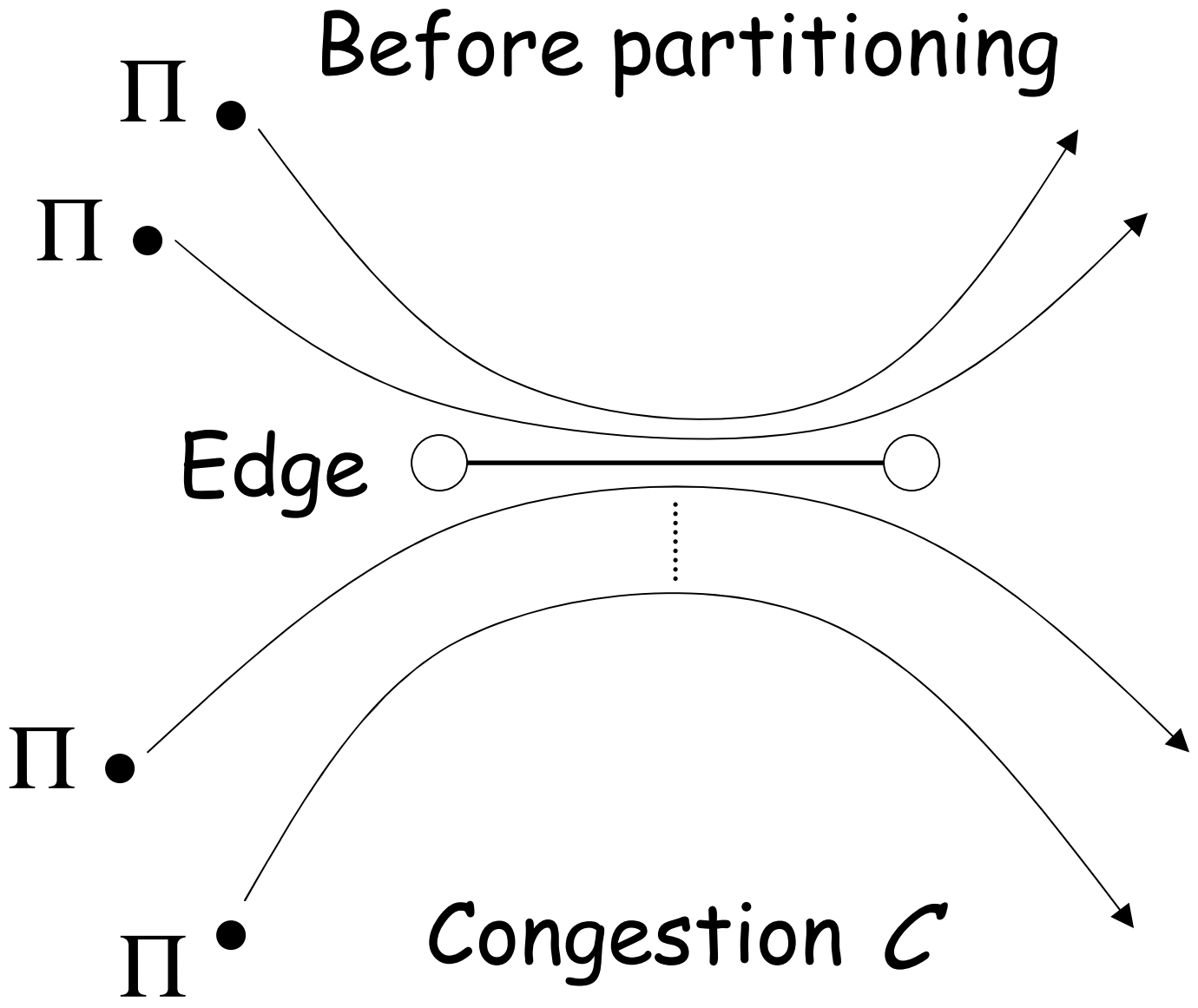
Dilation $\leq D$

Partition the packets randomly and uniformly into C sets

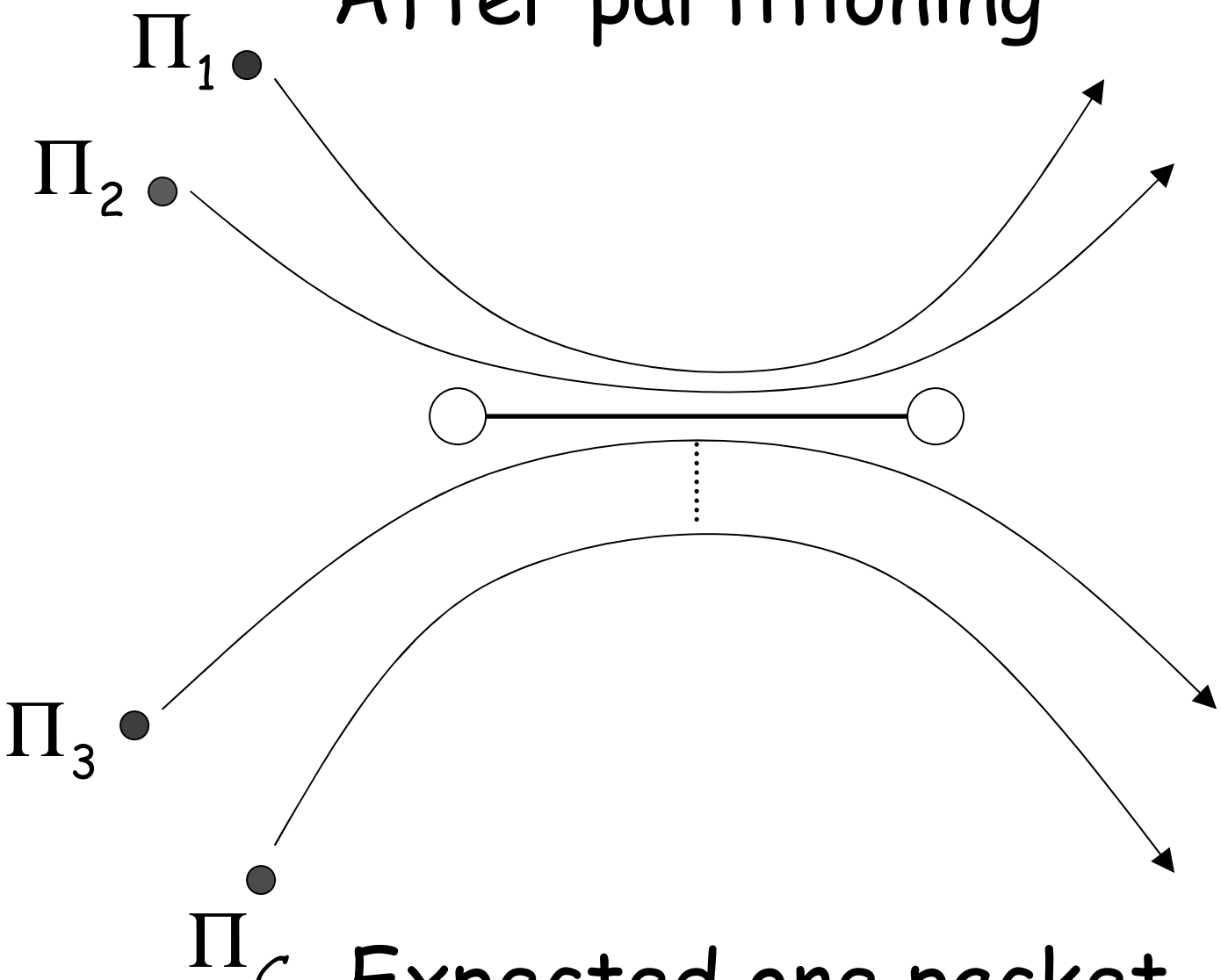


Benefit: congestion drops

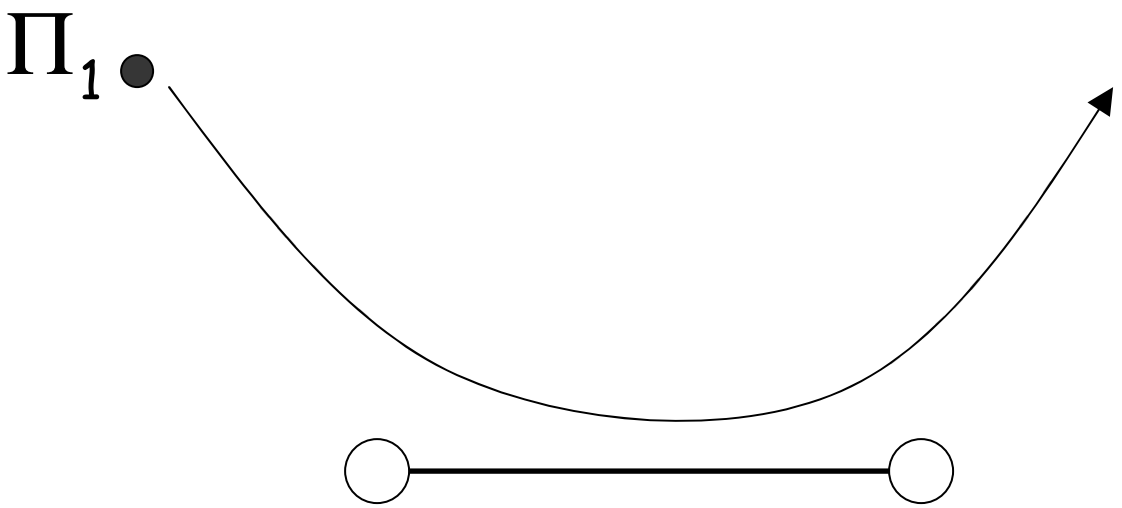



Before partitioning
Π
Π
Edge
Π
Π
Congestion C

After partitioning



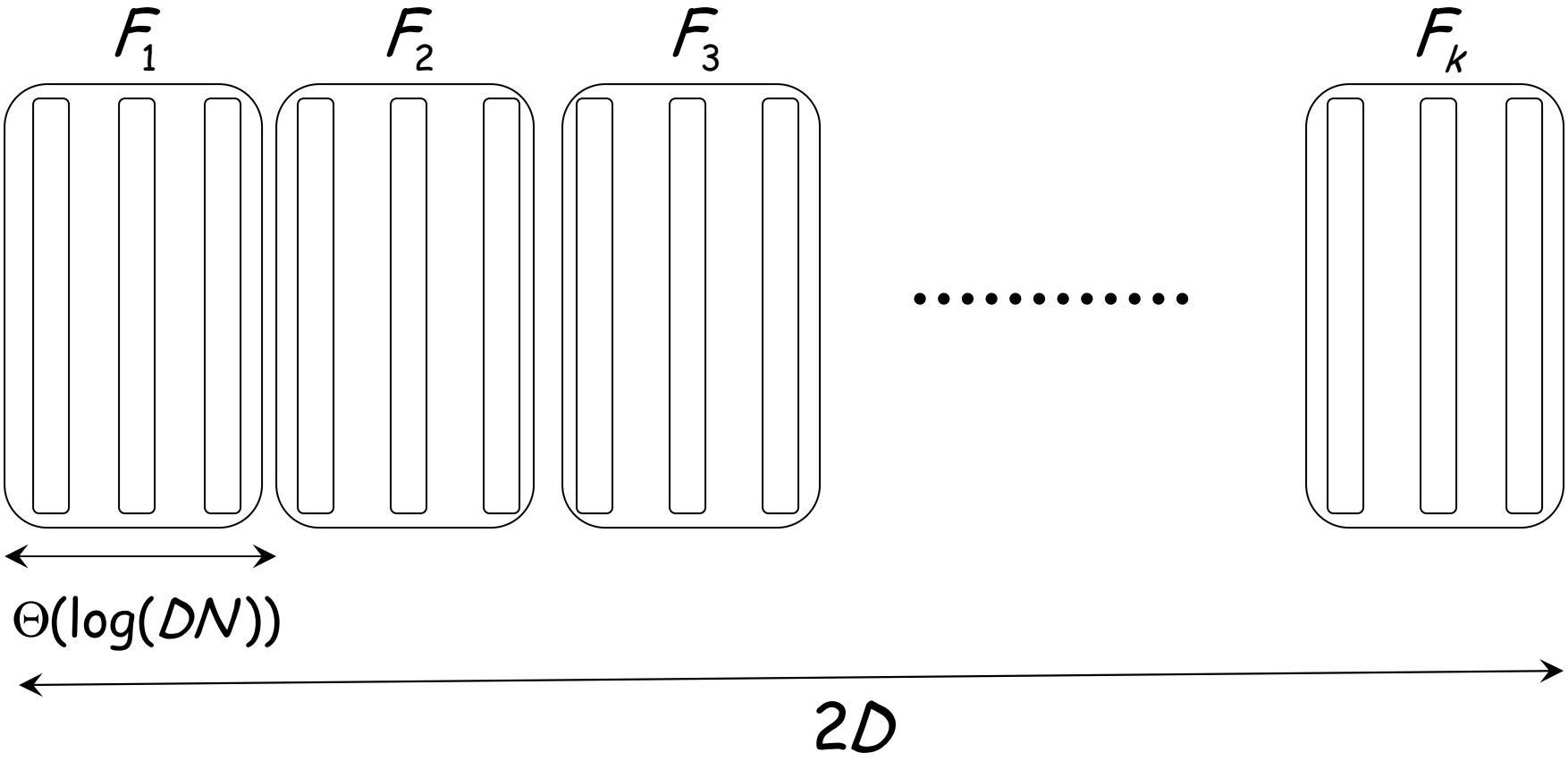
Expected one packet from each packet set



Expected Congestion 1

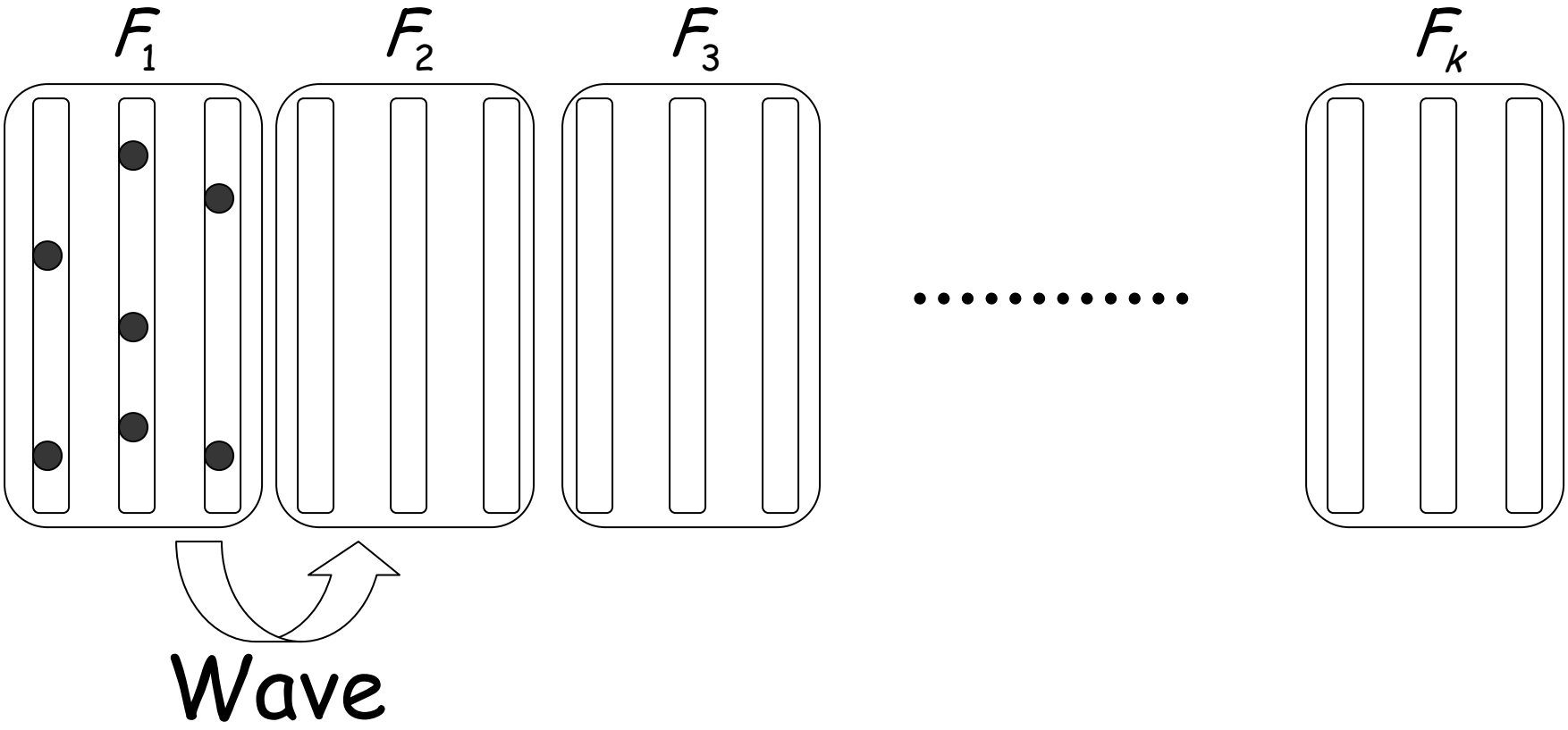
(Congestion $\log(DN)$ w.h.p.)

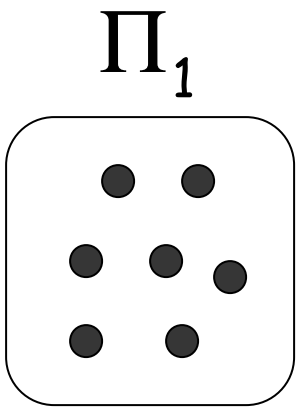
We partition the levels into **frames**



number of frames: $\Theta\left(\frac{D}{\log(DN)}\right)$

We send packets from frame to frame





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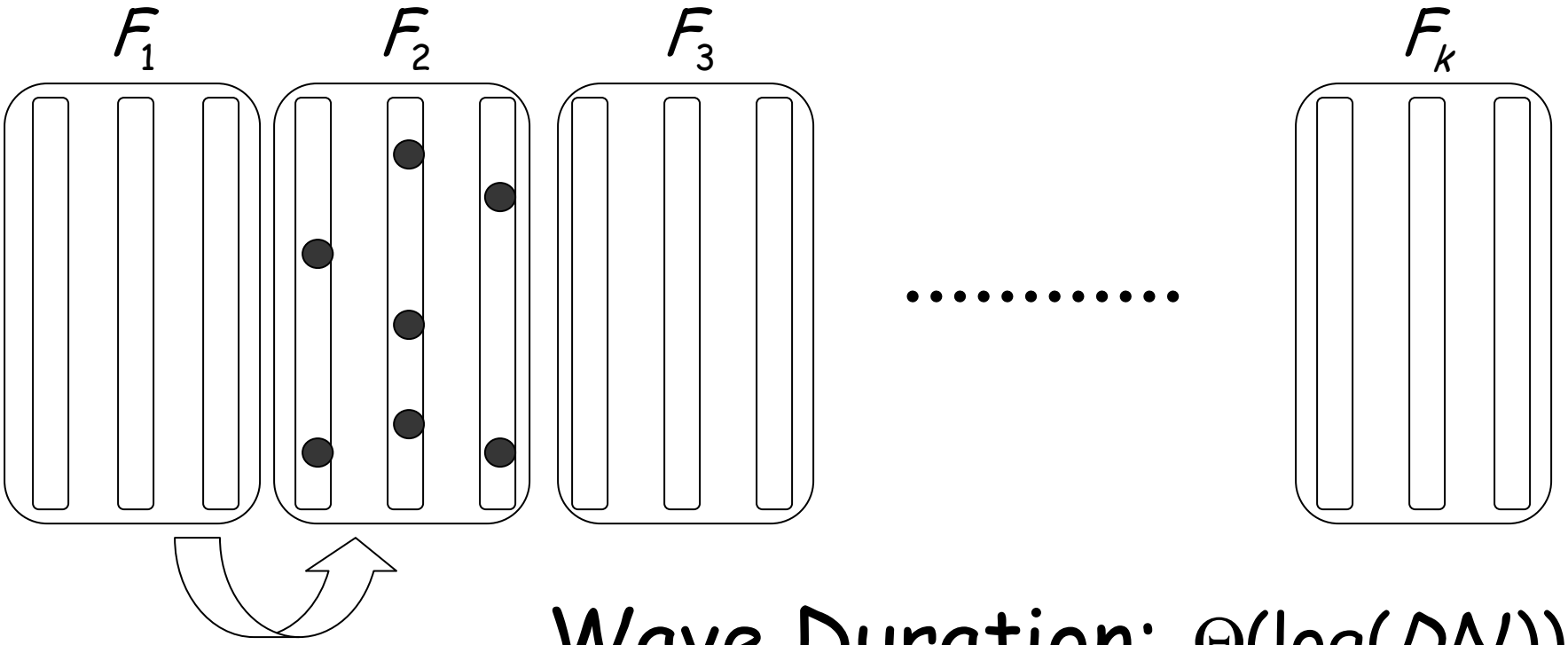
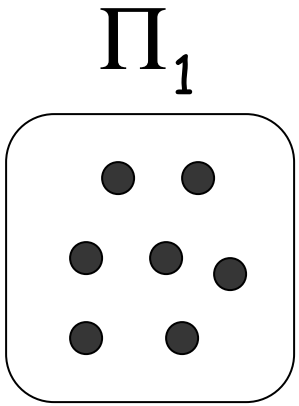


Wave

Wave Duration: $\Theta(\log(DN))$



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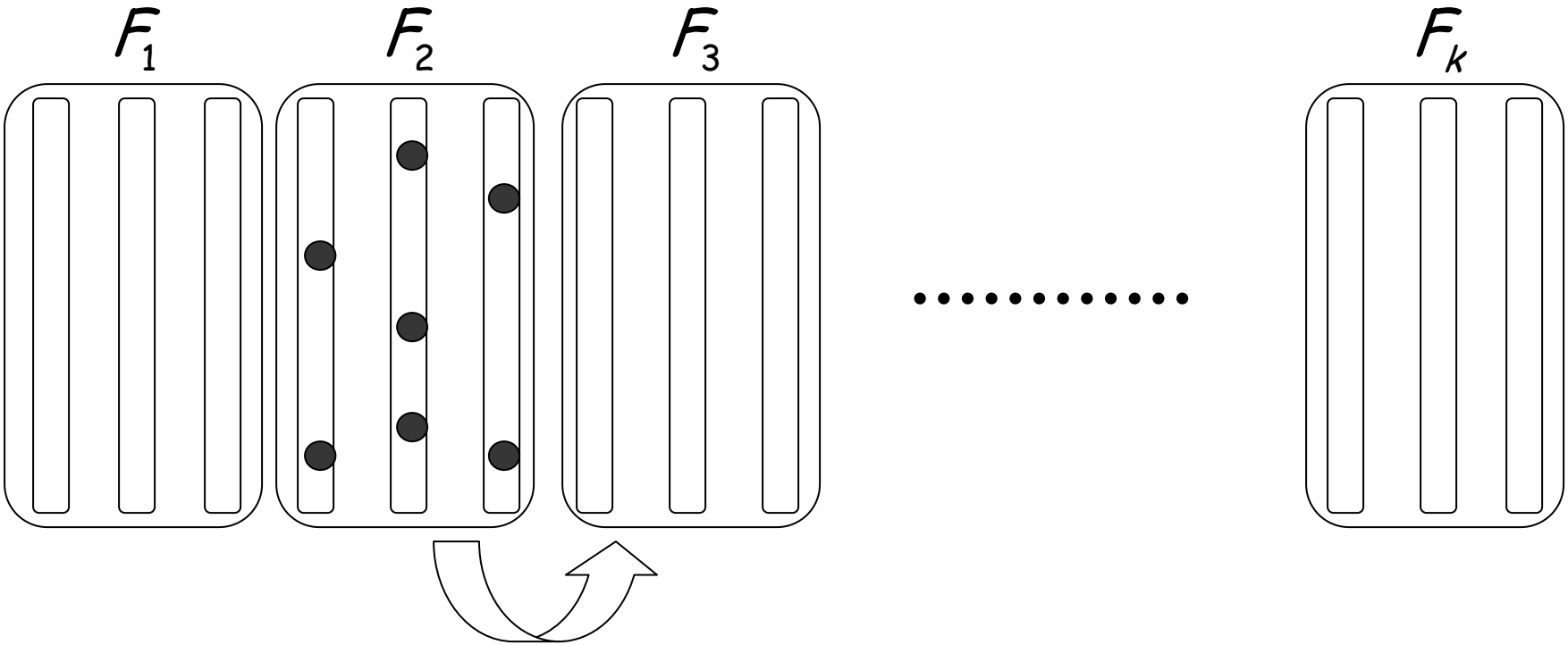

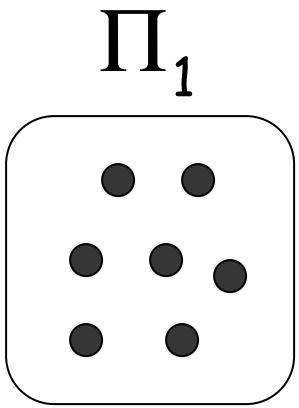

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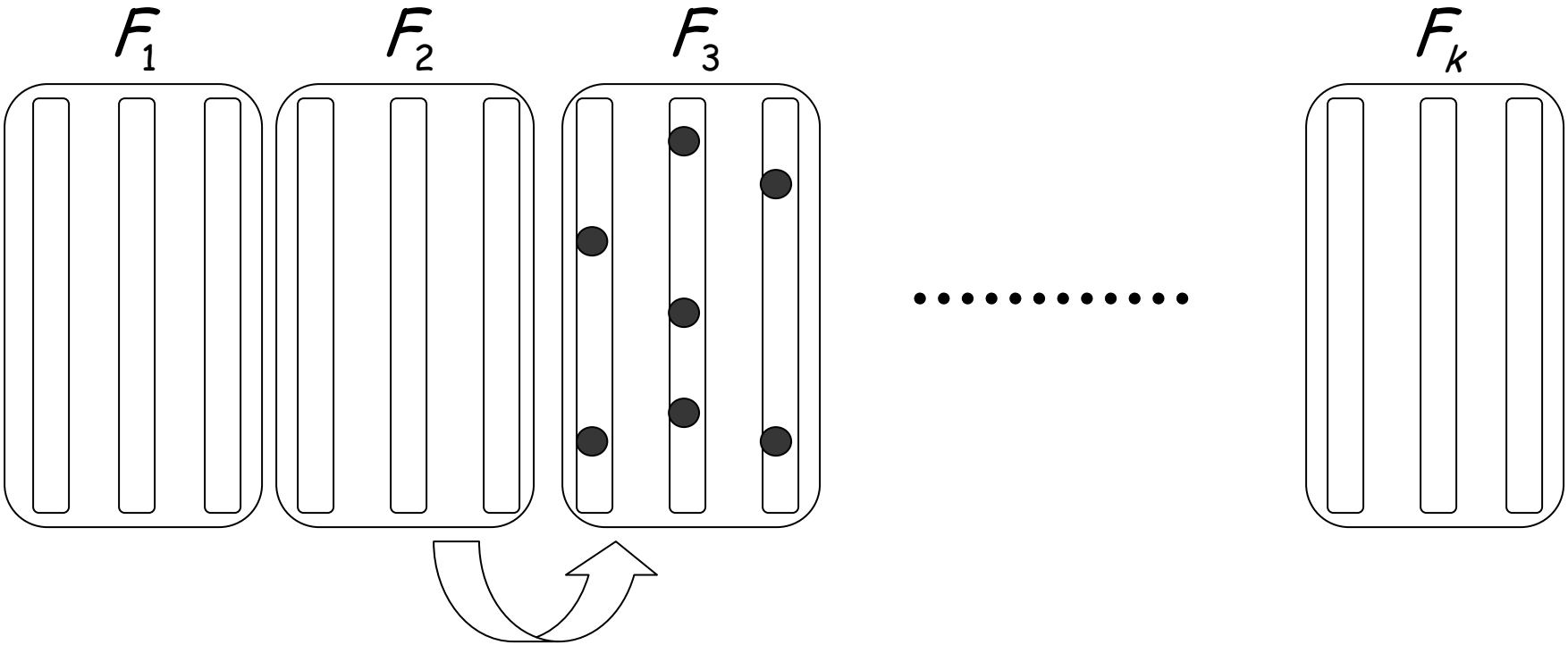

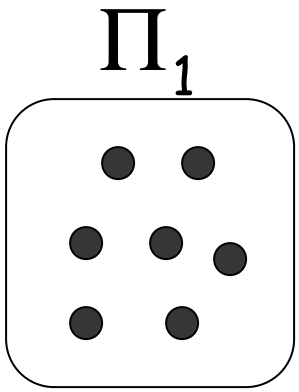

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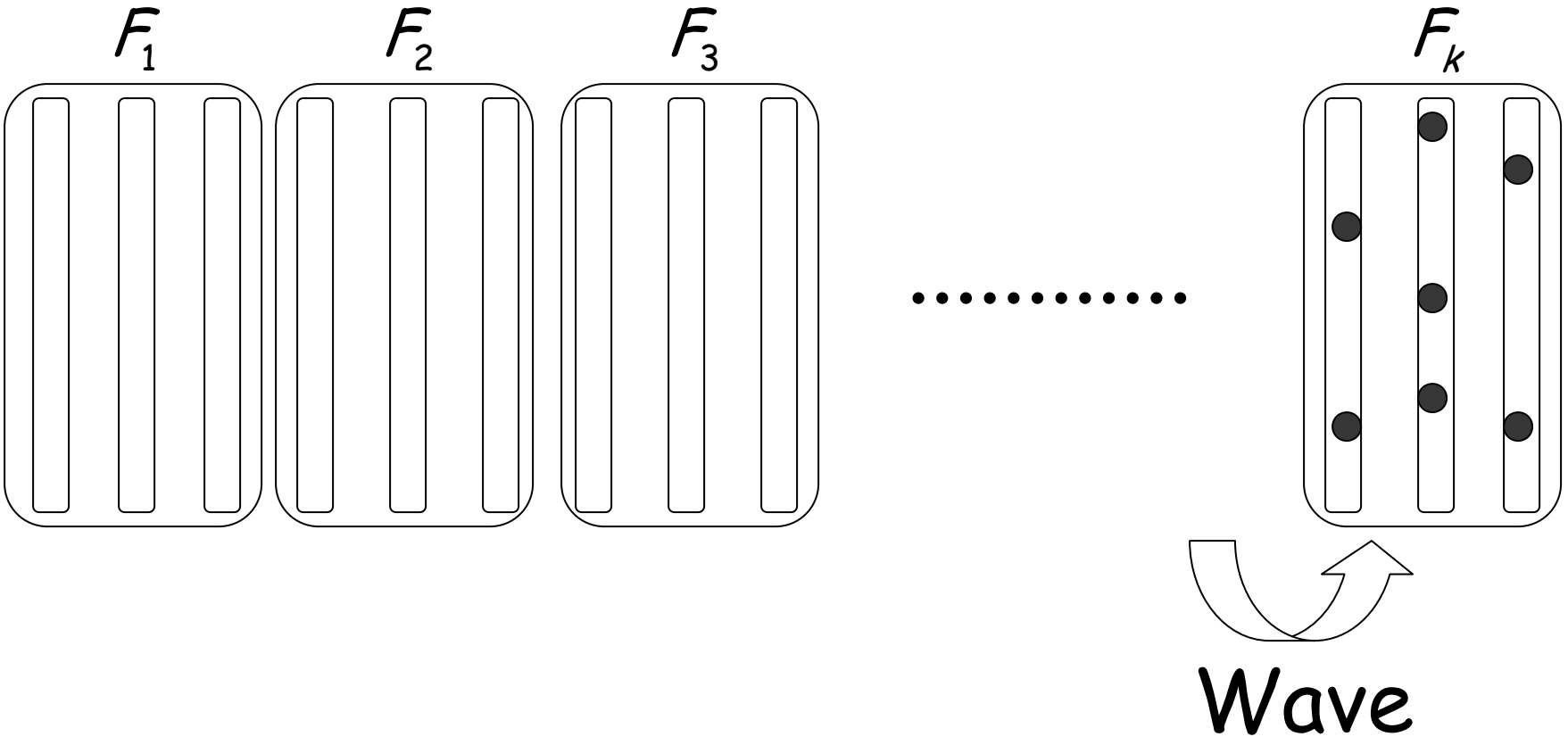
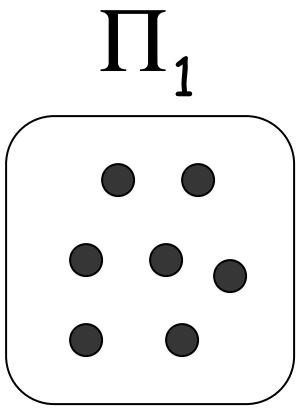
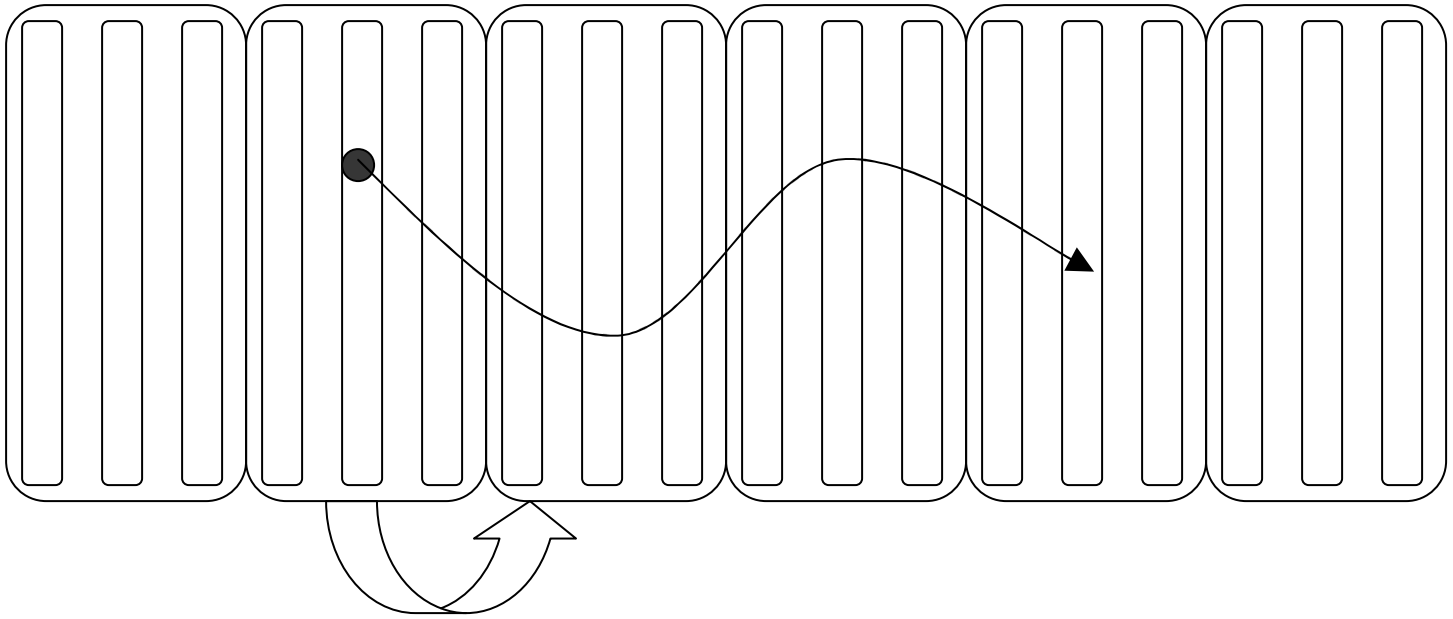
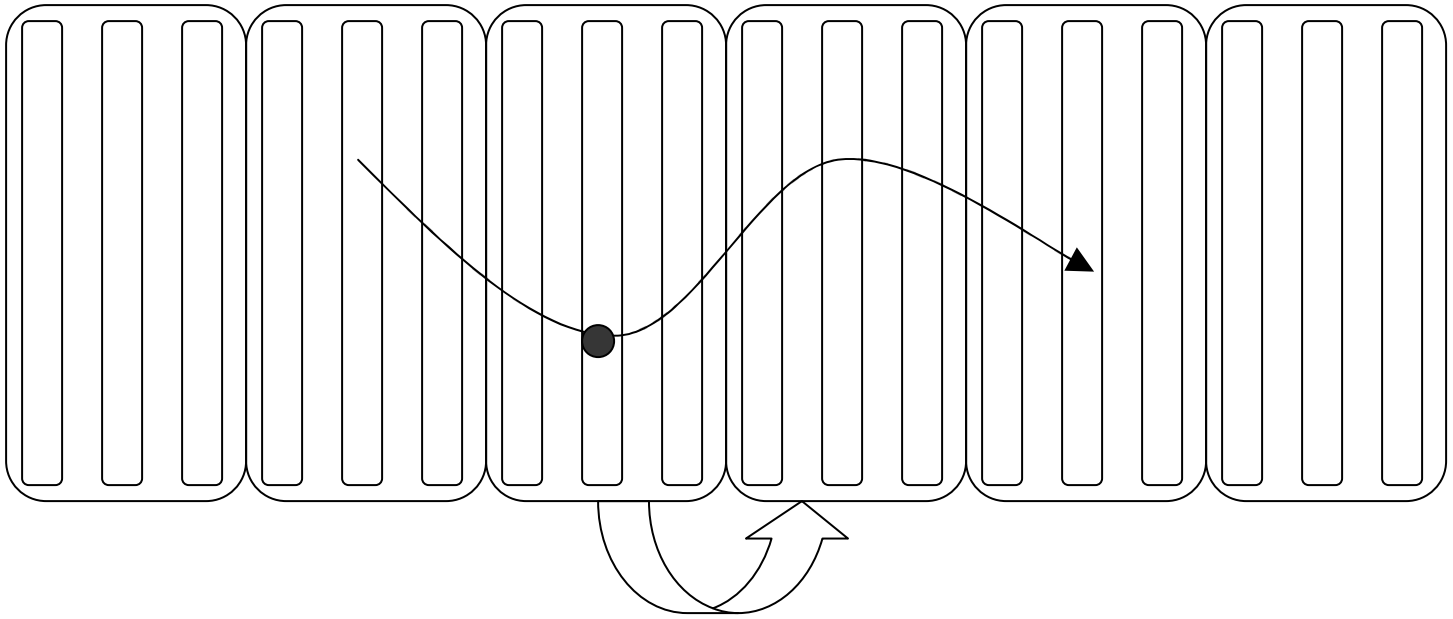






A packet follows its path from source to Destination along the wave



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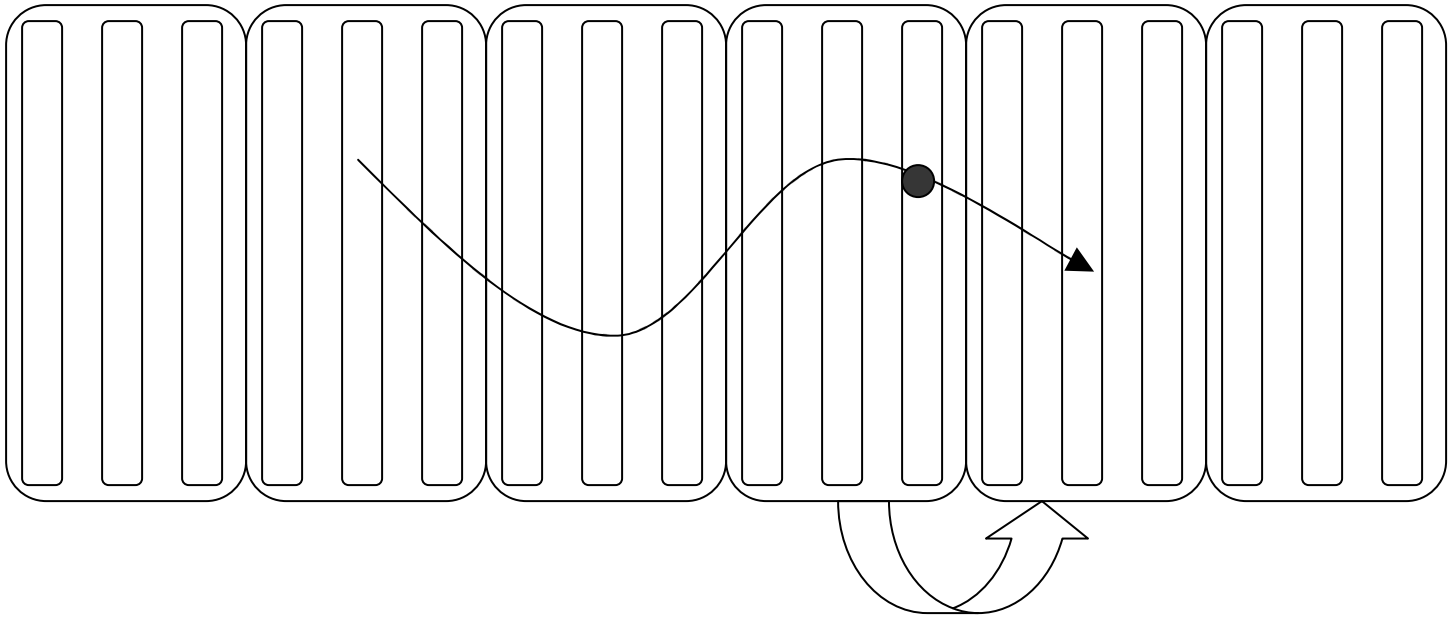

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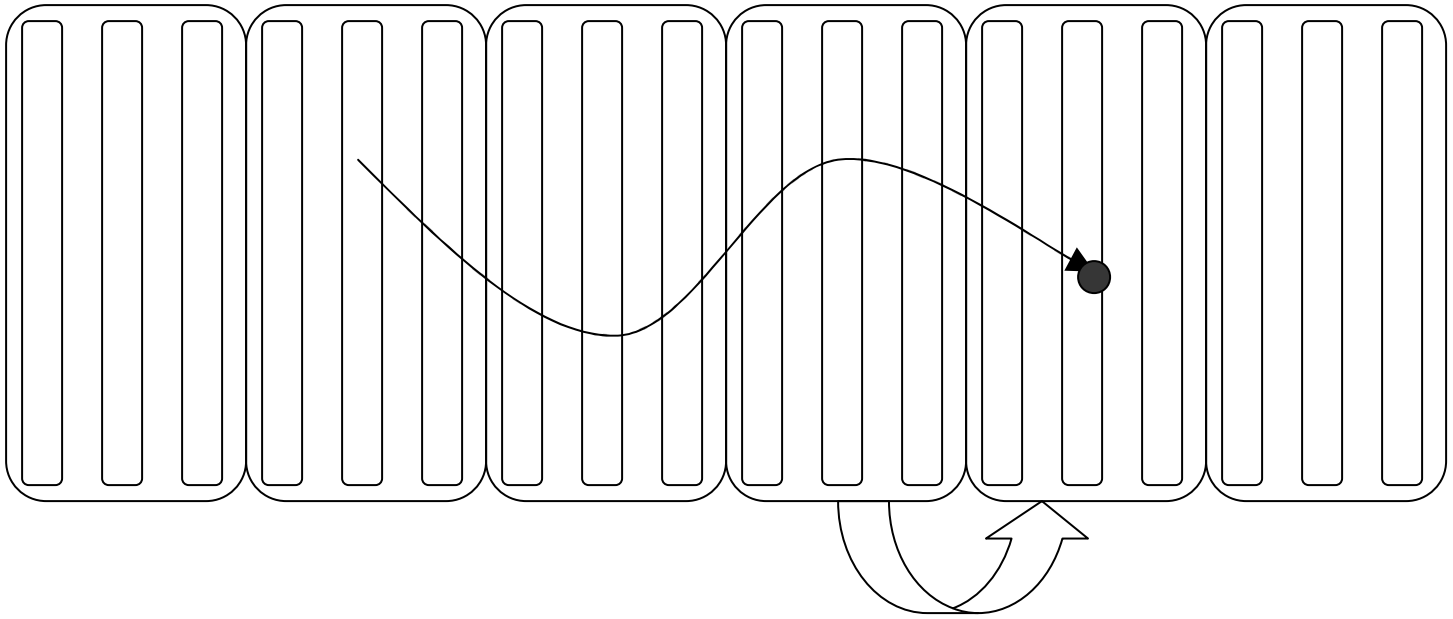
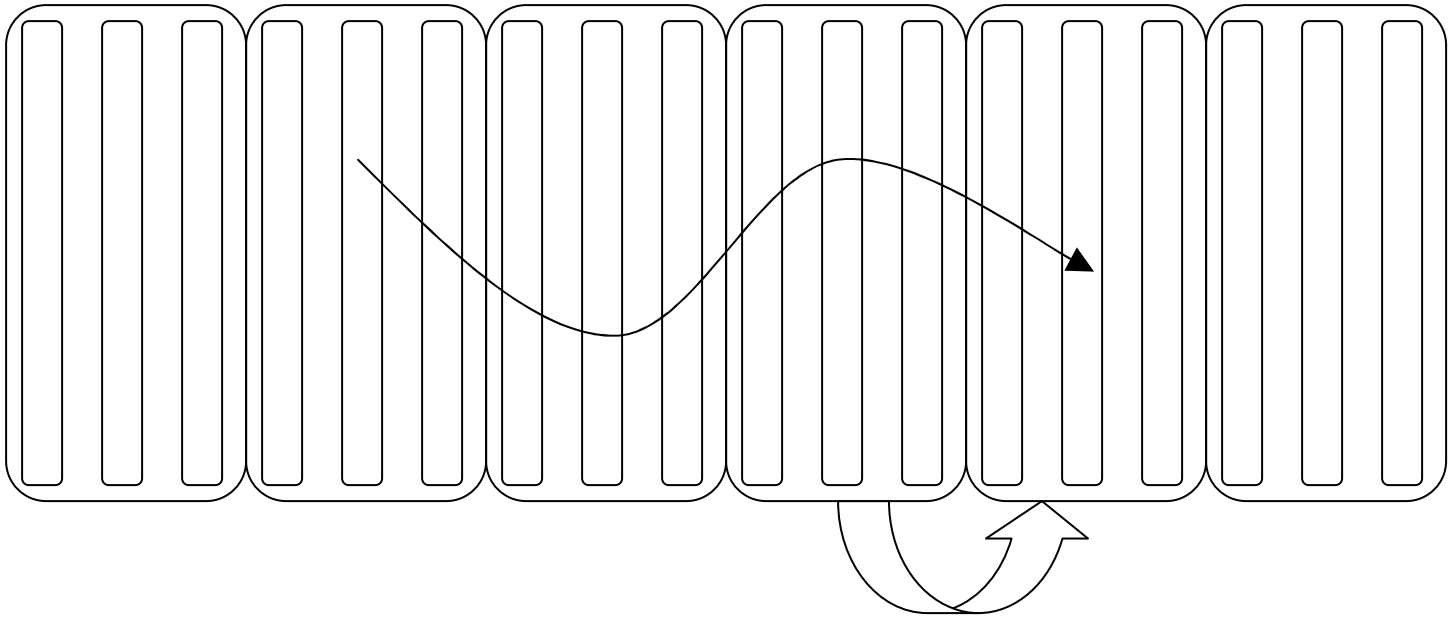
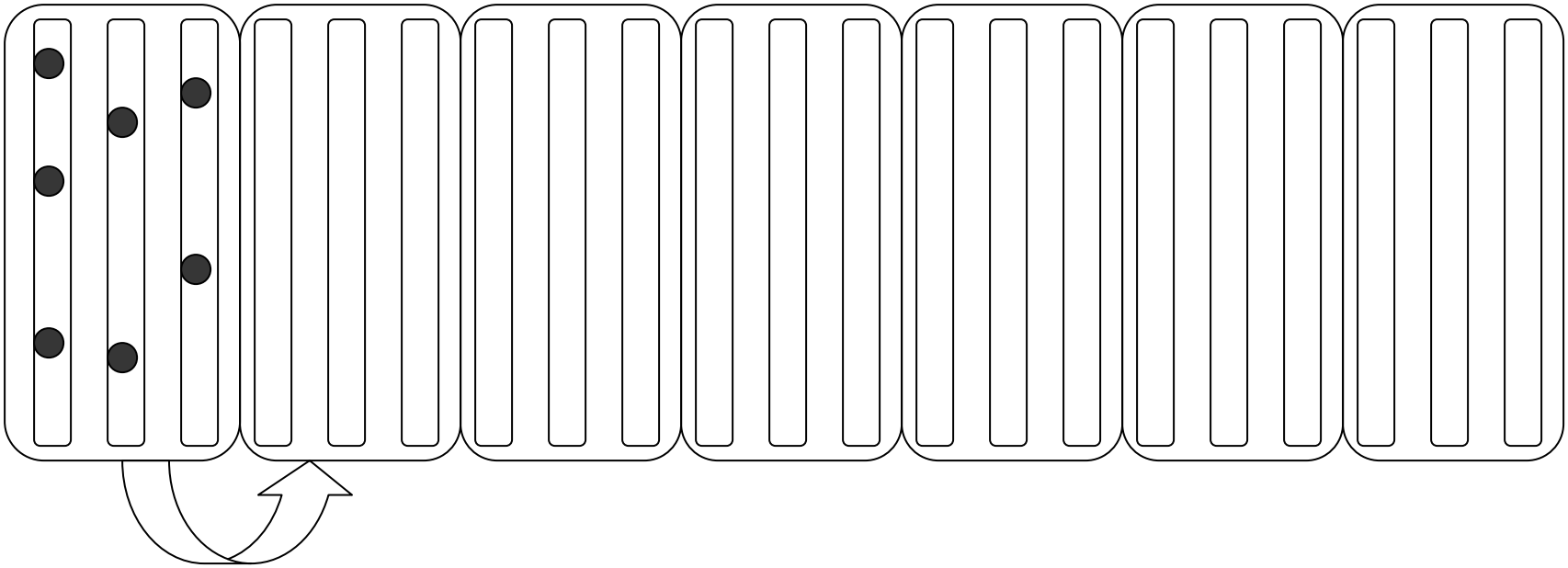


A packet follows its path from source to Destination along the wave



A packet follows its path from source to Destination along the wave

Sending packets of different packet sets simultaneously



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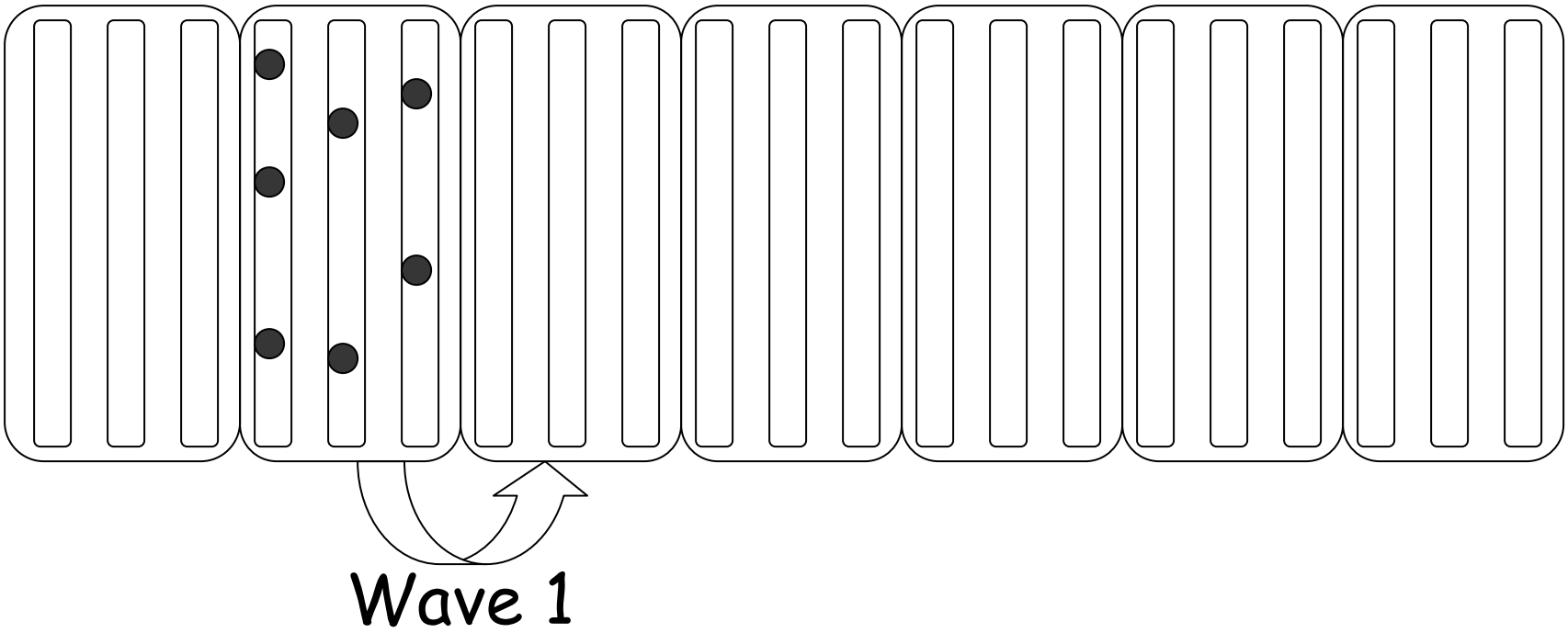

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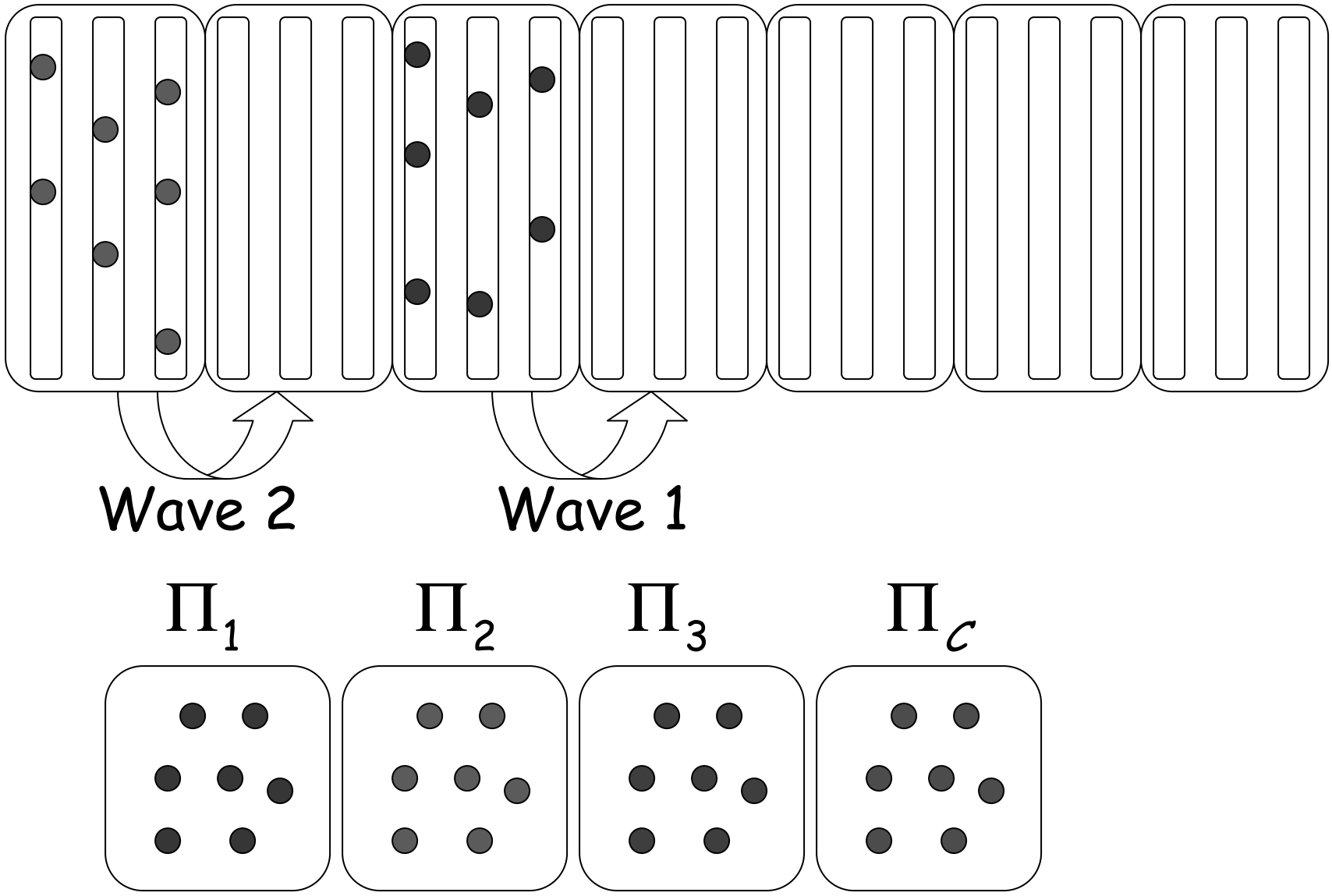

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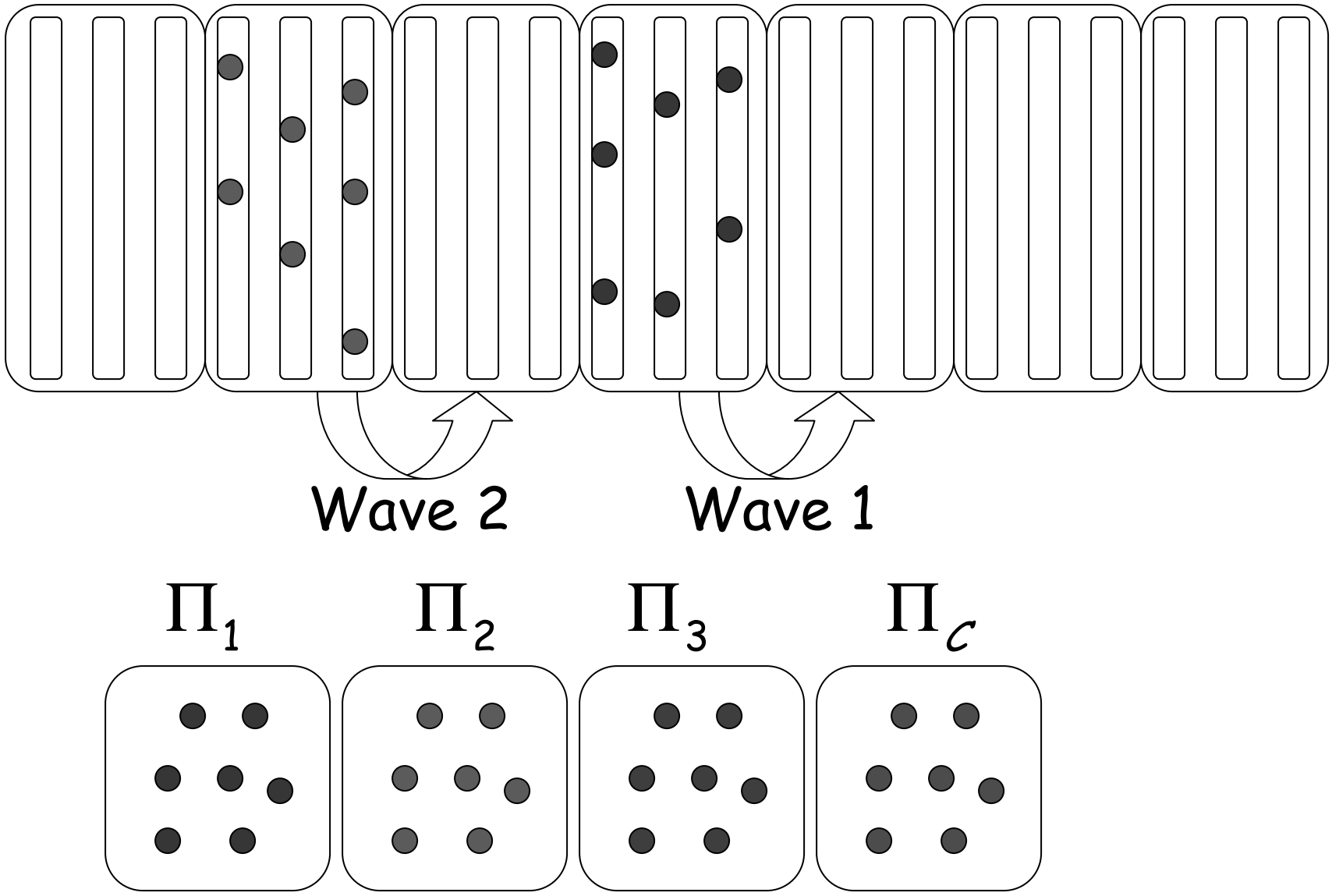

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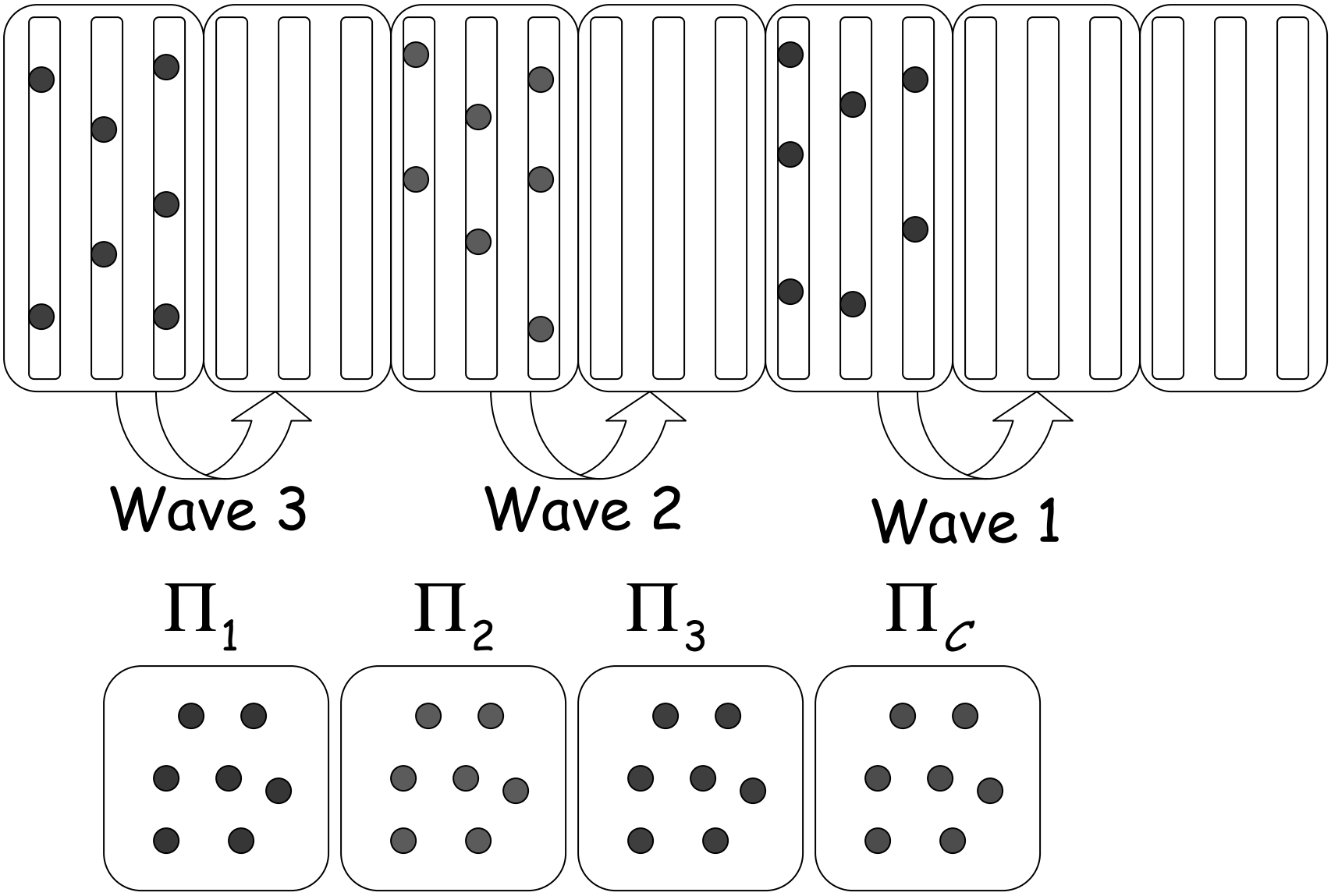
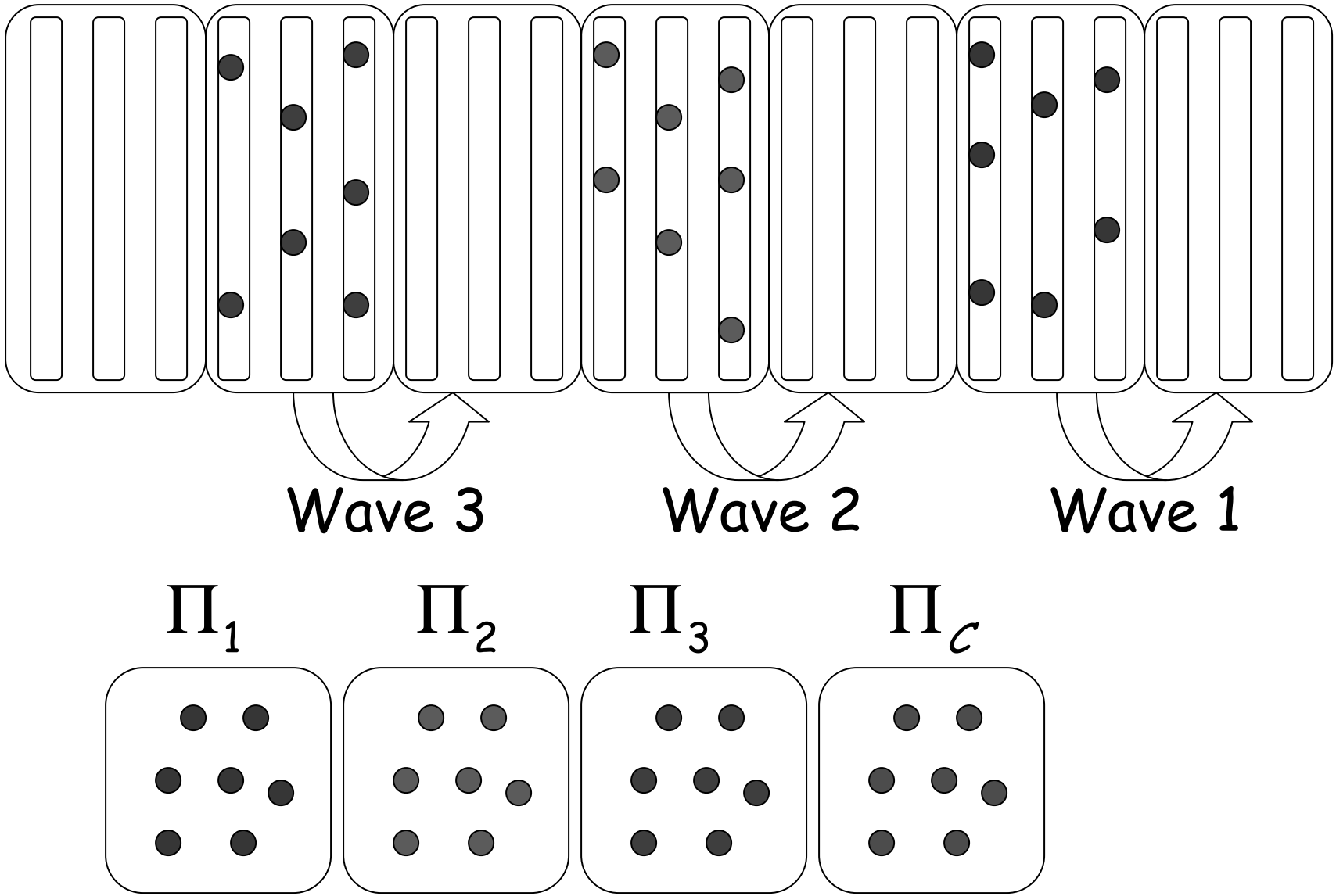
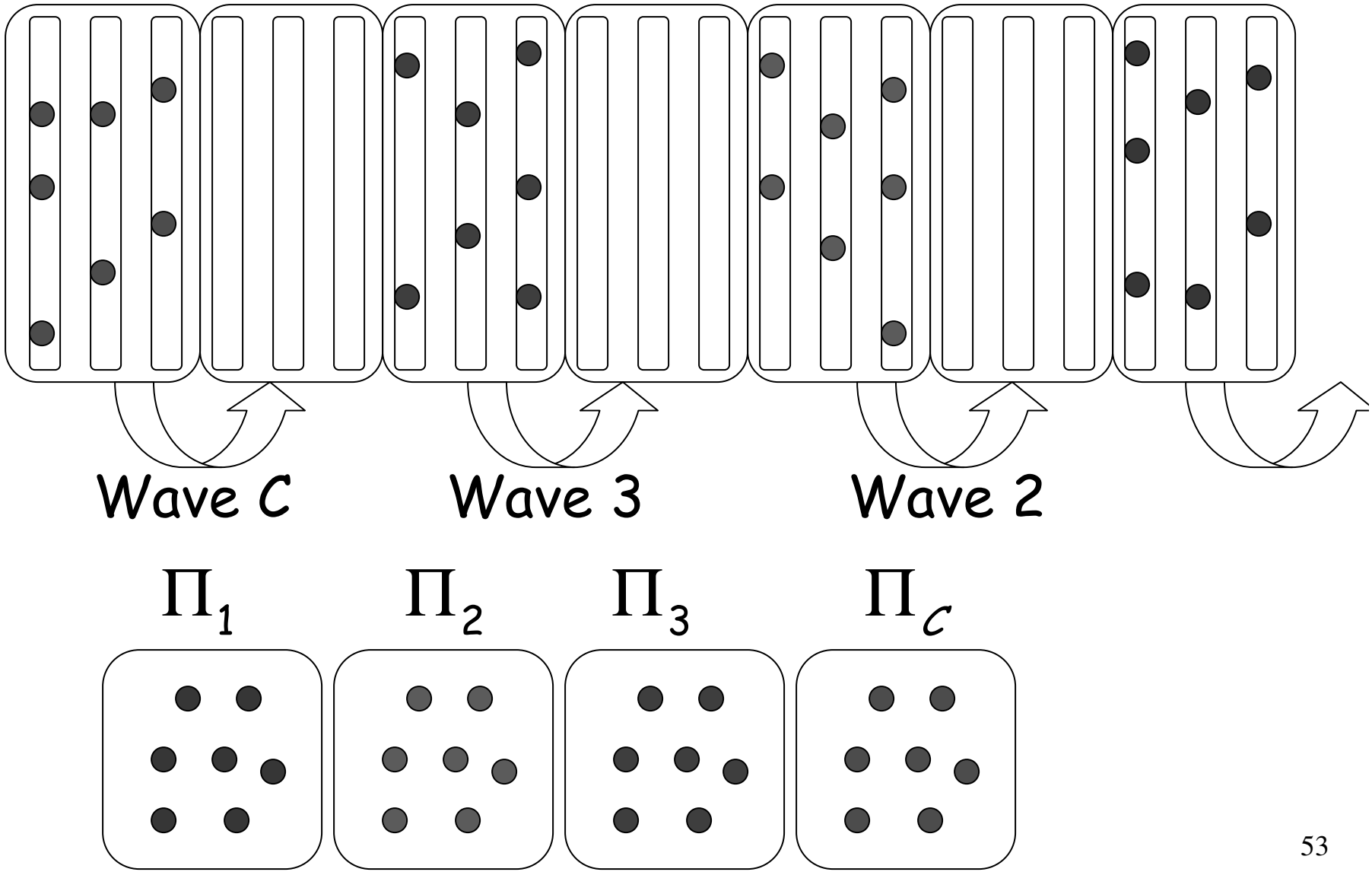


Sending packets of different packet sets simultaneously



Sending packets of different packet sets simultaneously



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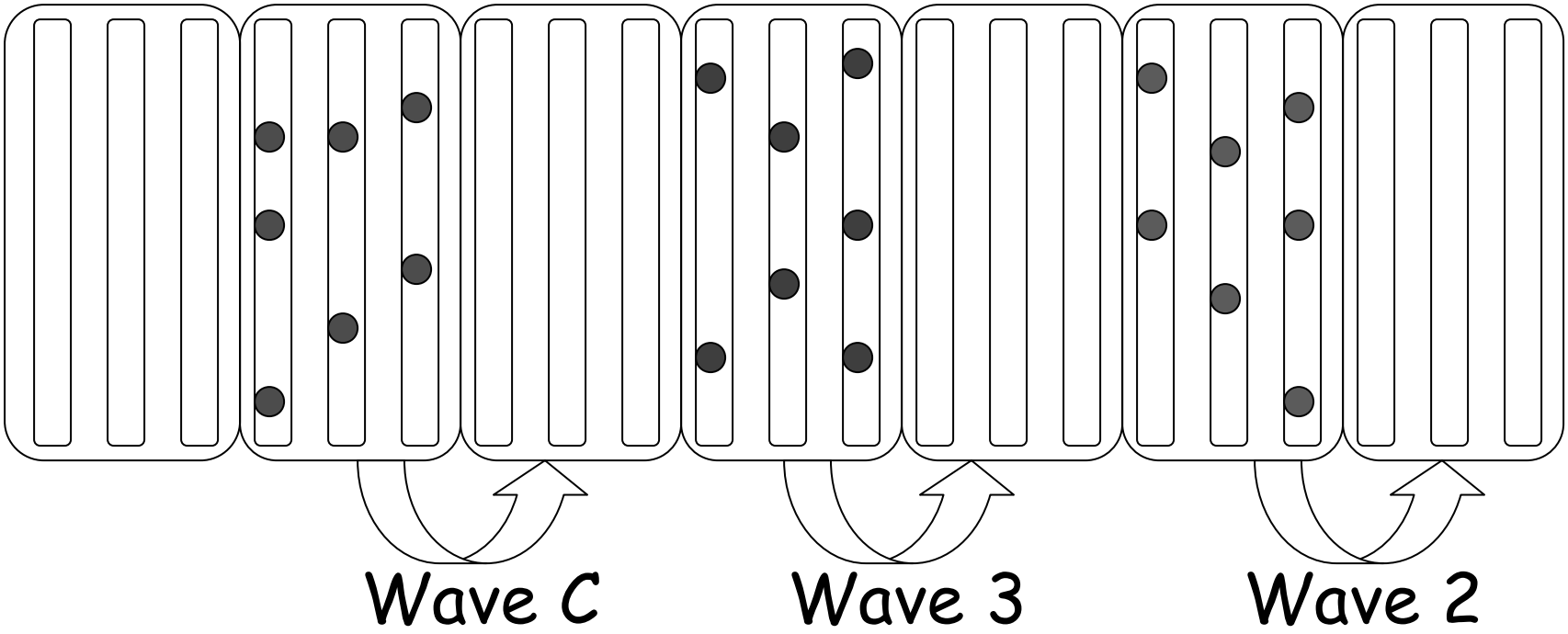

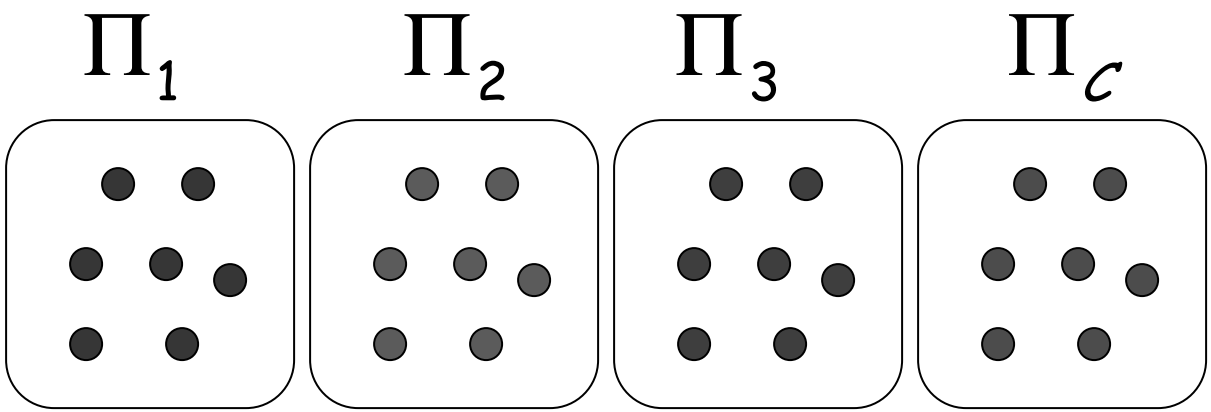

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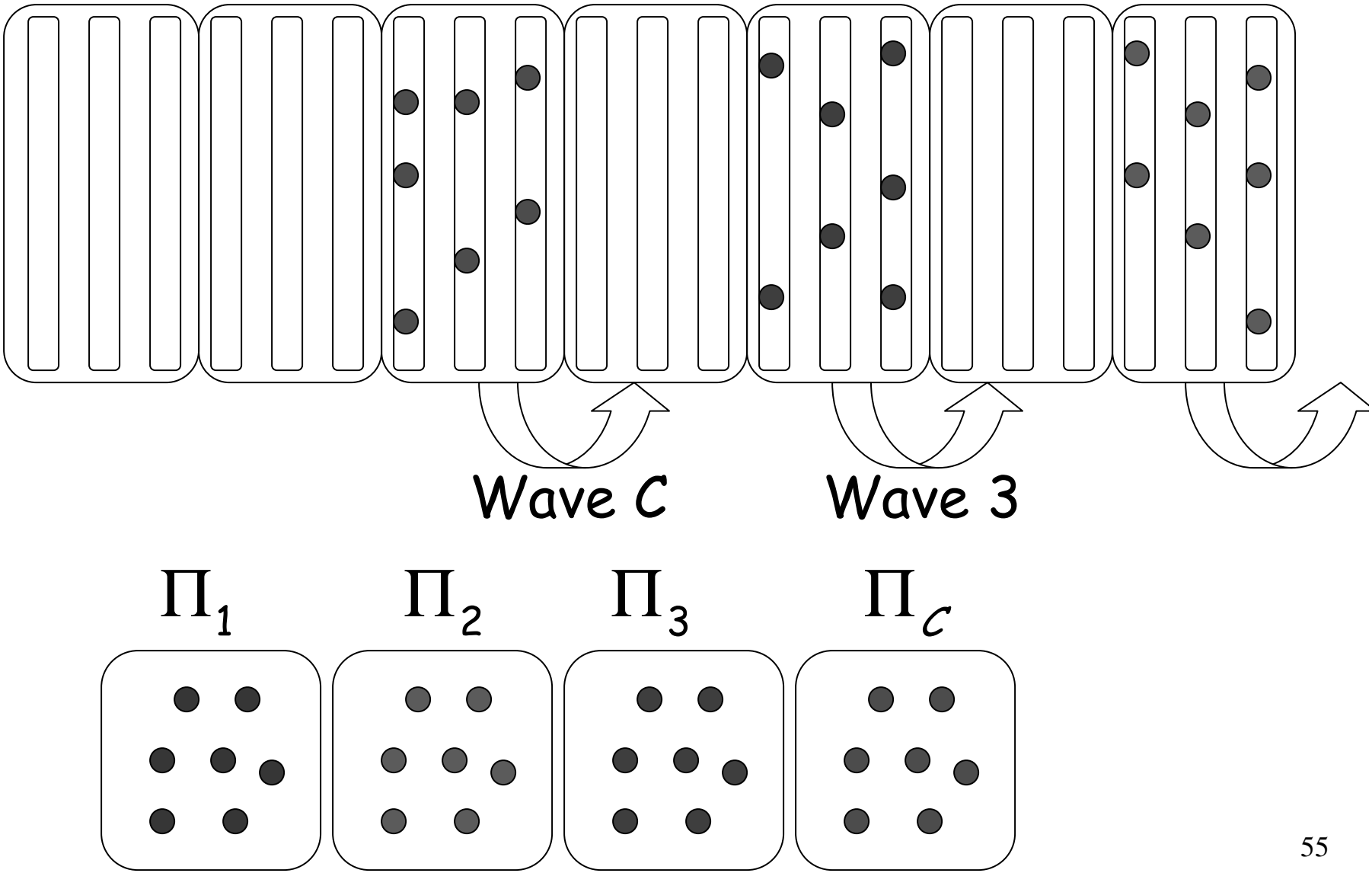
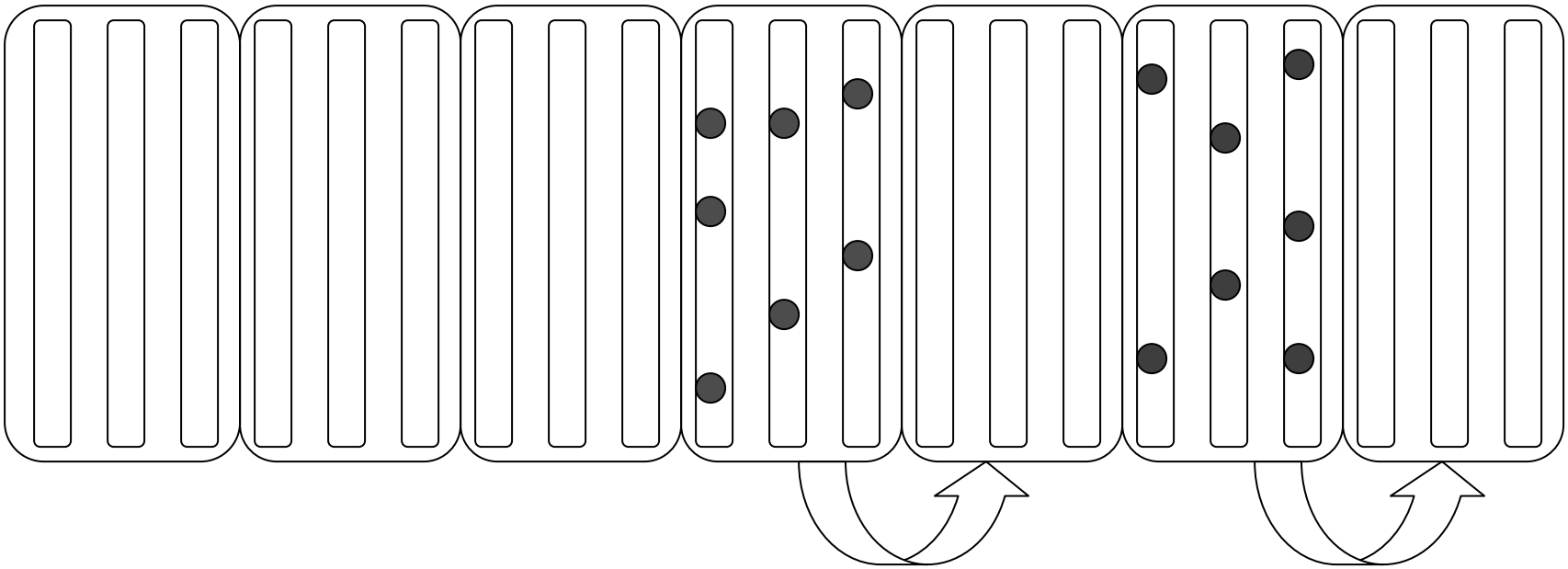
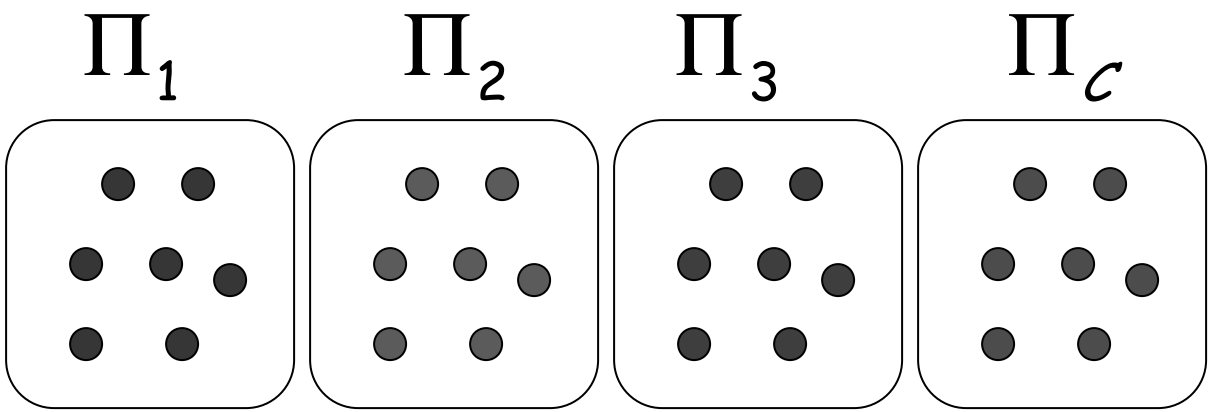


Sending packets of different packet sets simultaneously





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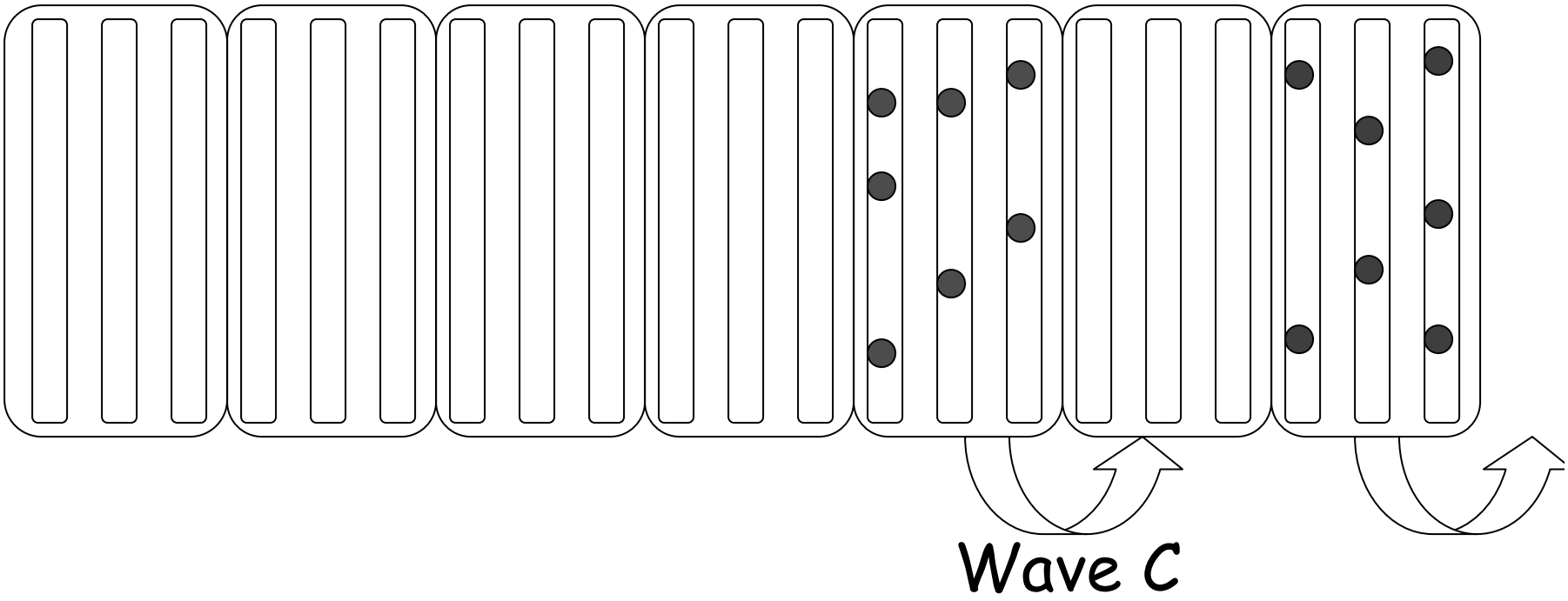

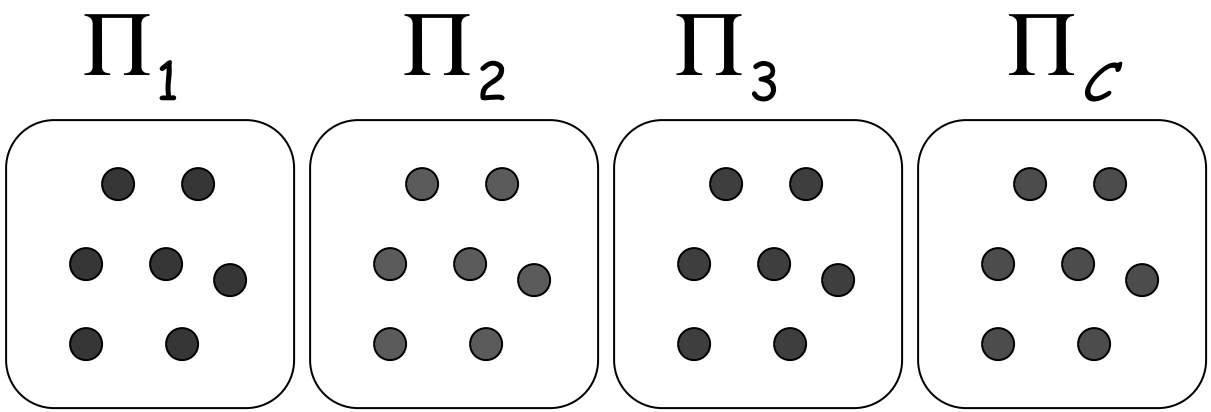

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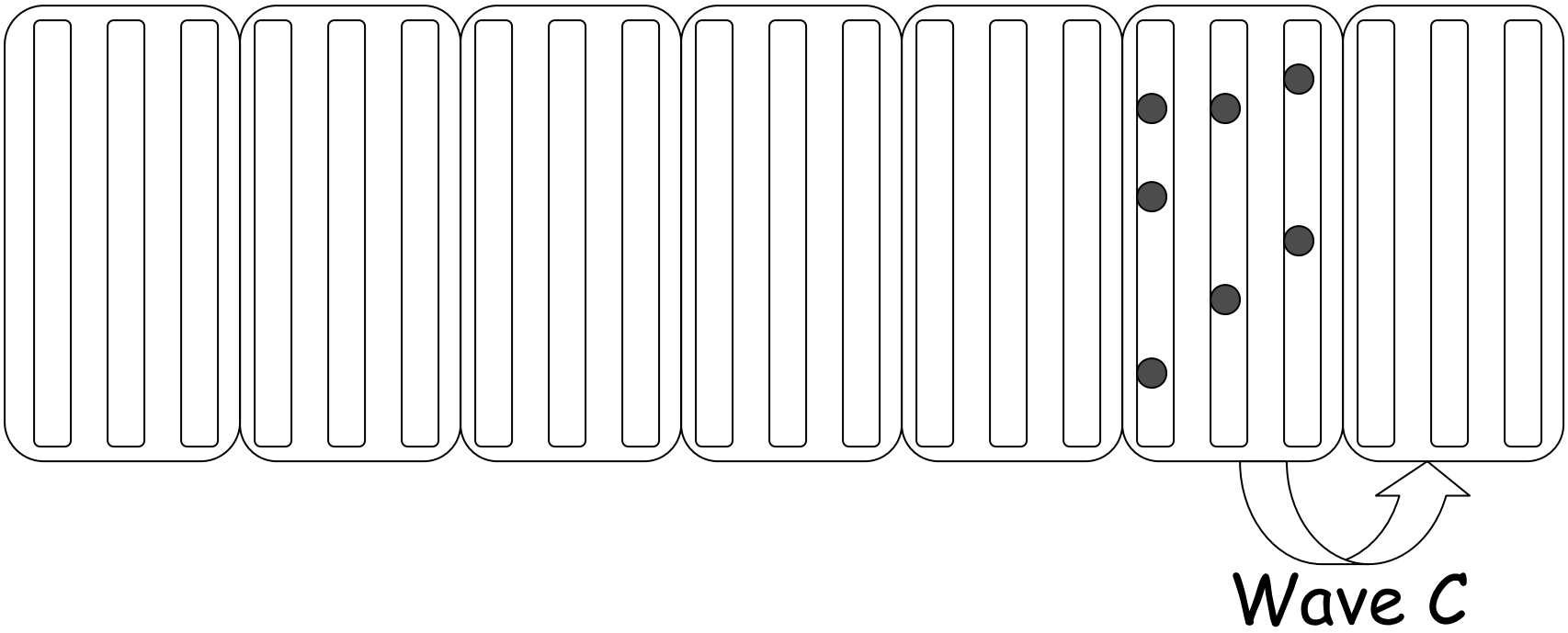

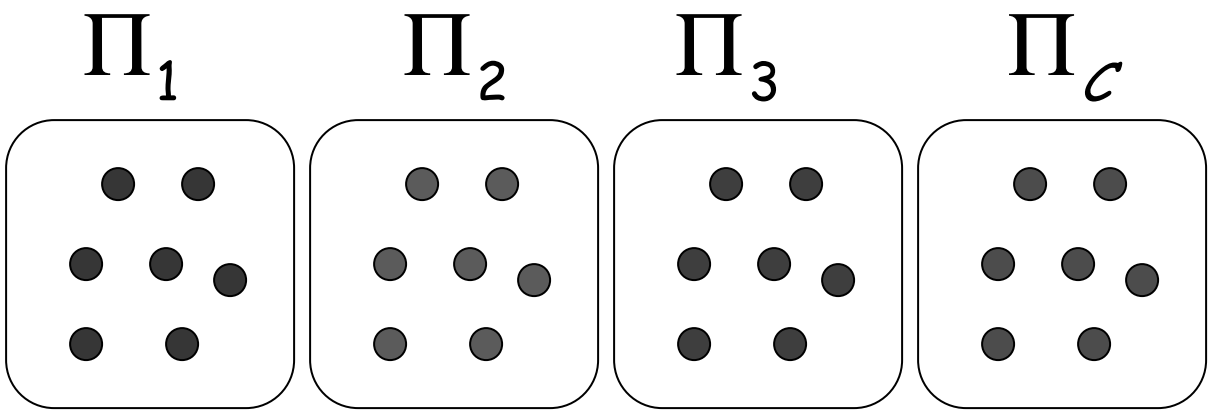

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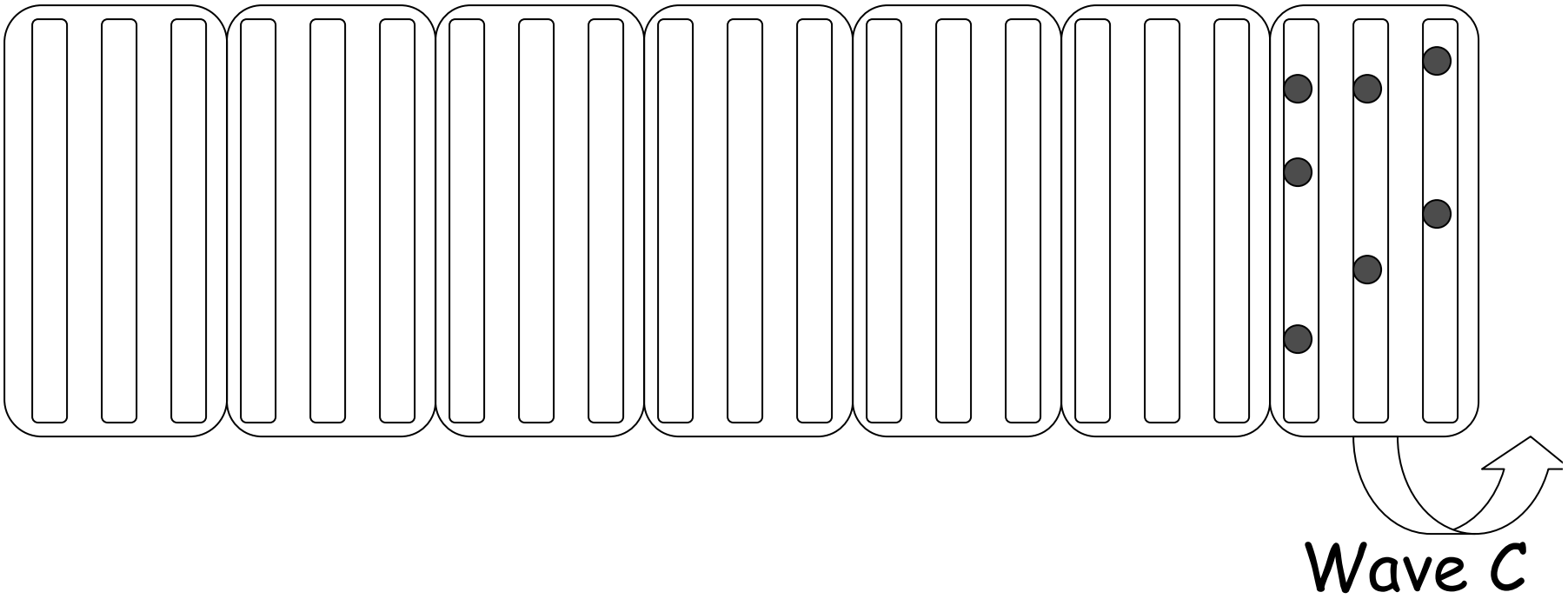
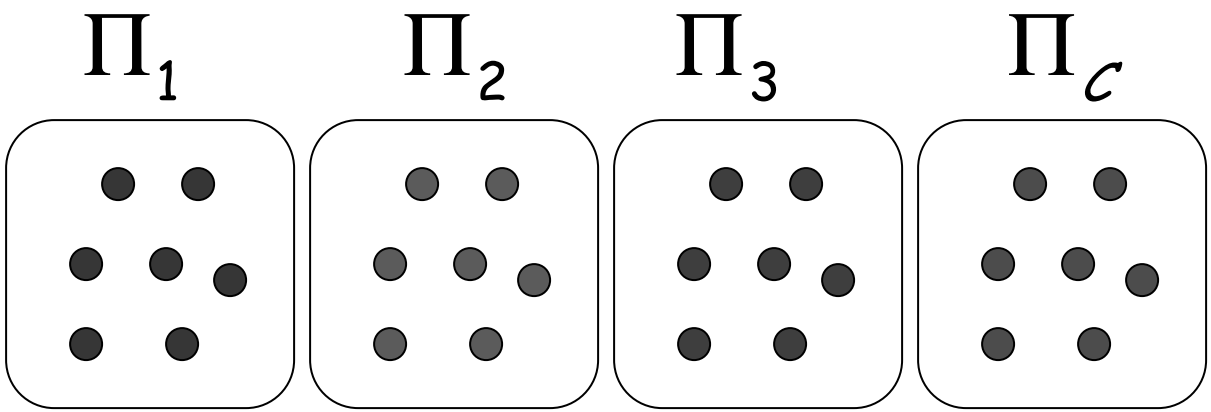
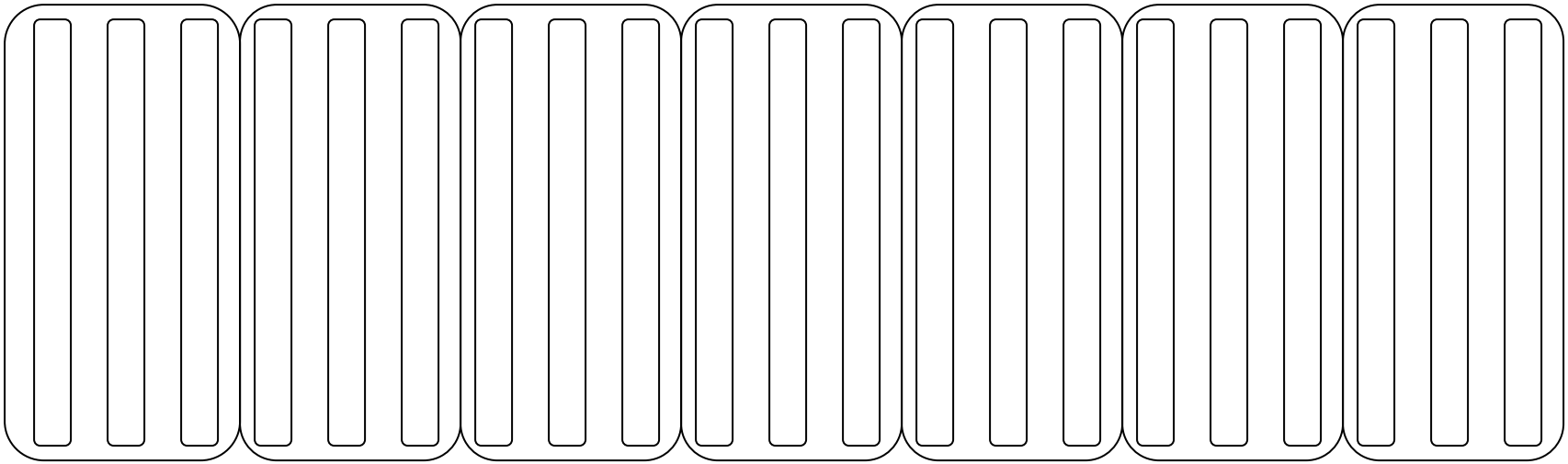
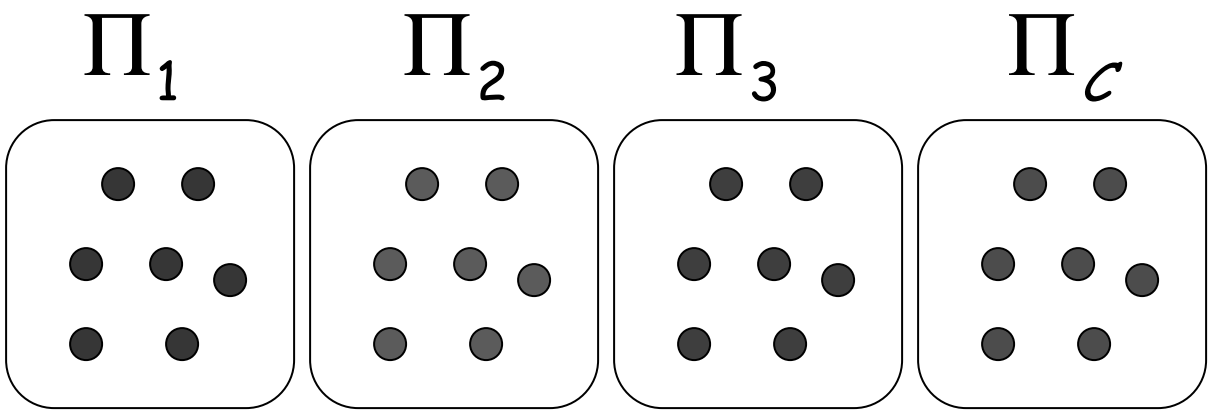




All packets have been absorbed!





Routing Time = Time until last wave C leaves the network

Wave duration: $2C\log(DN)$ + #frames: $\frac{D}{\log(DN)}$ Wave duration: $\log(DN) = \Theta(C\log(DN) + D)$

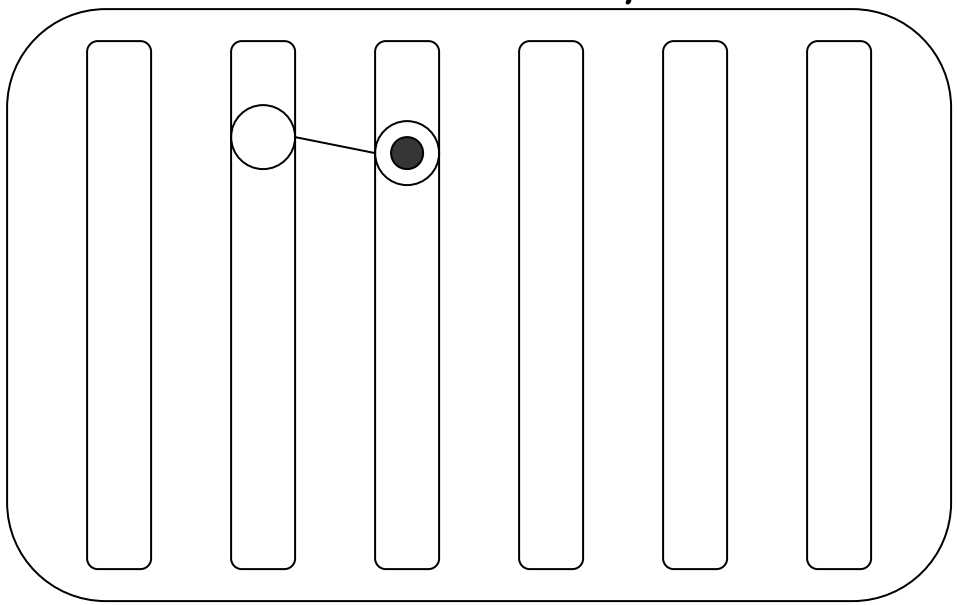
Time when wave C enters the network

Time that wave C needs to traverse the network

Oscilation

Simulates buffering

Frame F_i

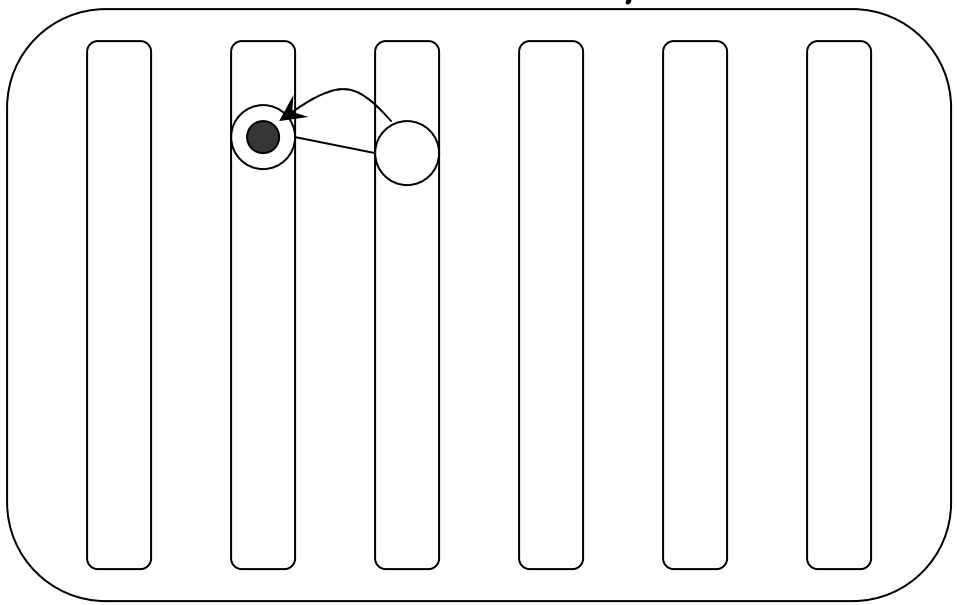


Time t

Oscilation

Simulates buffering

Frame F_i

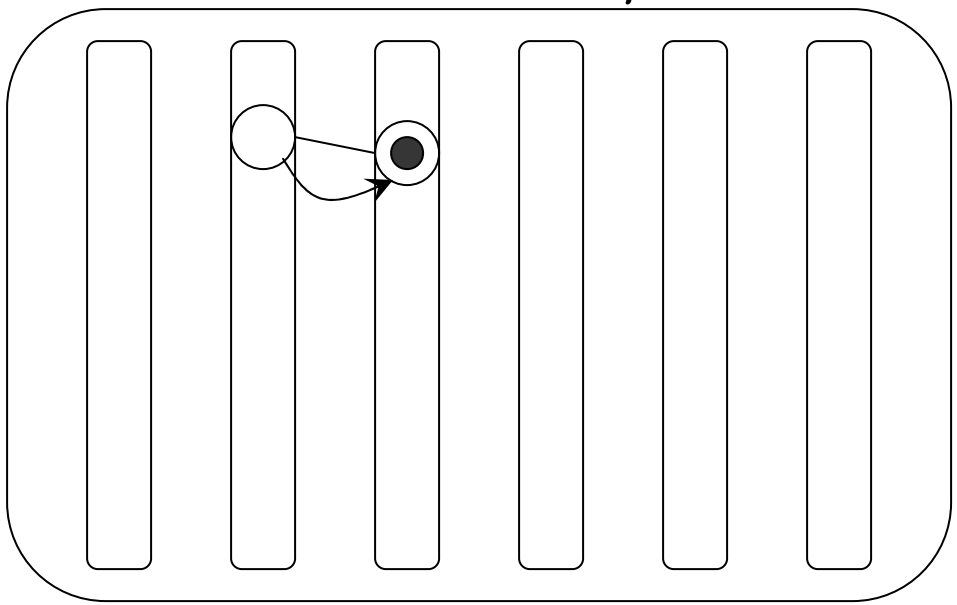


Time $t+1$

Oscilation

Simulates buffering

Frame F_i

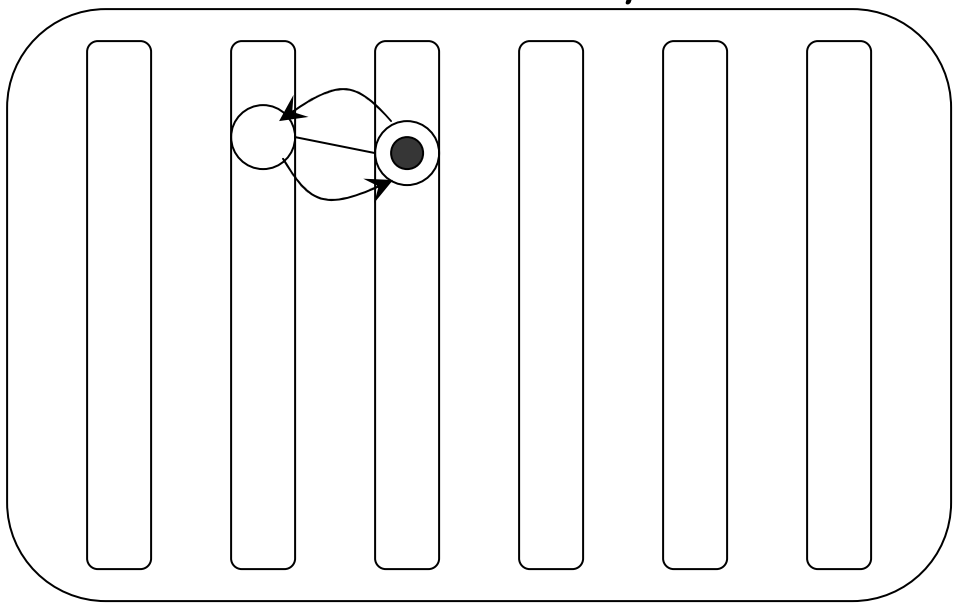


Time $t+2$

Oscilation

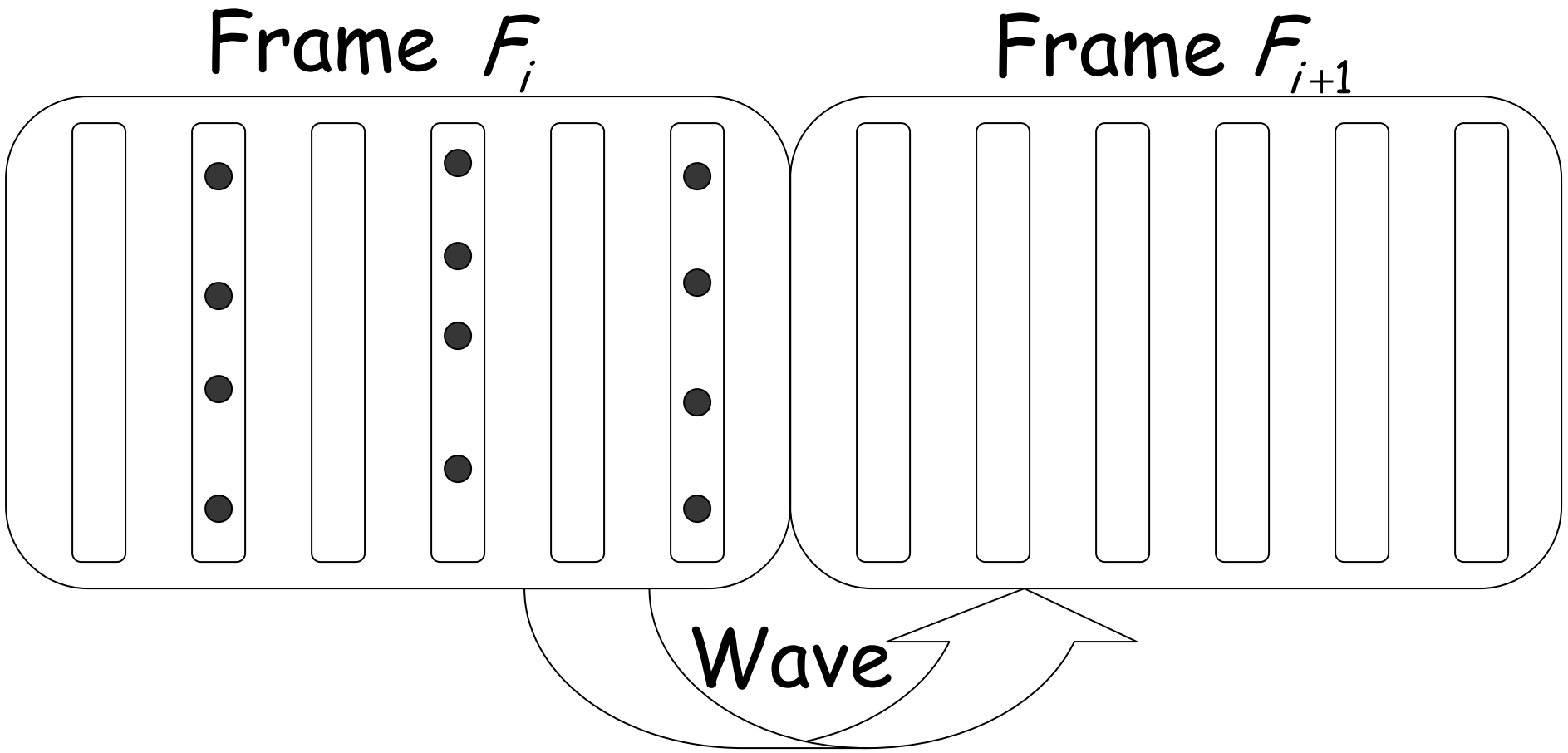
Simulates buffering

Frame F_i

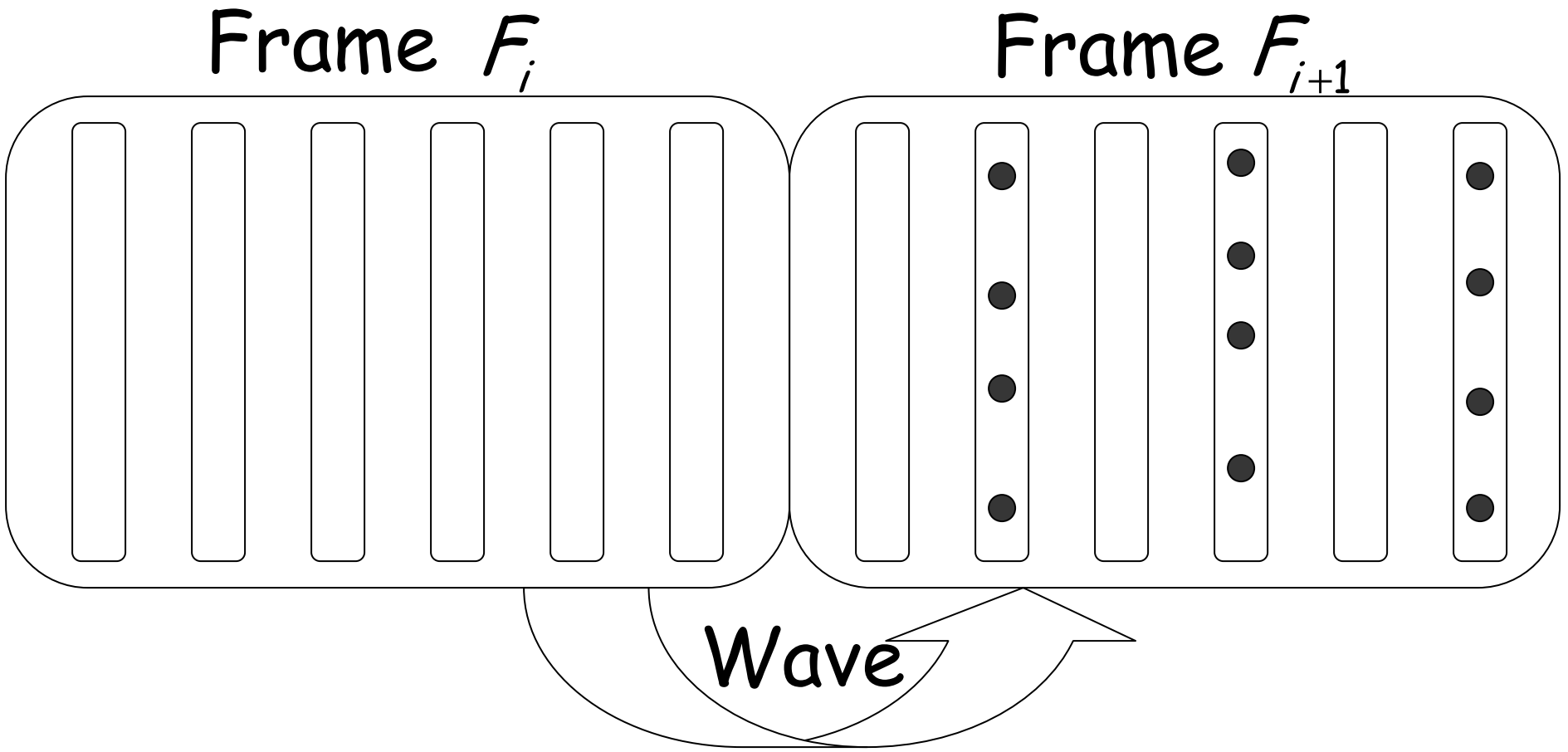


Time $t+3, t+4, \ldots$

Packet propagation during a wave

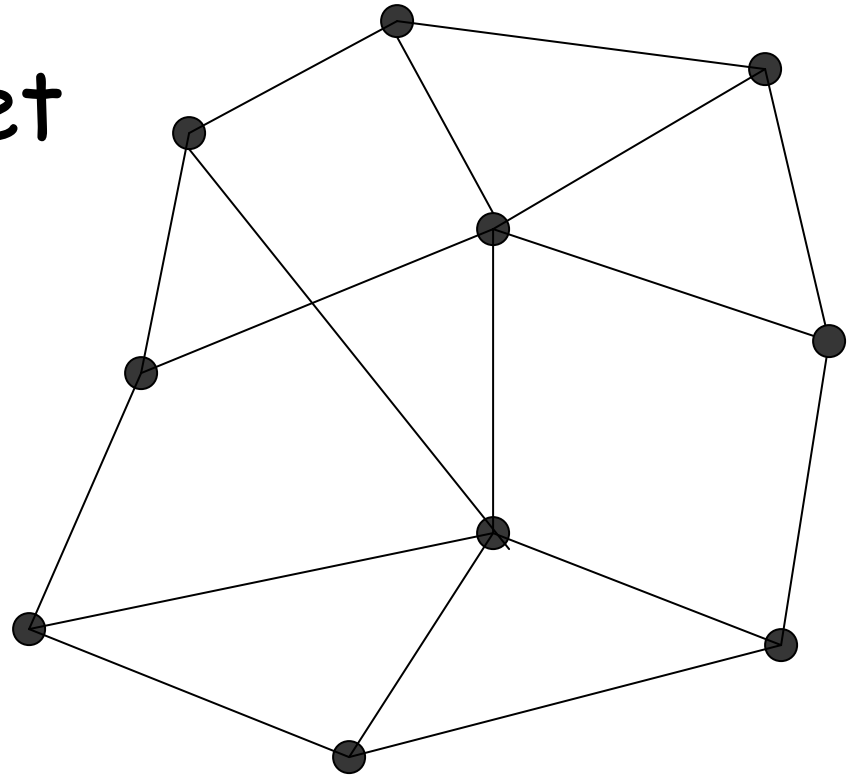


Packet propagation during a wave



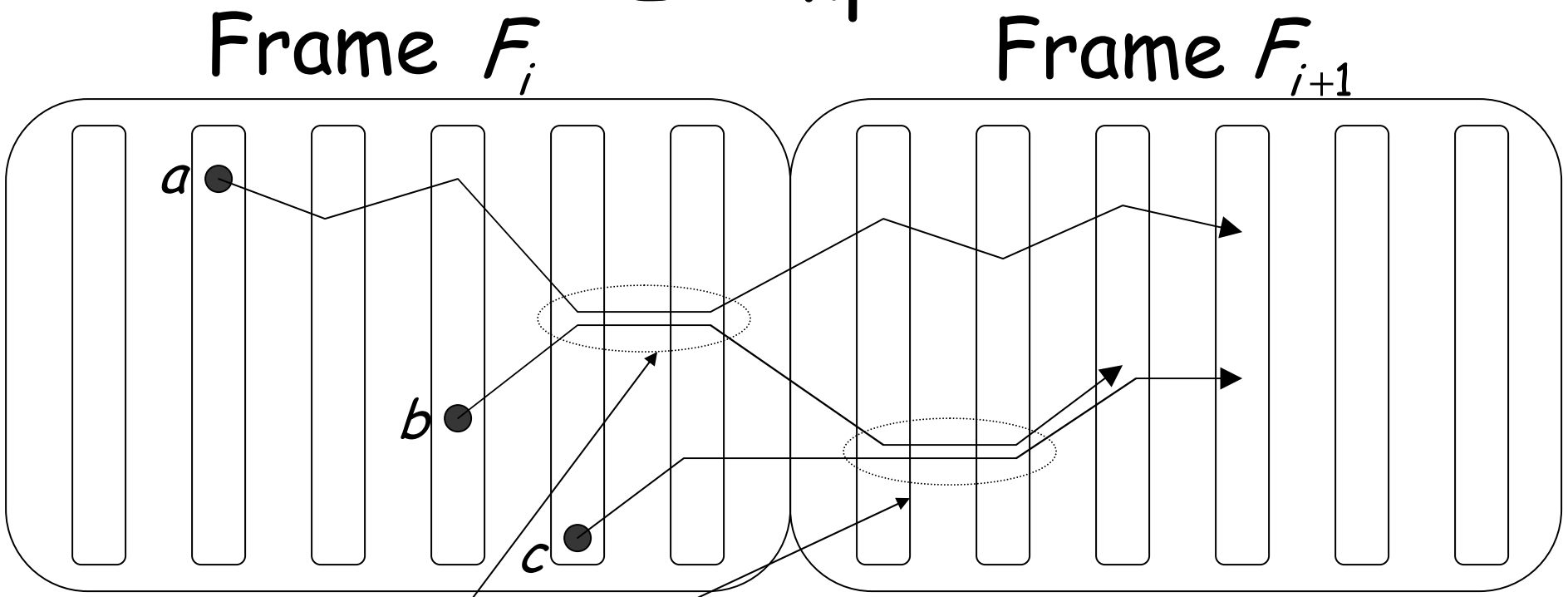
Conflict Graph

Each node is a packet



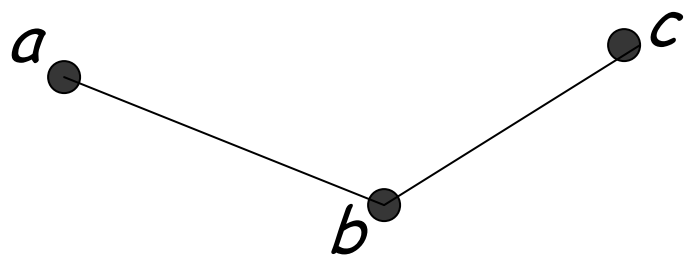
Two packets are adjacent
if their paths use a common edge in the frames F_i and F_{i+1}

Example



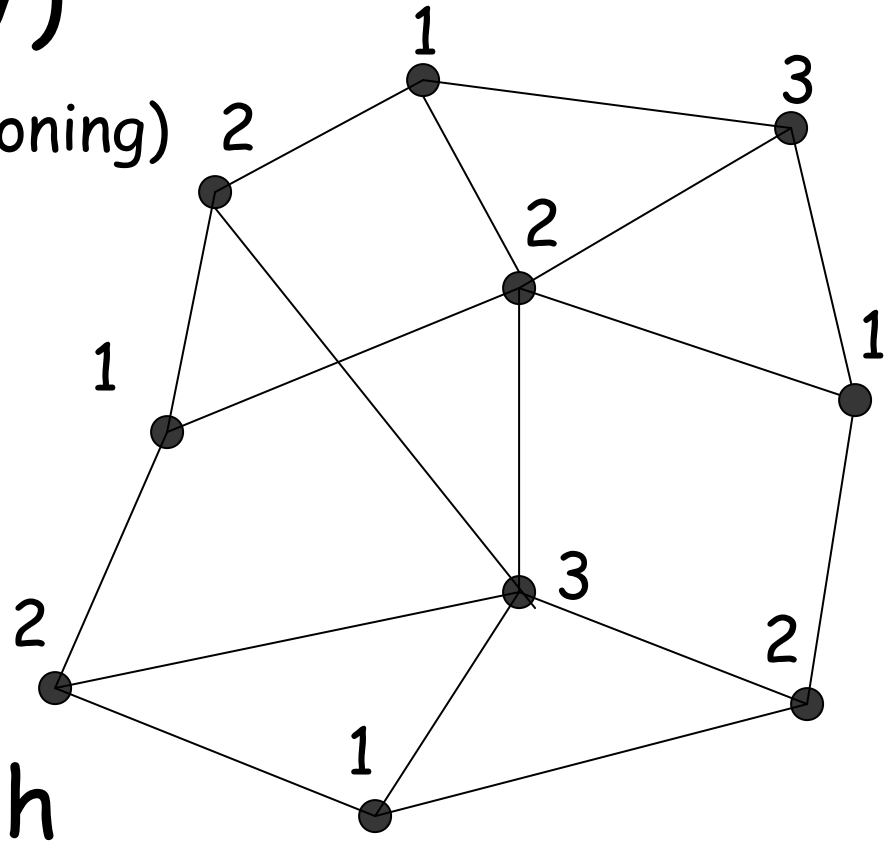
Share edge

Conflict graph



The degree of any node is bounded by $\log(DN)$

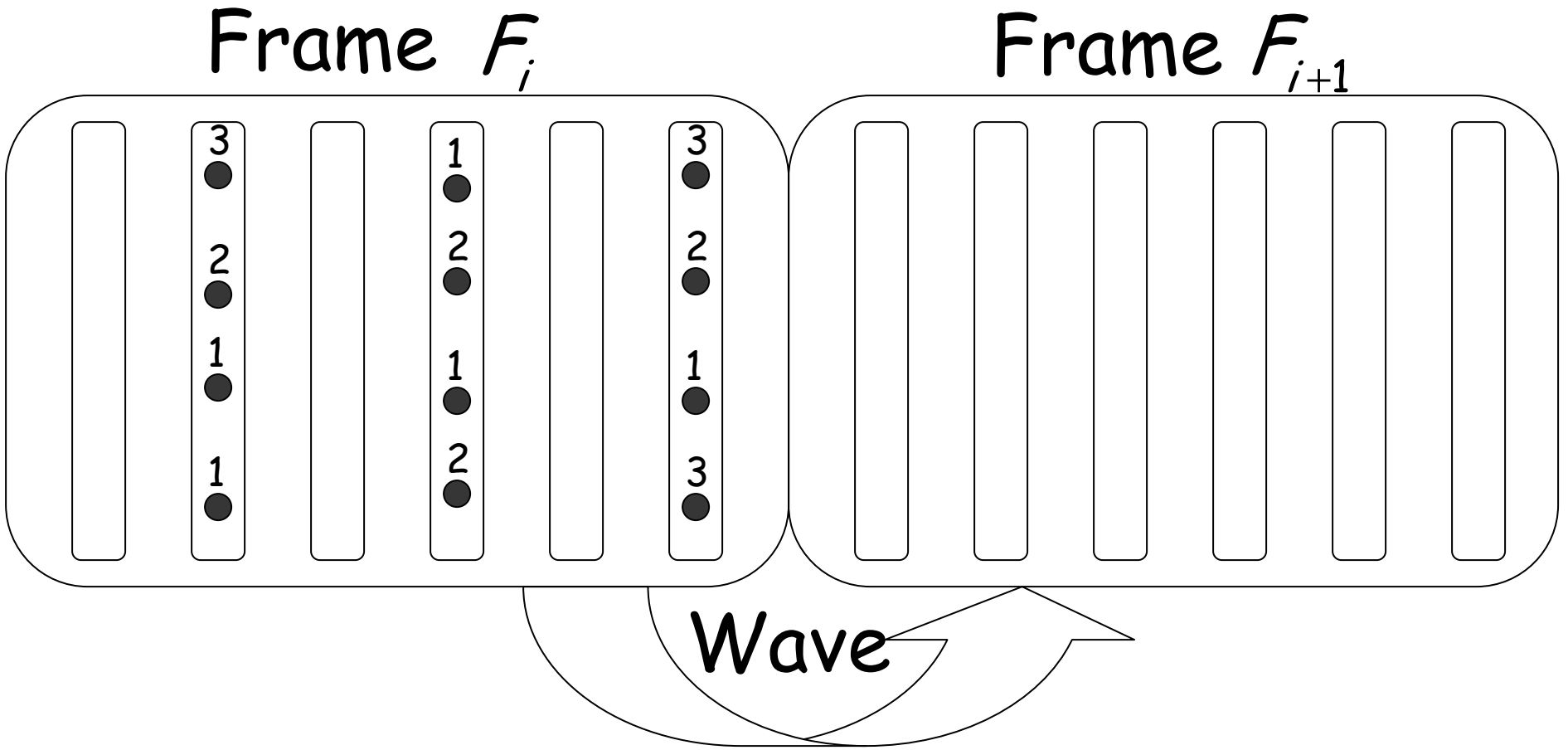
(A consequence of packet partitioning)



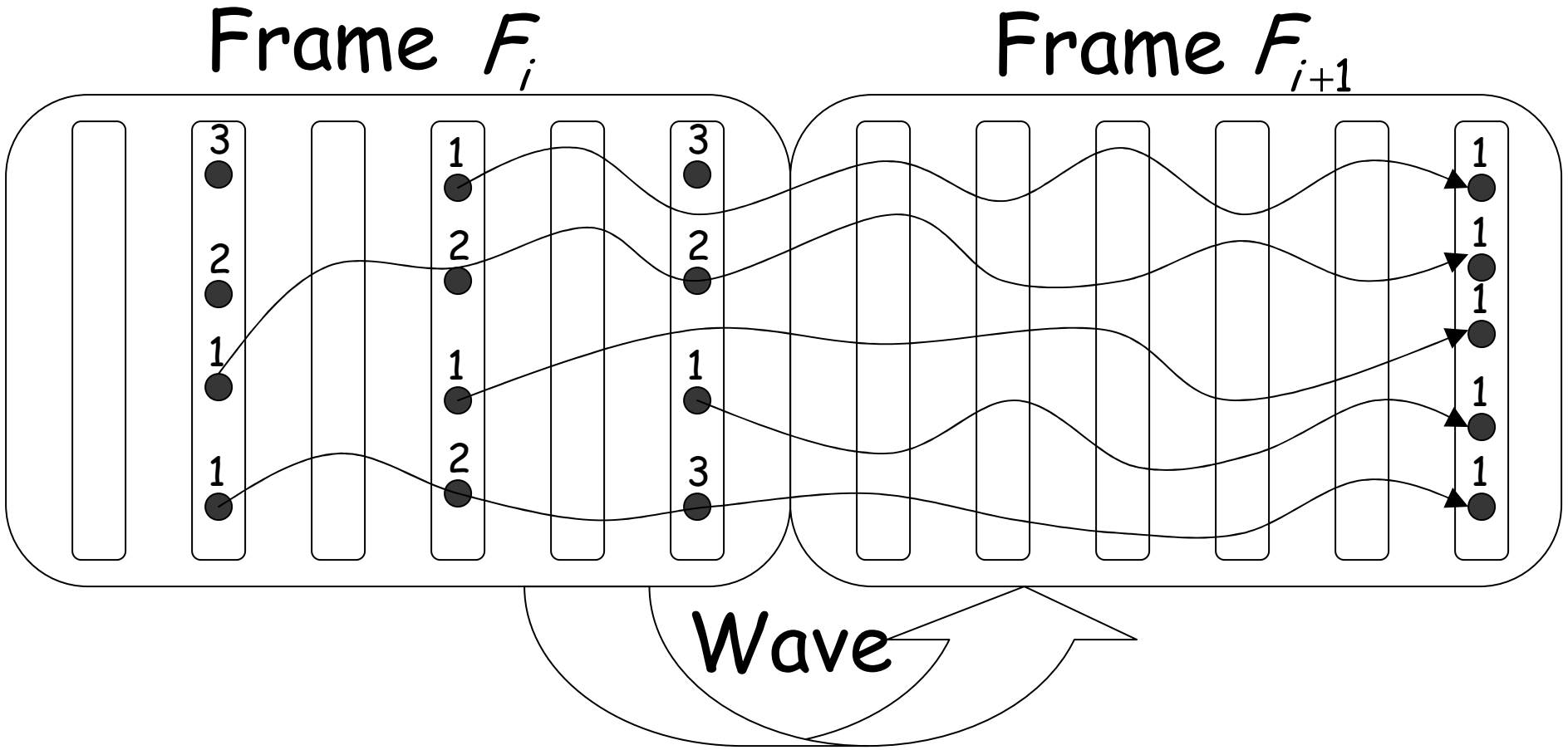
Thus the conflict graph can be colored with

$$\chi = \log(DN) + 1 \quad \text{colors}$$

We send packets of each color seperately



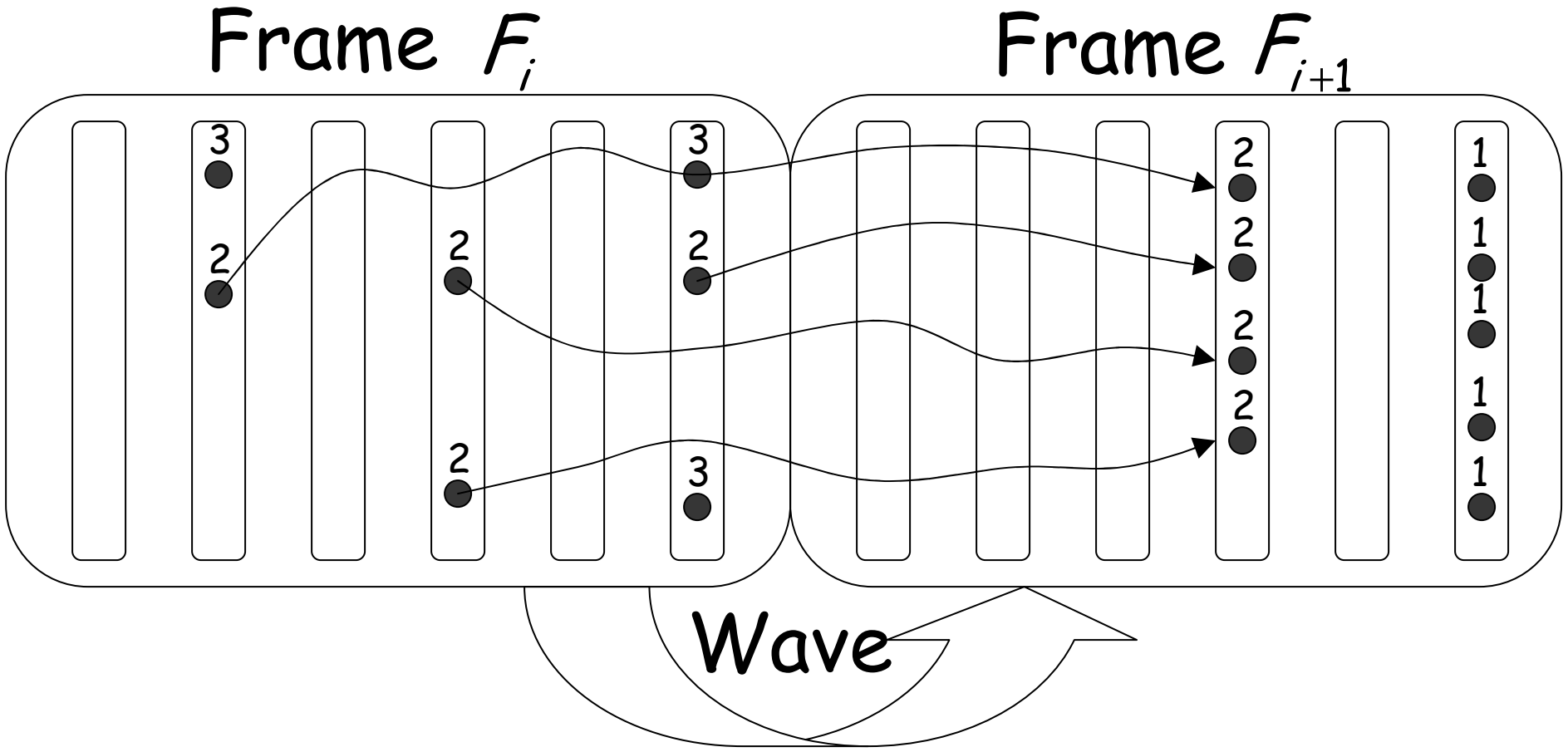
First send packets of color 1



Packet paths don't conflict

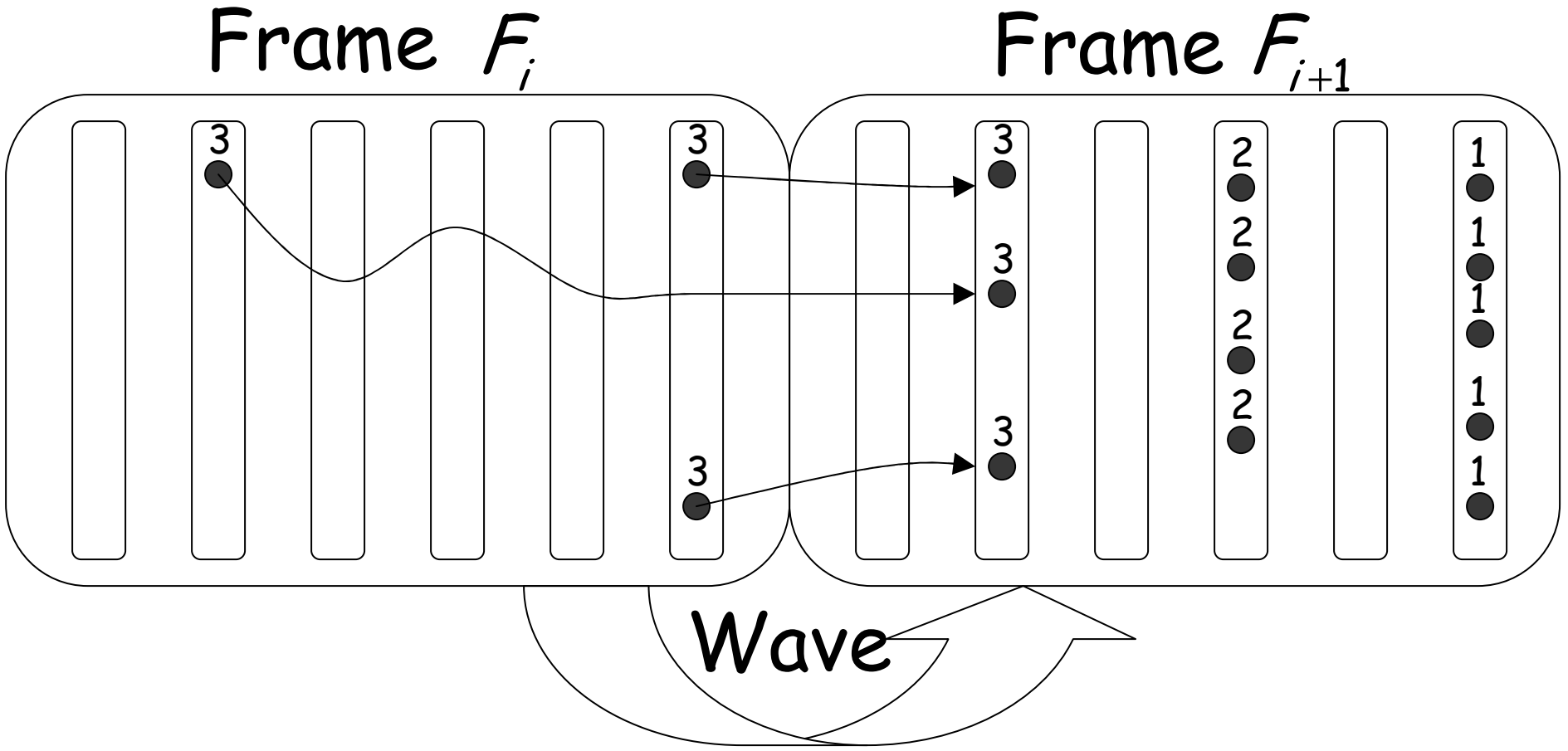
Time needed: $\Theta(\log(DN))$

Similarly, send packets of color 2



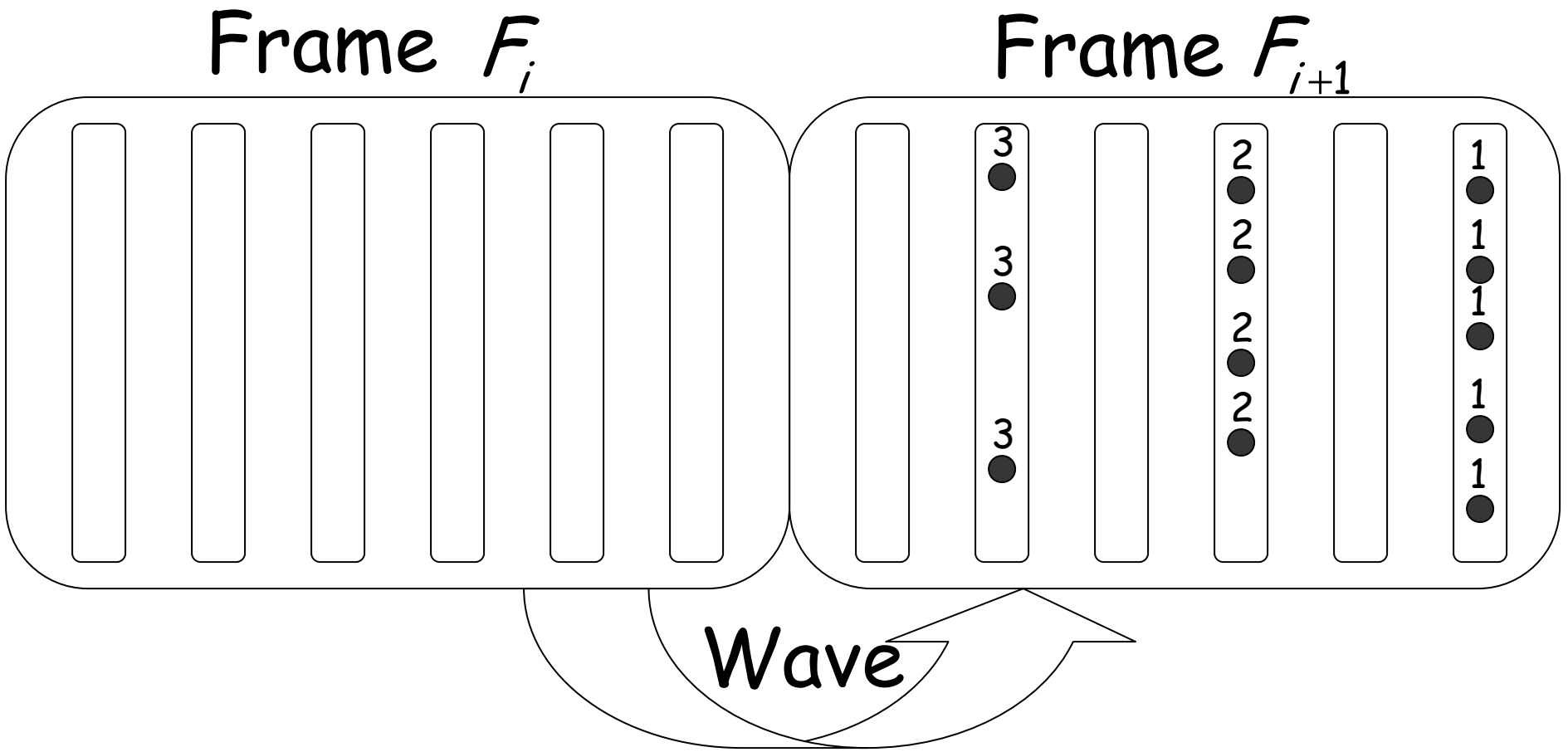
Packet paths don't conflict

Similarly, send packets of color 3



Packet paths don't conflict

All packets have been delivered



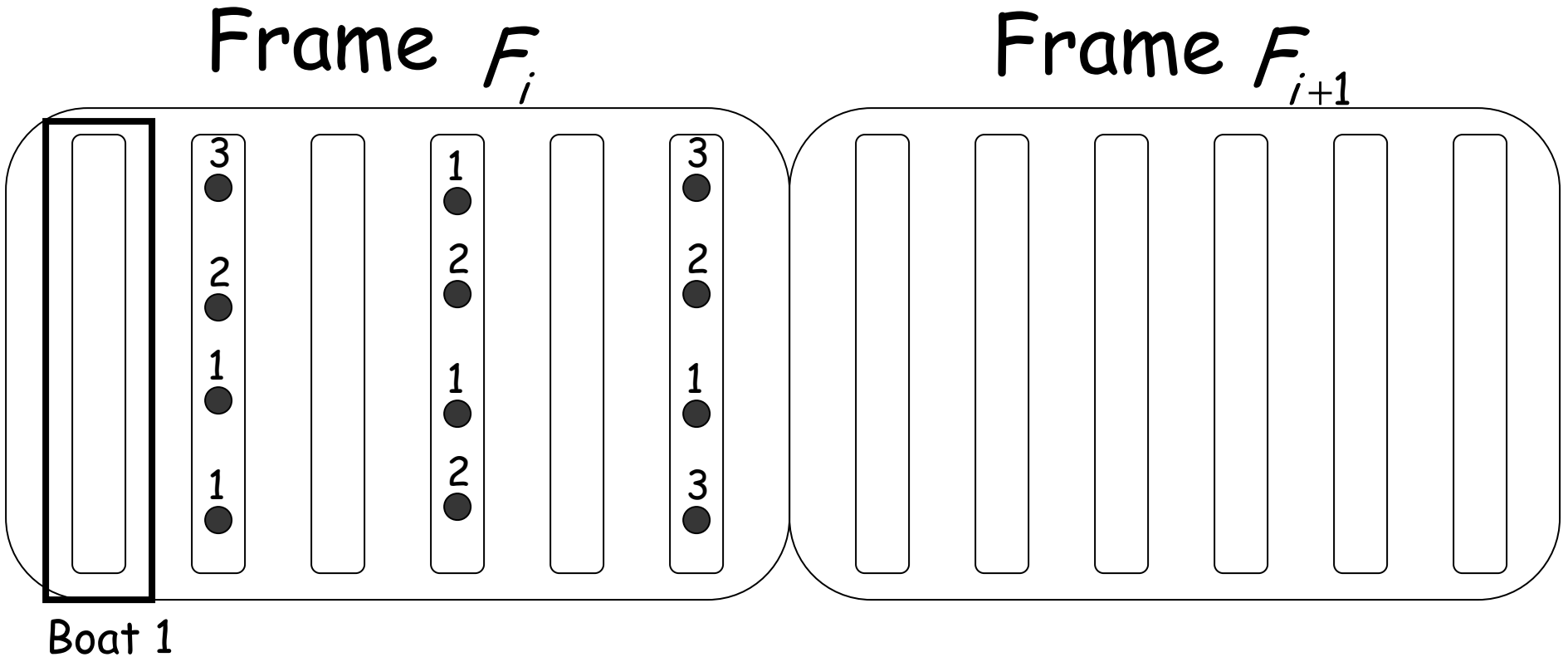
Wave time: Colors X 2 Frame size

$$\log(DN) \times 2\log(DN) = \Theta(\log^2(DN))$$

We can speed up the process
by pipelining different colors:

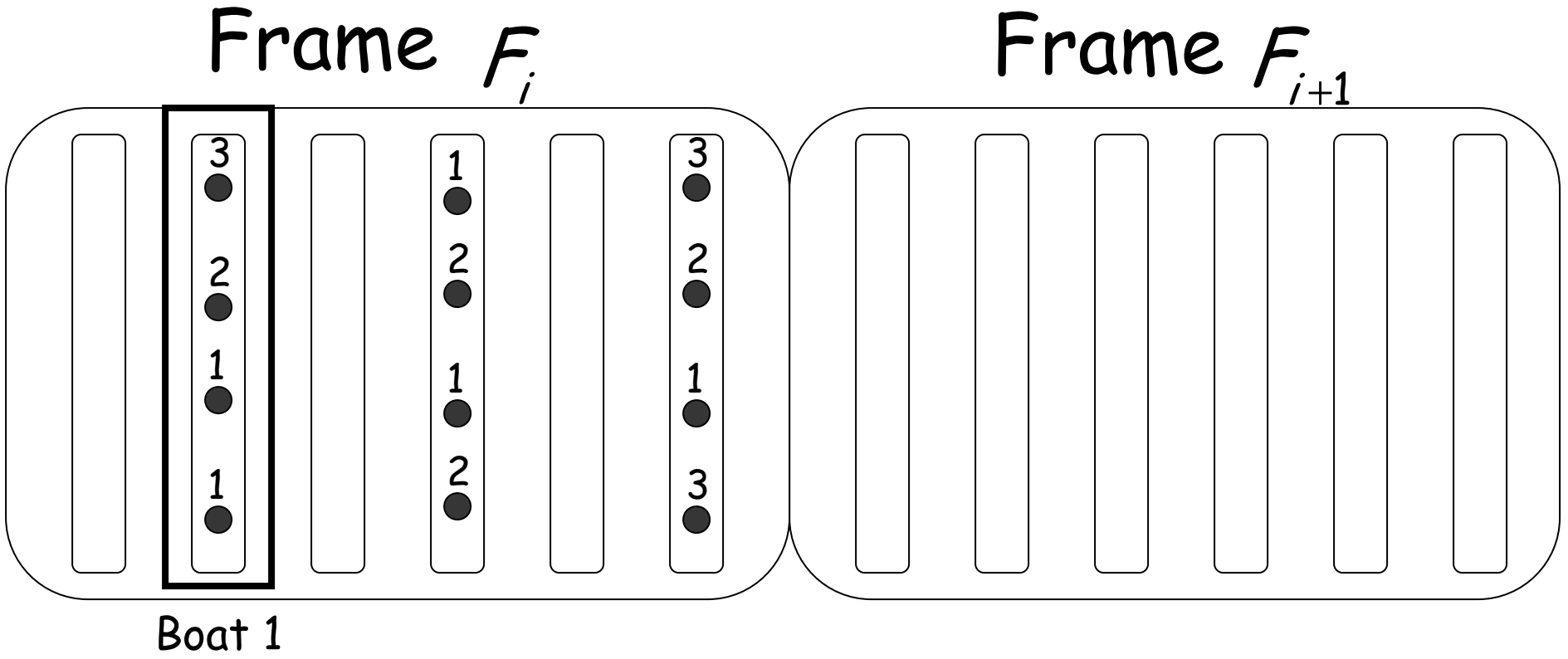
$$\Theta(\log(DN))$$

Pipelining using Boats



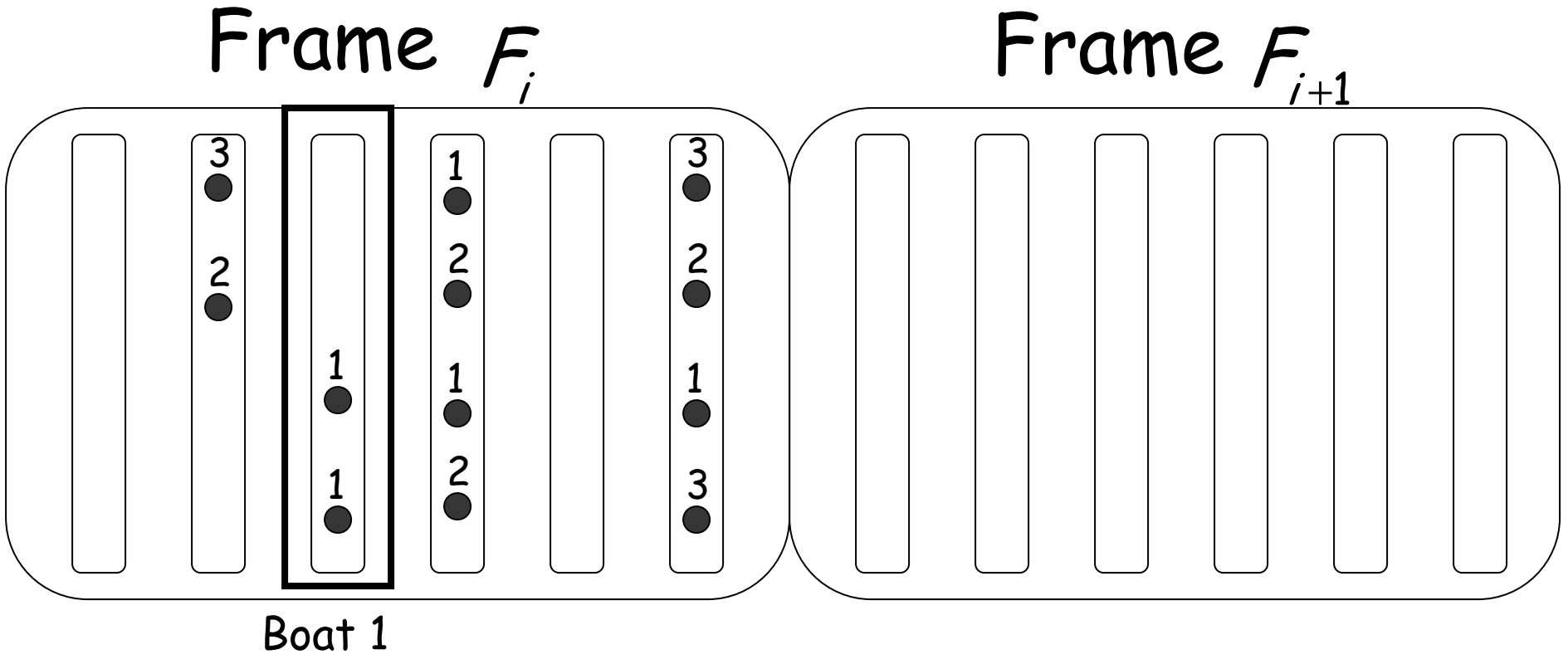
Packets of color i follow boat i

Pipelining using Boats



Packets of color i follow boat i

Pipelining using Boats



Packets of color i follow boat i

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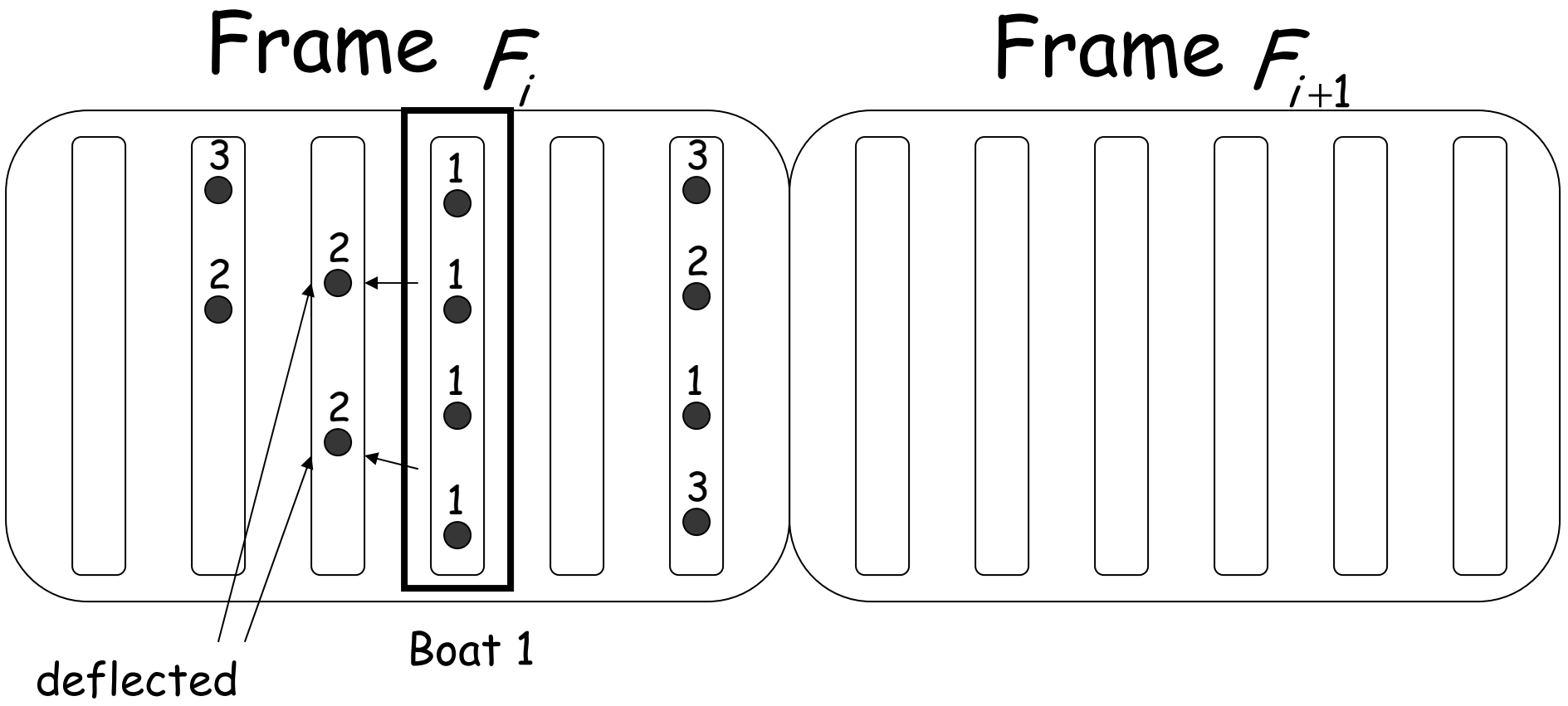

Packets of color i follow boat i

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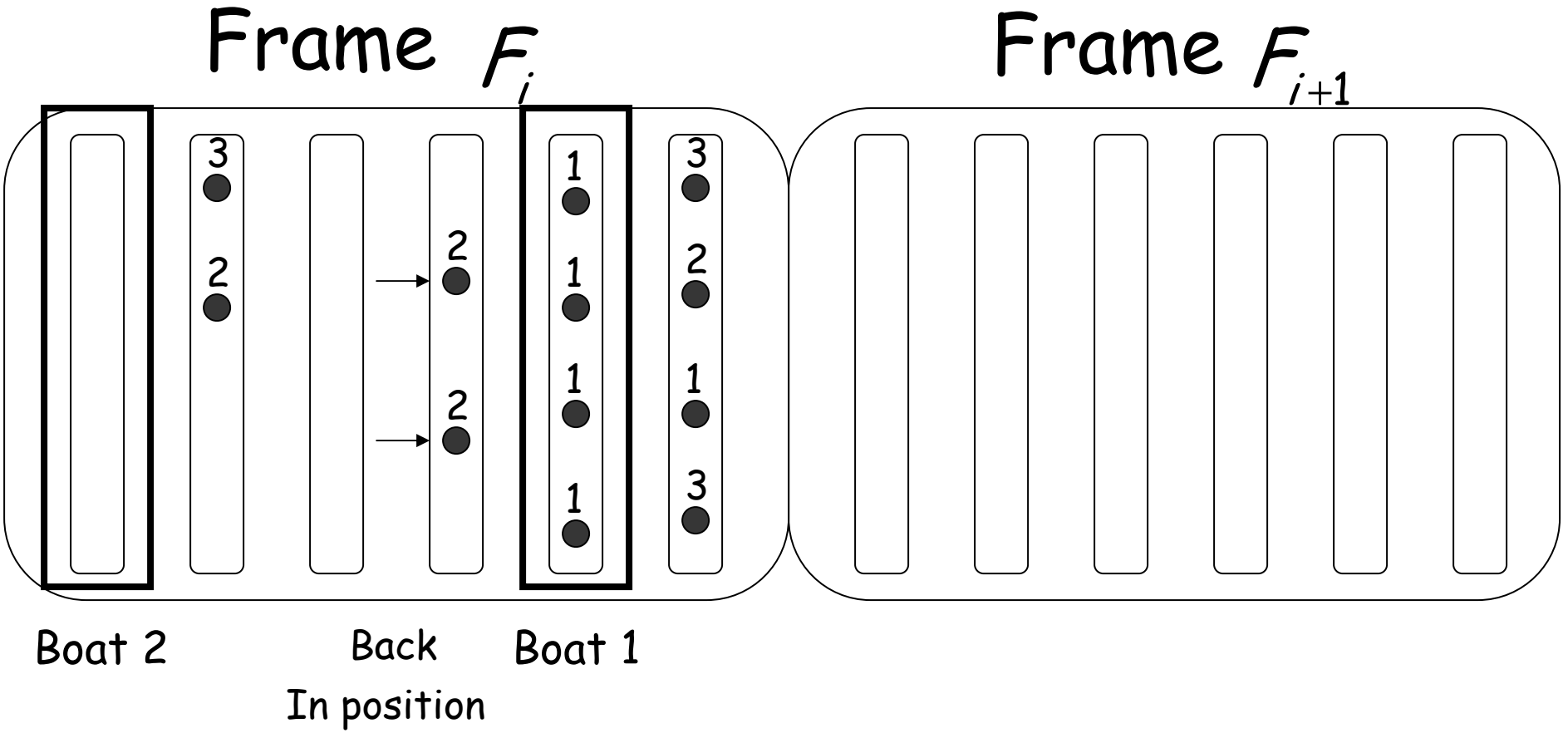

Packets of color i follow boat i

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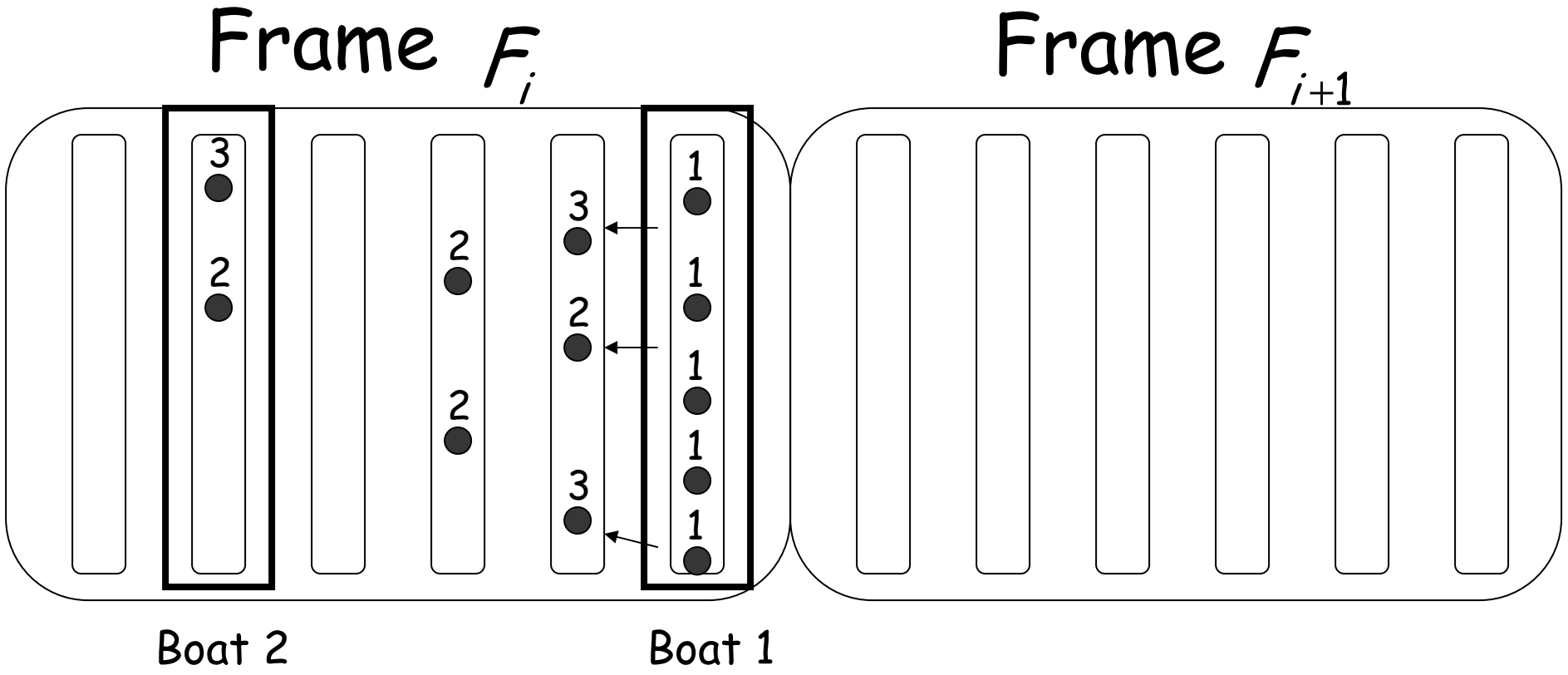

Packets of color i follow boat i

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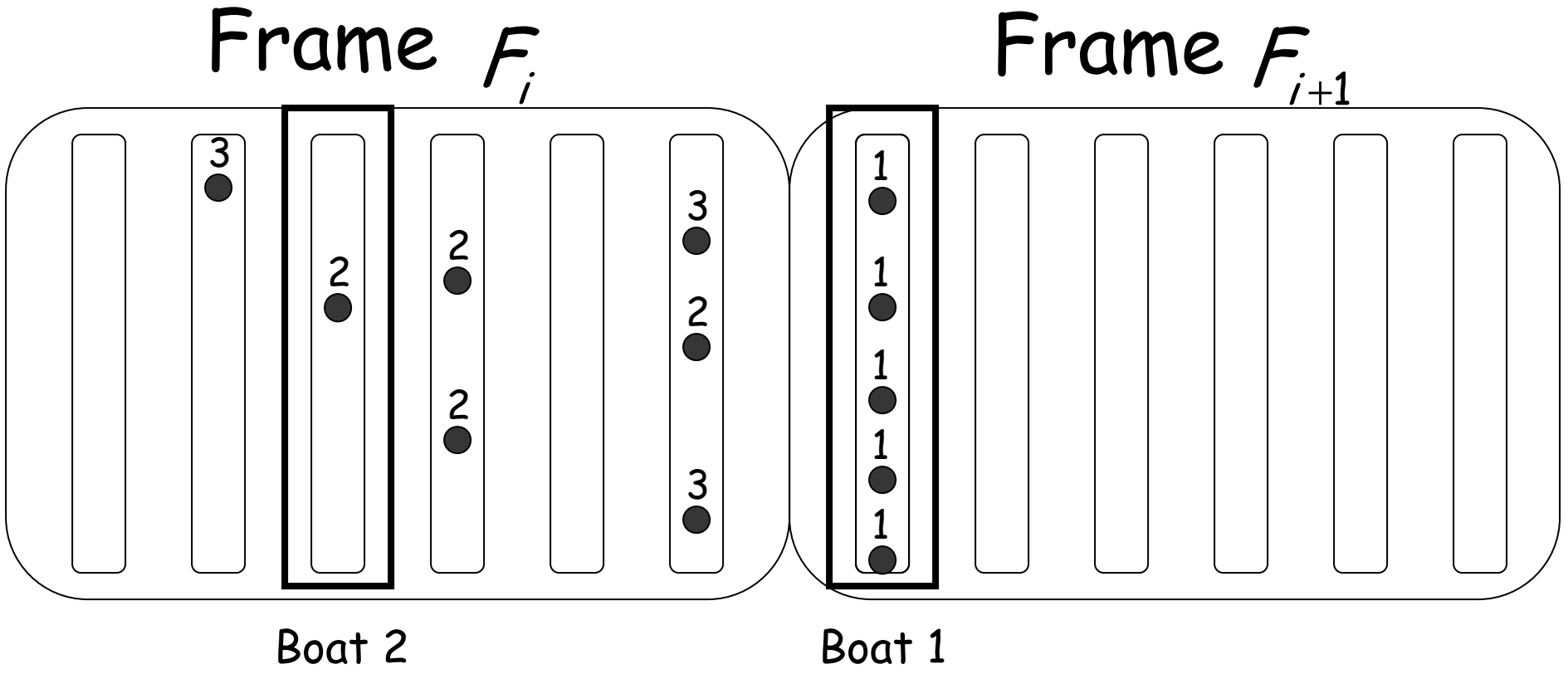

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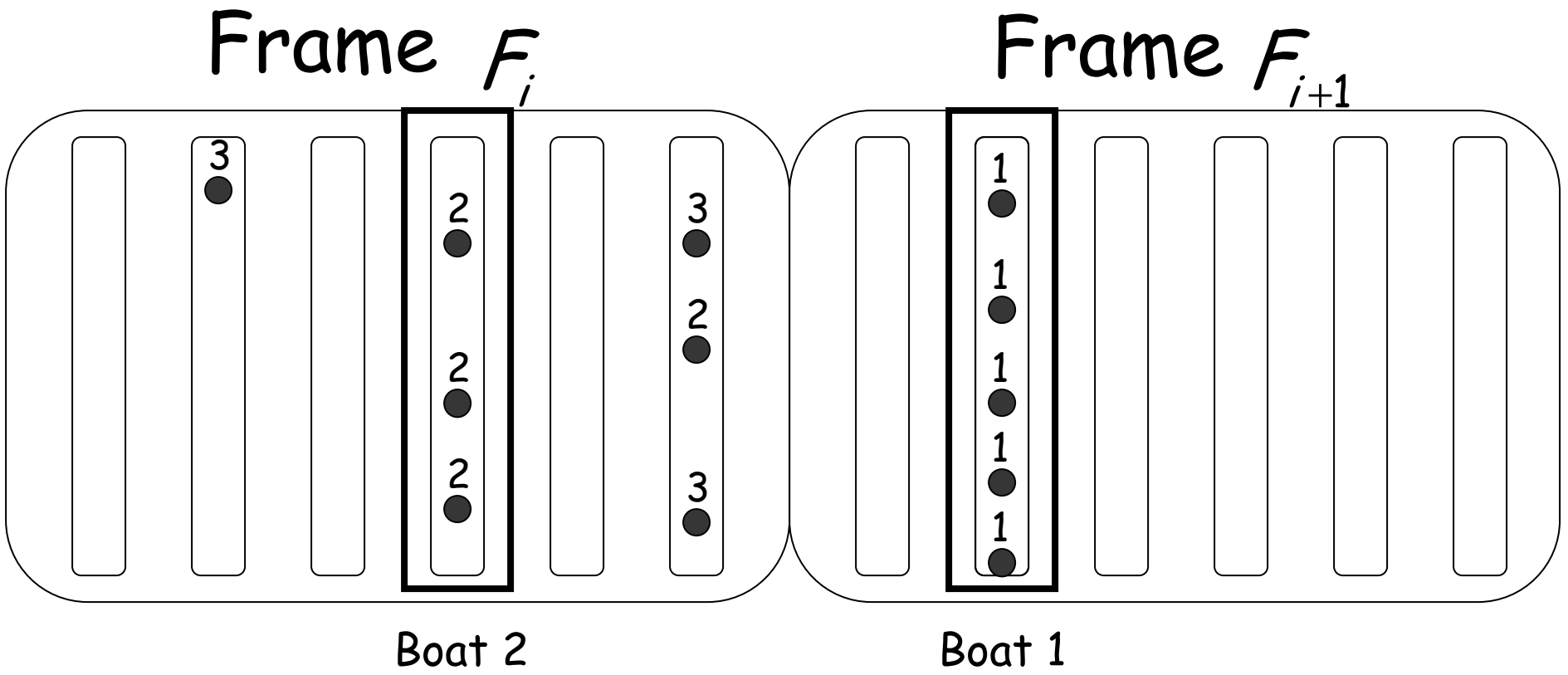

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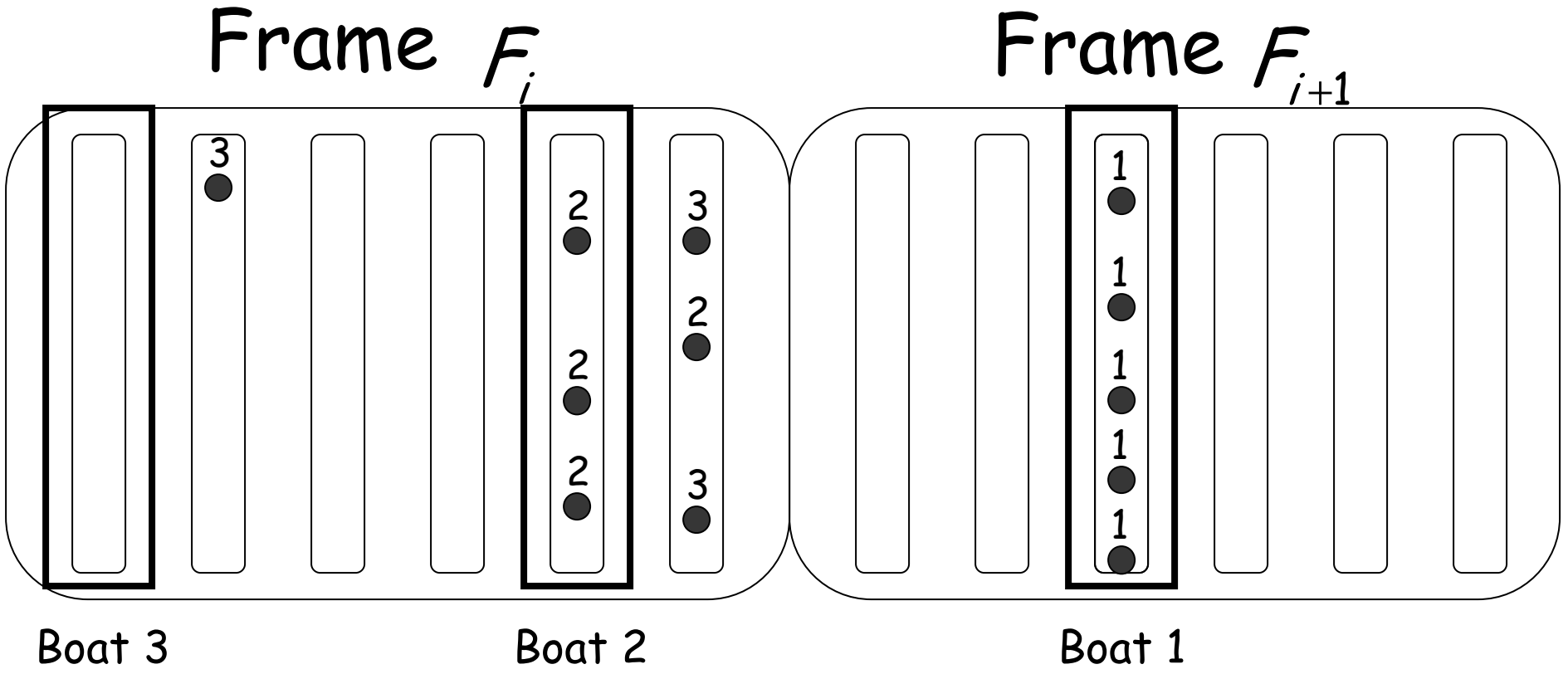
Pipelining using Boats



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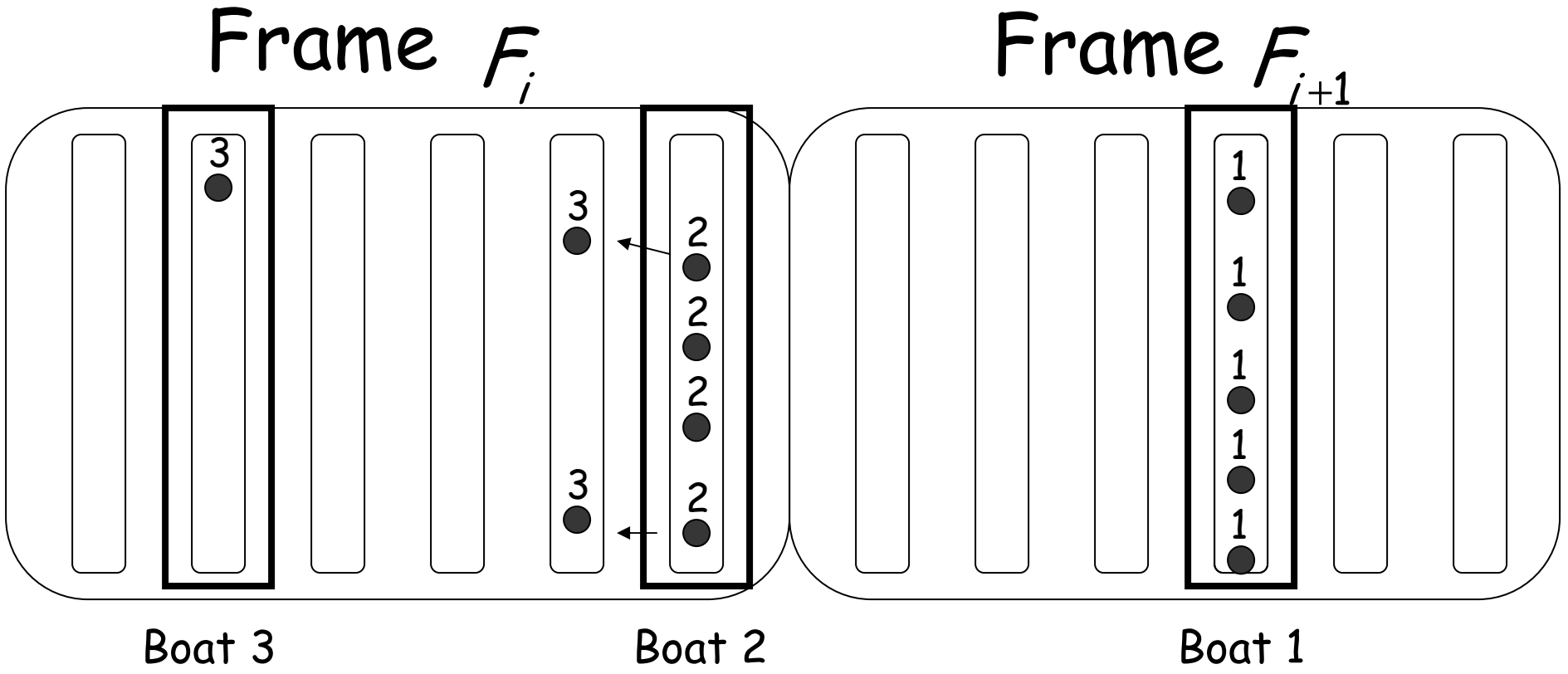

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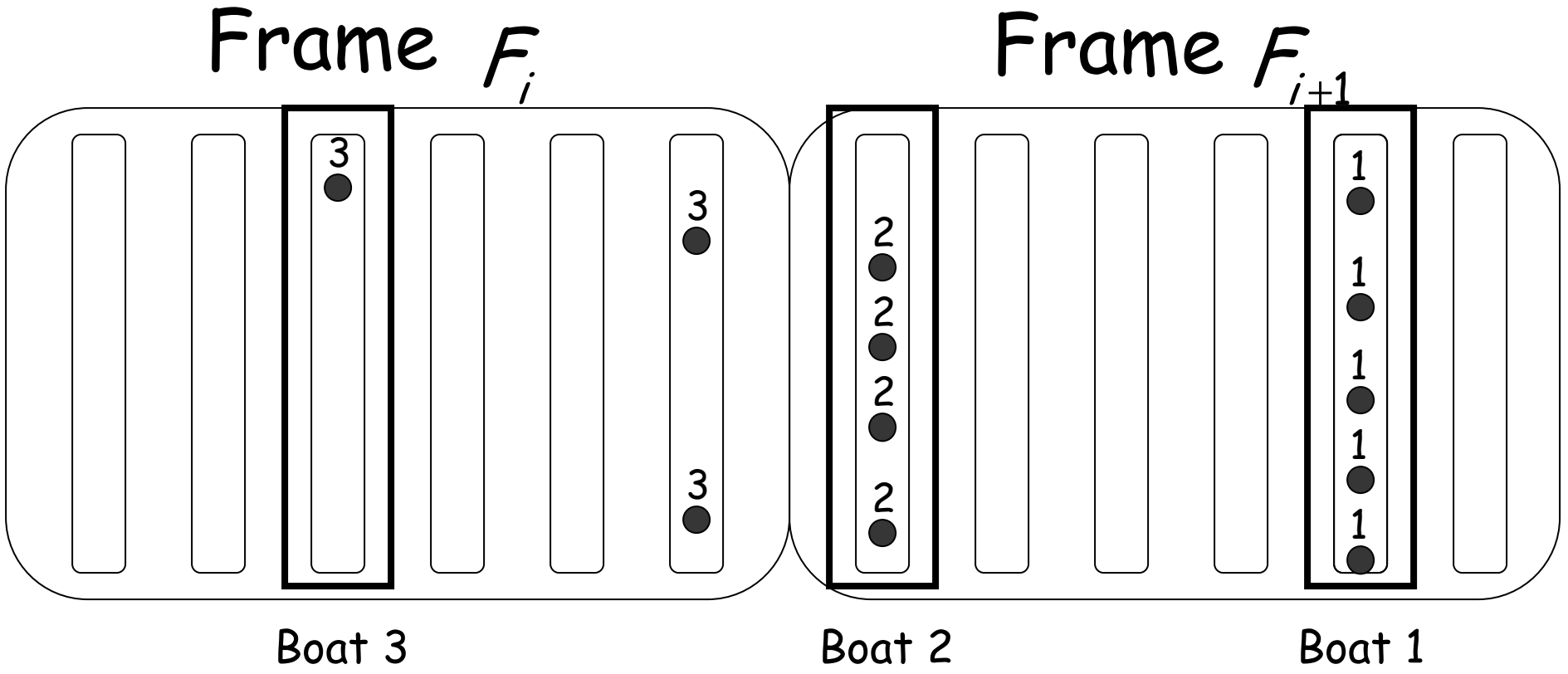

Packets of color i follow boat i

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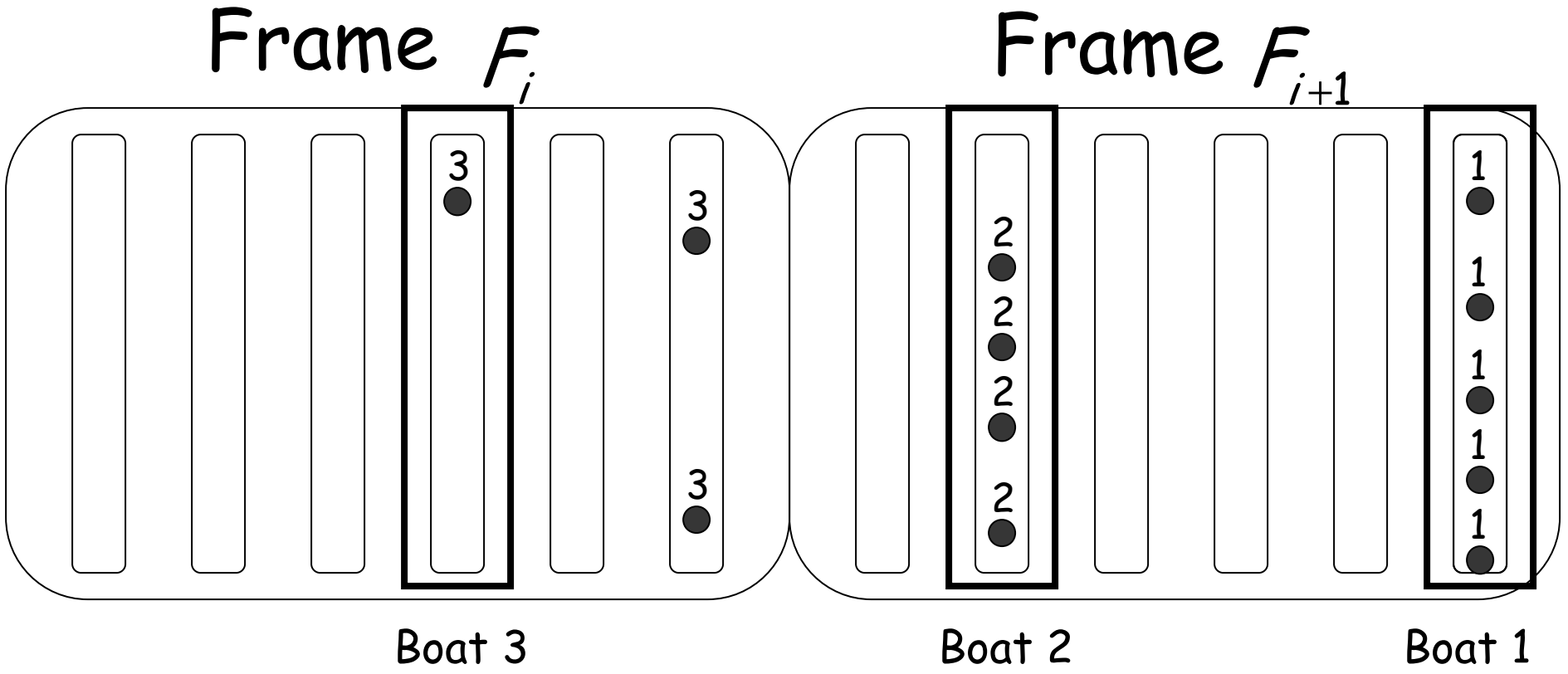

Packets of color i follow boat i

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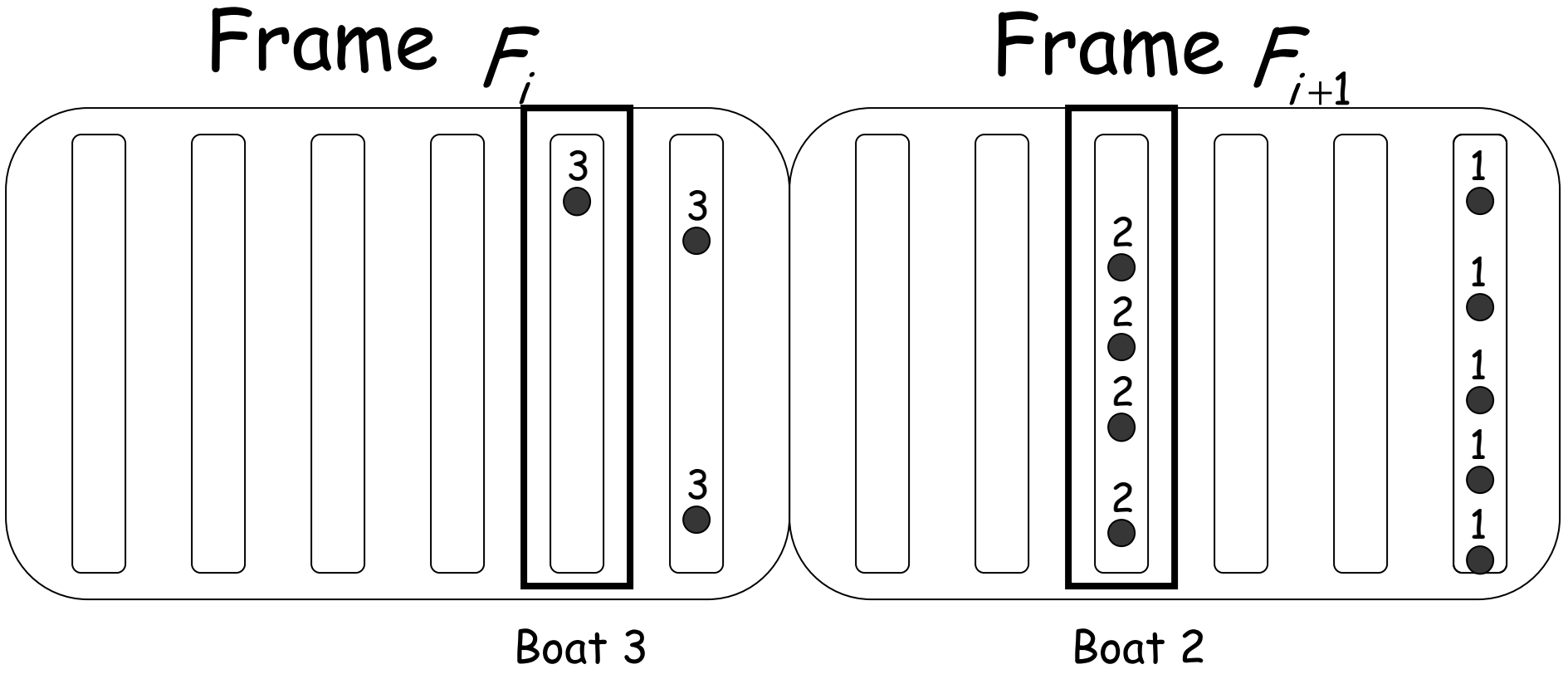

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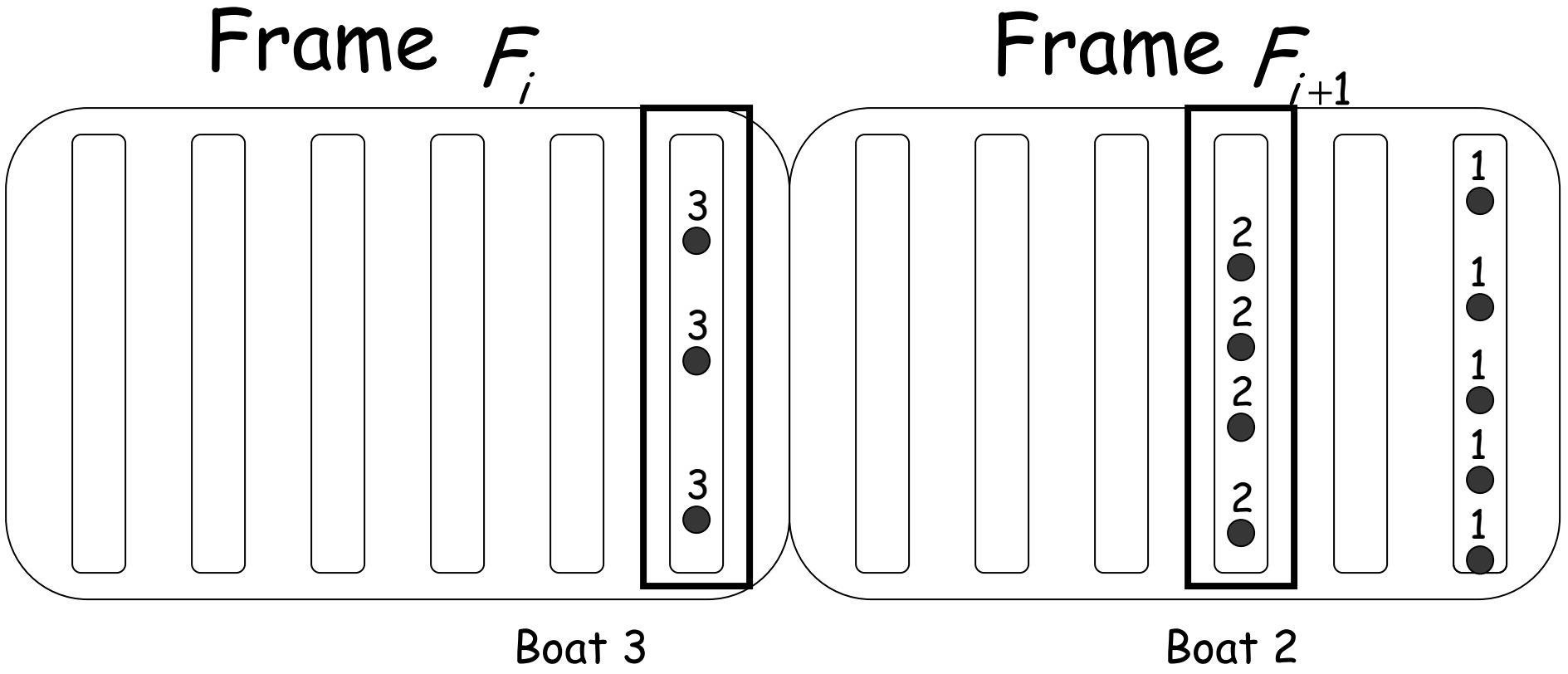

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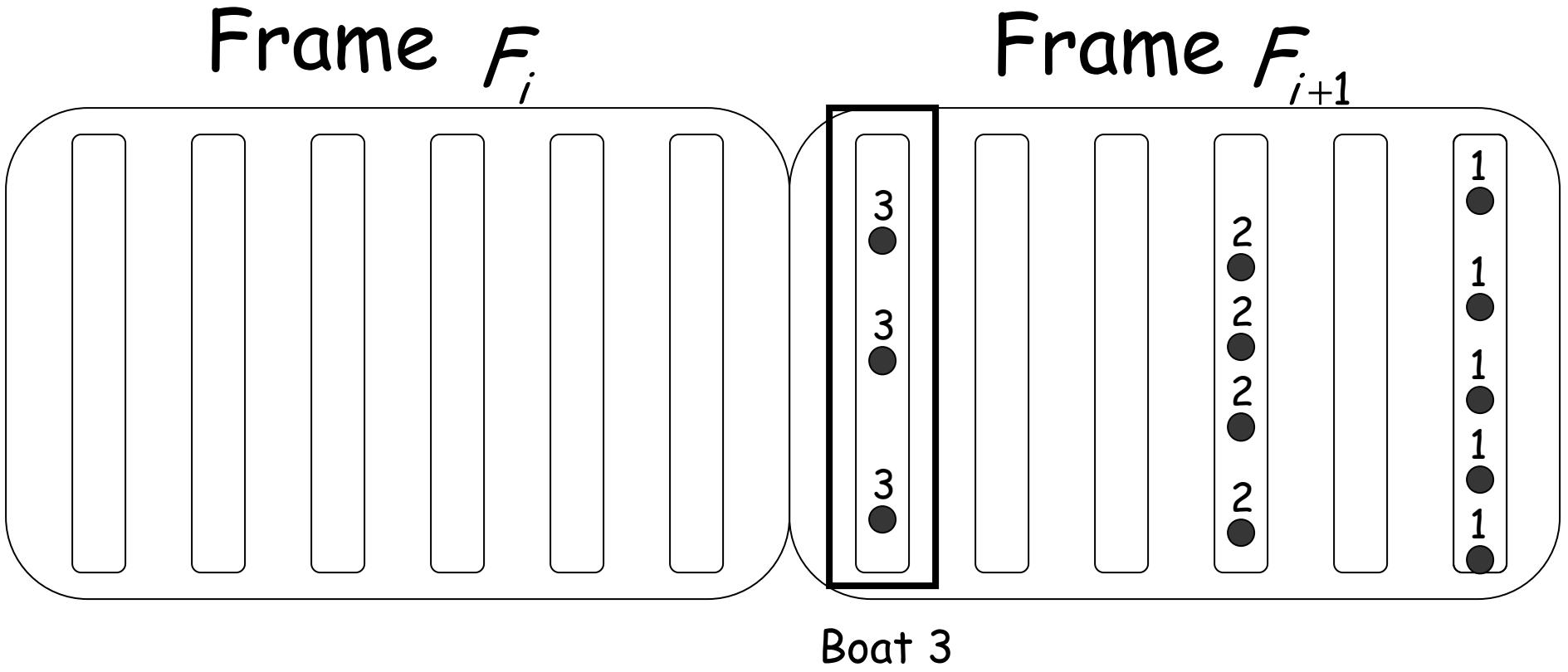

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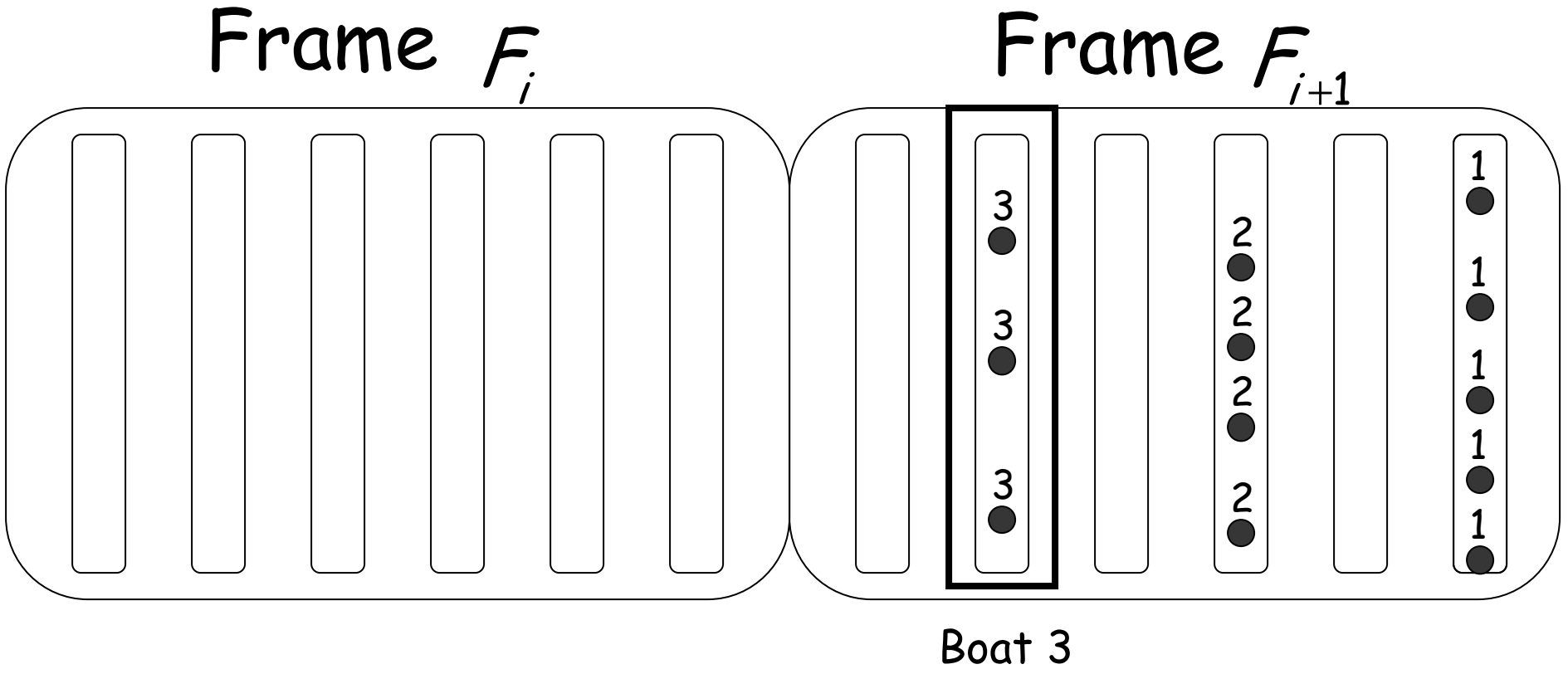

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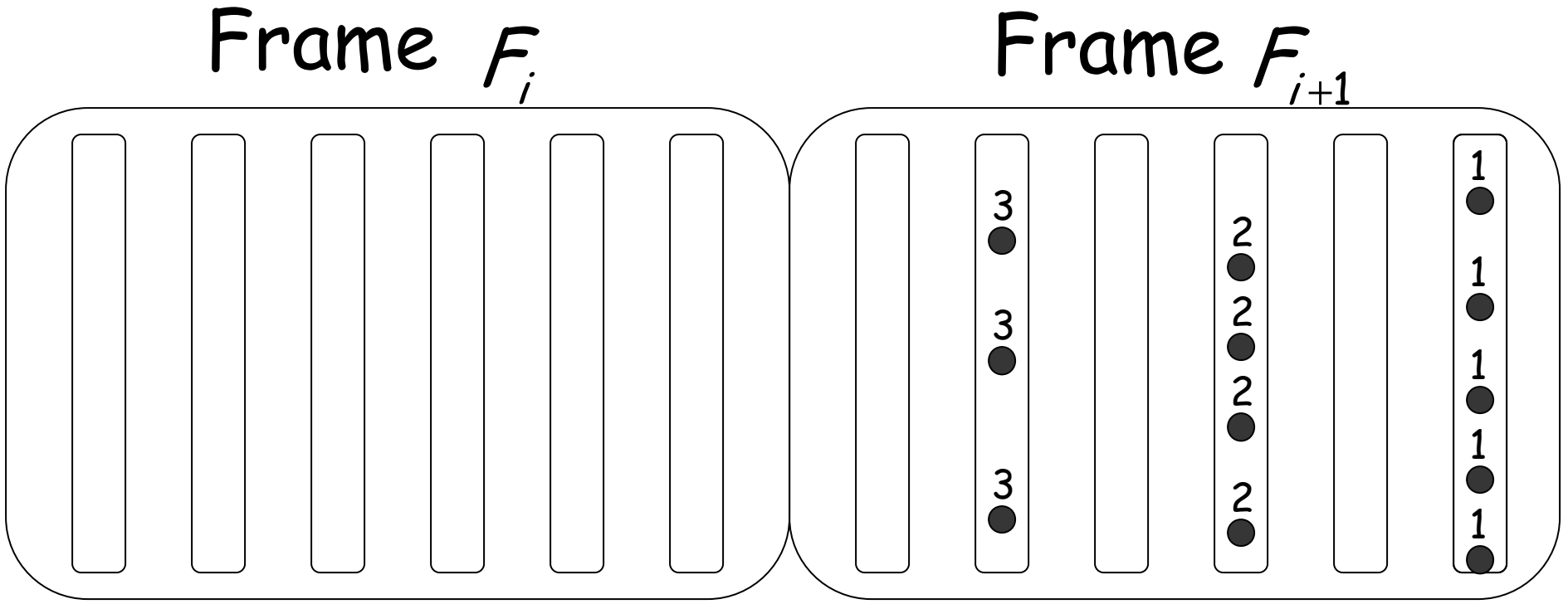

Packets of color i follow boat i

Wave time: Time until last boat reaches target level

Number of colors

Frame size

$$4\chi + 2\log(DN)$$
$$= 4\log(DN) + \log(DN)$$
$$= \Theta(\log(DN))$$

Talk Outline

- Introduction
- Centralized Algorithm
- Distributed Algorithm
- Conclusion

In the distributed version,
we assume that every node knows
parameters C, D, N

Nodes do not know the packet paths,
except for the packets in them.

The distributed algorithm
is the same with the centralized,
except for one thing:

The conflict graph is colored in a distributed manner

Distributed coloring – Basic Idea

During a wave:

1. each packet chooses a random color Between 0 and 2log(DN)

2. each packet assumes the color is correct and follows the respective boat

3. For packets that conflict, the process repeats

This process repeats a logarithmic number of times, thus it gives an extra logarithmic factor in the performance

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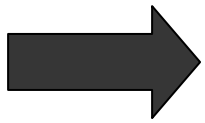

Conclusion

- Centralized algorithm $O(C \cdot \log(DN) + D)$

  Uses conflict graph coloring

- Distributed Algorithm

  Worse by a logarithmic factor

  Implements distributed coloring