Callibration of Multi-Factor Vasicek Interest Rate Models

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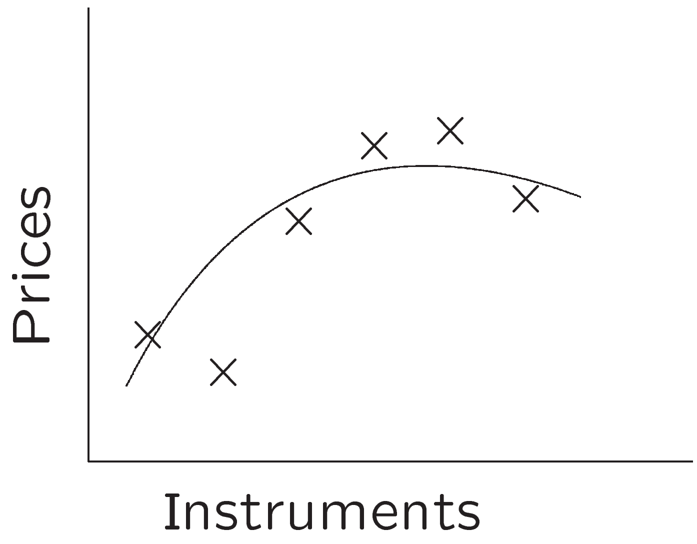
GOAL

- We have a set of instruments (eg interest rate swaps, caps, swaptions) and market prices – **Historical Data**.



- Assume a model: **Vasicek-k factor** model, which gives the interest rate dynamics.

- To the extent that the model describes the market dynamics, obtain the fair value of the instruments today.

MODEL: VASICEK K-Factor DYNAMICS

- $dx_i(t) = k_i(m_i - x_i(t)) + \sigma_i dW_i(t)$
  $$i = 1, \ldots, K$$

  Patameters (Y): k_i, m_i, σ_i and ρ_{ij}. $\left[\frac{K(K+5)}{2}\right]$

  States (x): x_i. $[K]$

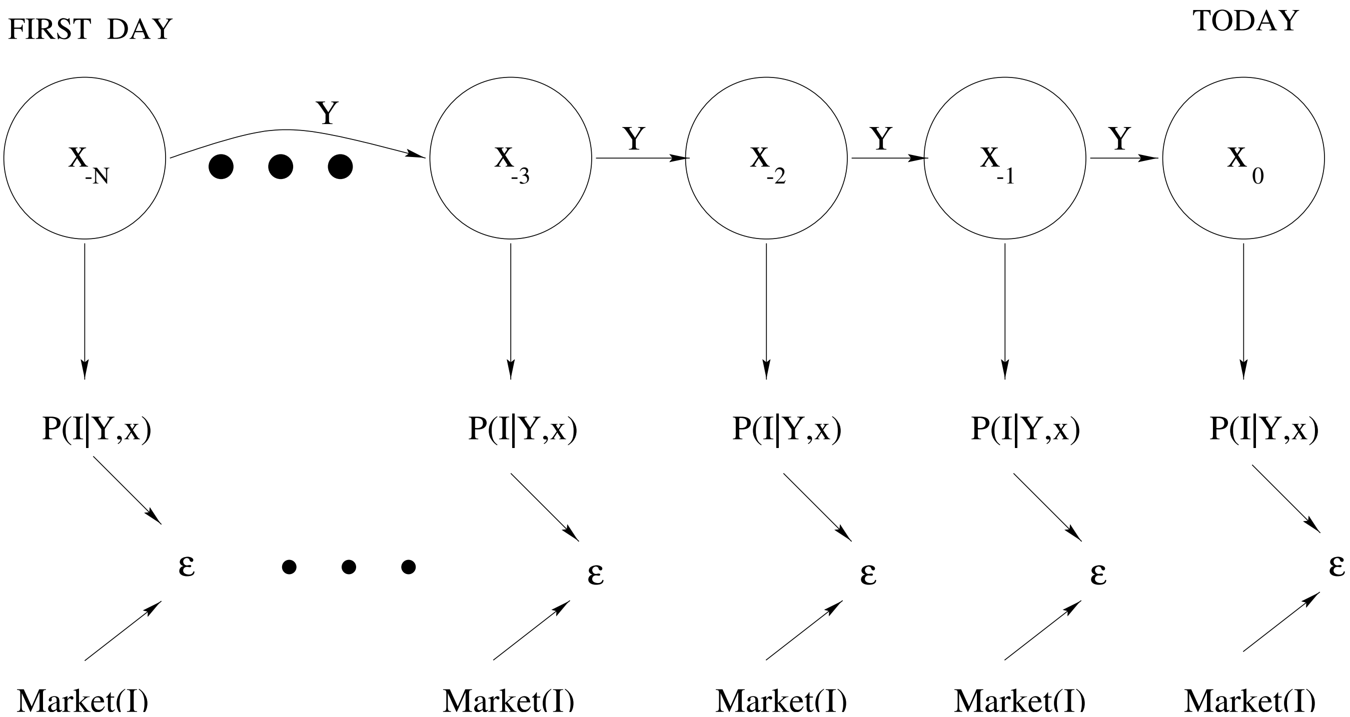
- Interest rate dynamics are given by

$$r(t) = \sum_{i=1}^{K} x_i(t)$$

- For any instriment, I, one can compute

$$Price(I|\mathbf{Y}, \mathbf{x})$$

PROBLEM STATEMENT



- Dynamics given by Vasicek model.
- ϵ is a noise "shock", assumed to have some distribution, à la maximum likelihood.
- Noise term represents (tradeable) mispricing that is expected to "revert" back to zero.

$$\boxed{\text{Determine } \mathbf{Y}, \mathbf{x}_0.}$$

COMMONLY USED APPROACH

- Use *plausible* values for long term parameters.

- *Callibrate* each day's states to that days prices.

- Is this is curve fitting?

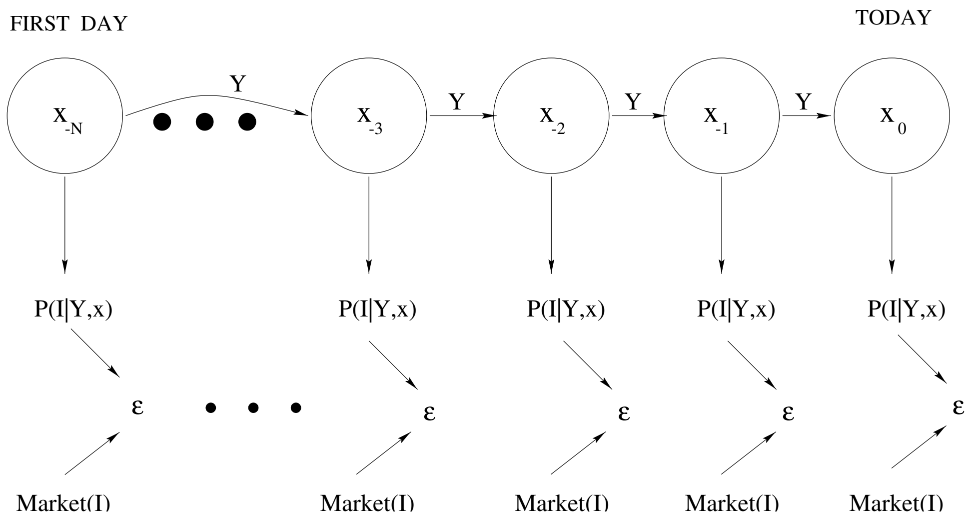
  - Fast

  - Does the callibration have any "economic" meaning?

  - Use small models, which may not realistically describe the dynamics.

  - As a result, can't "really" trade on the model-market discrepancies.

HIDDEN MARKOV MODEL



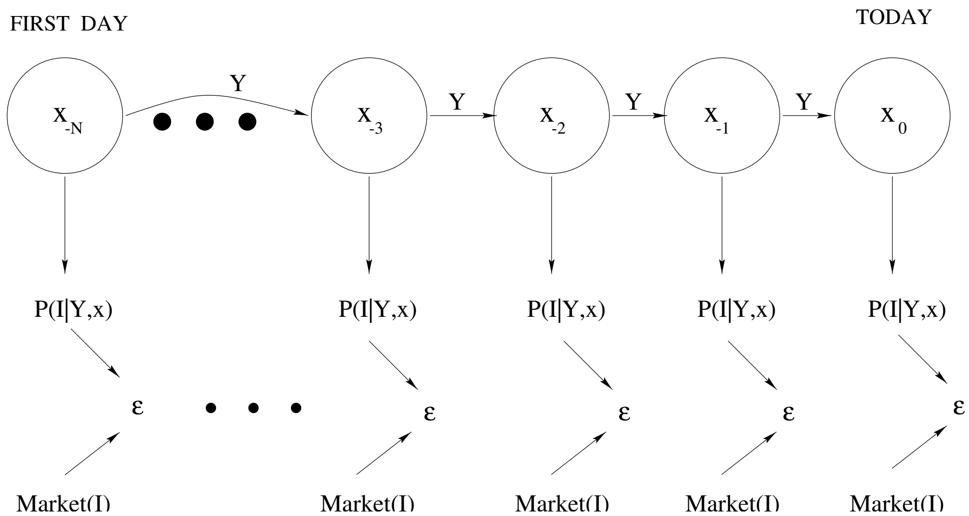
- Naturally posed as a HMM with infinite dimensional state space and known non-linearity.

- Unknowns: $\mathbf{Y}$, $\mathbf{x}_i$: LOTS of them.

- Optimize $Prob$[Unknowns|Data].

- Hard: Many variables, and Pricing function is complex.

E-M APPROACH



- Break down into two easier problems.

  1. Fix $\mathbf{x}_i$'s, and optimize $\mathbf{Y}$.

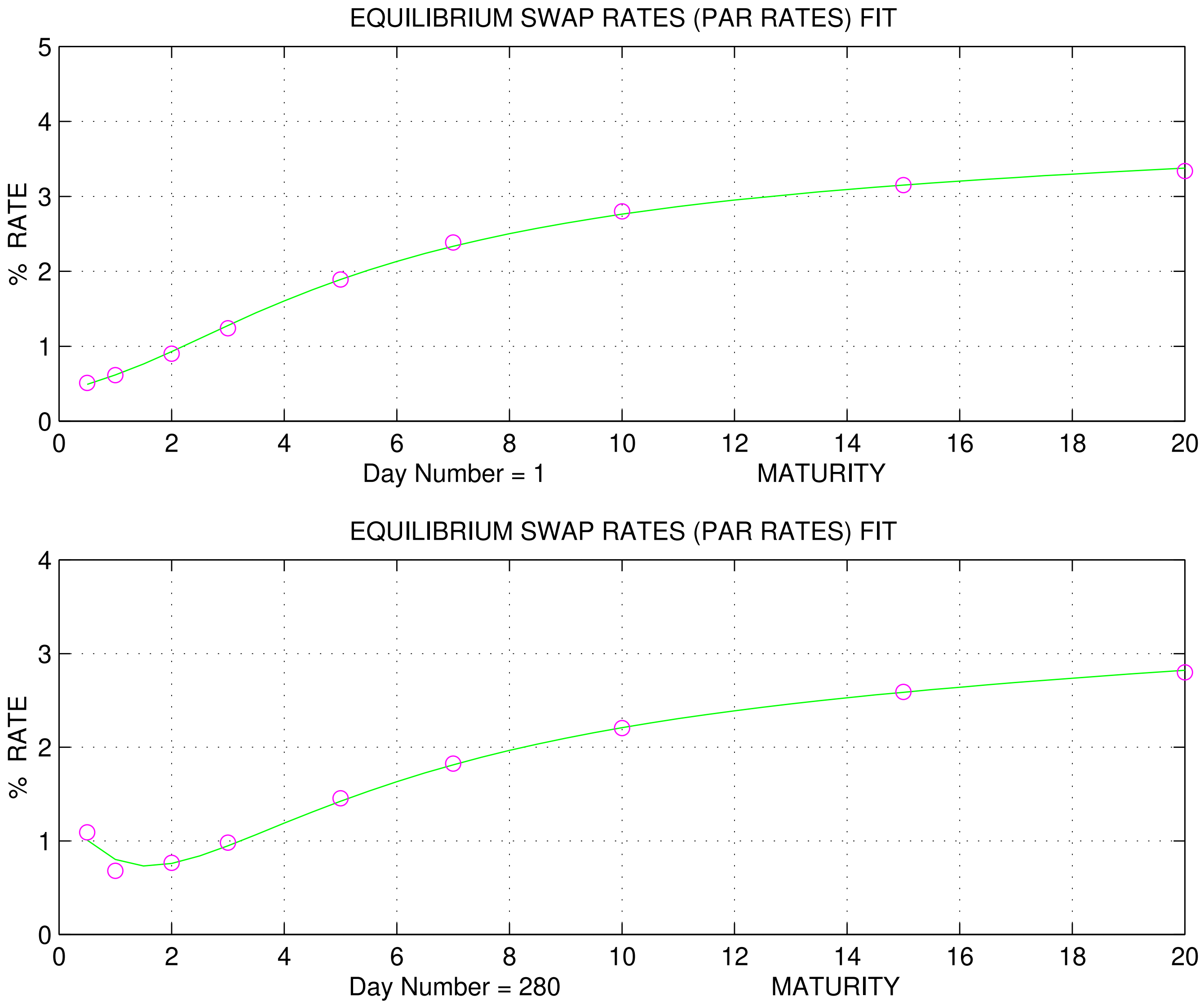
  2. Fix $\mathbf{Y}$, and optimize $\mathbf{x}_i$'s.

  3. Iterate.

- Can solve in a sequential manner (convergence?).

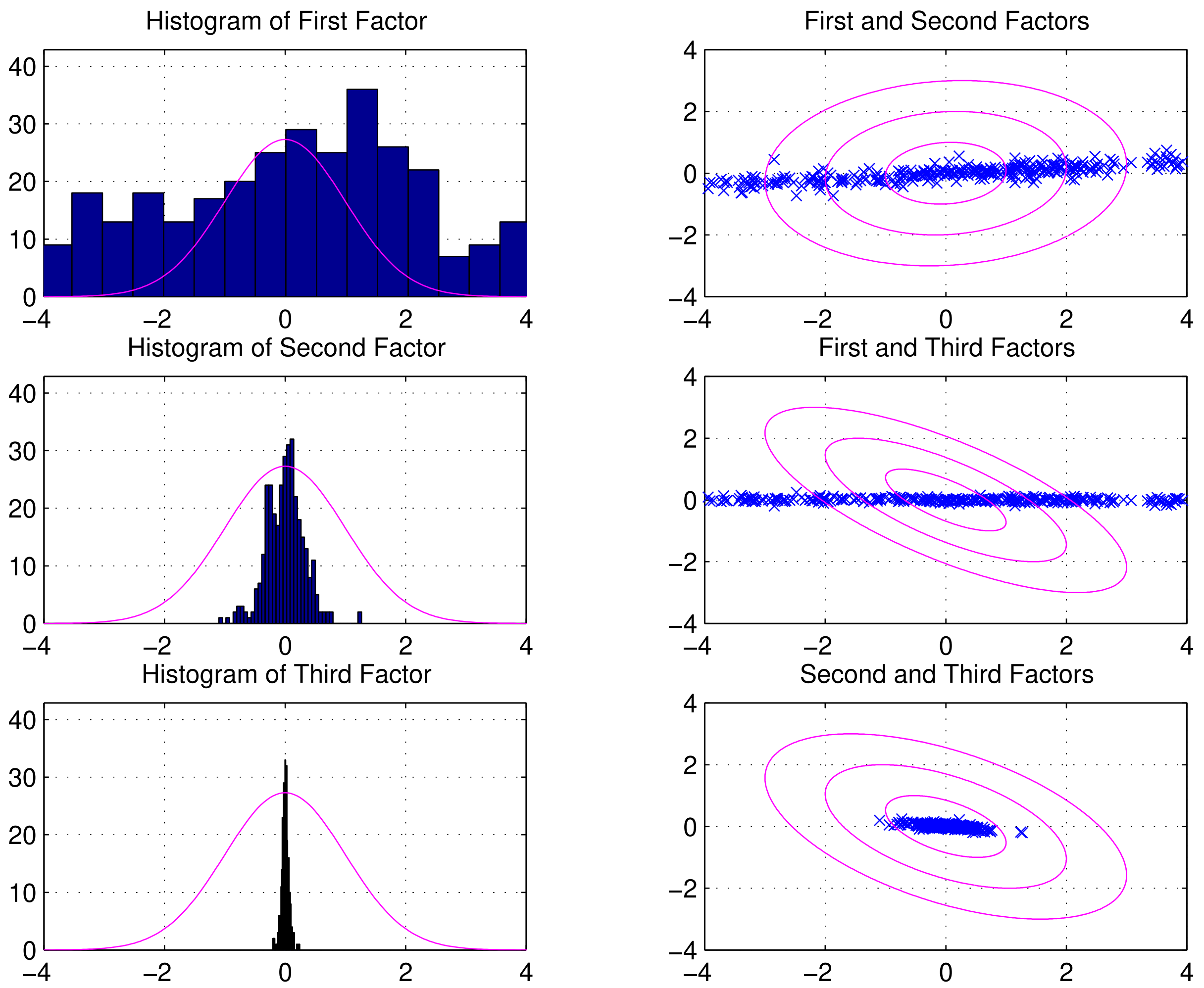
- Finite iterations: order is important, especially at end points.

FITS TO SWAP DATA



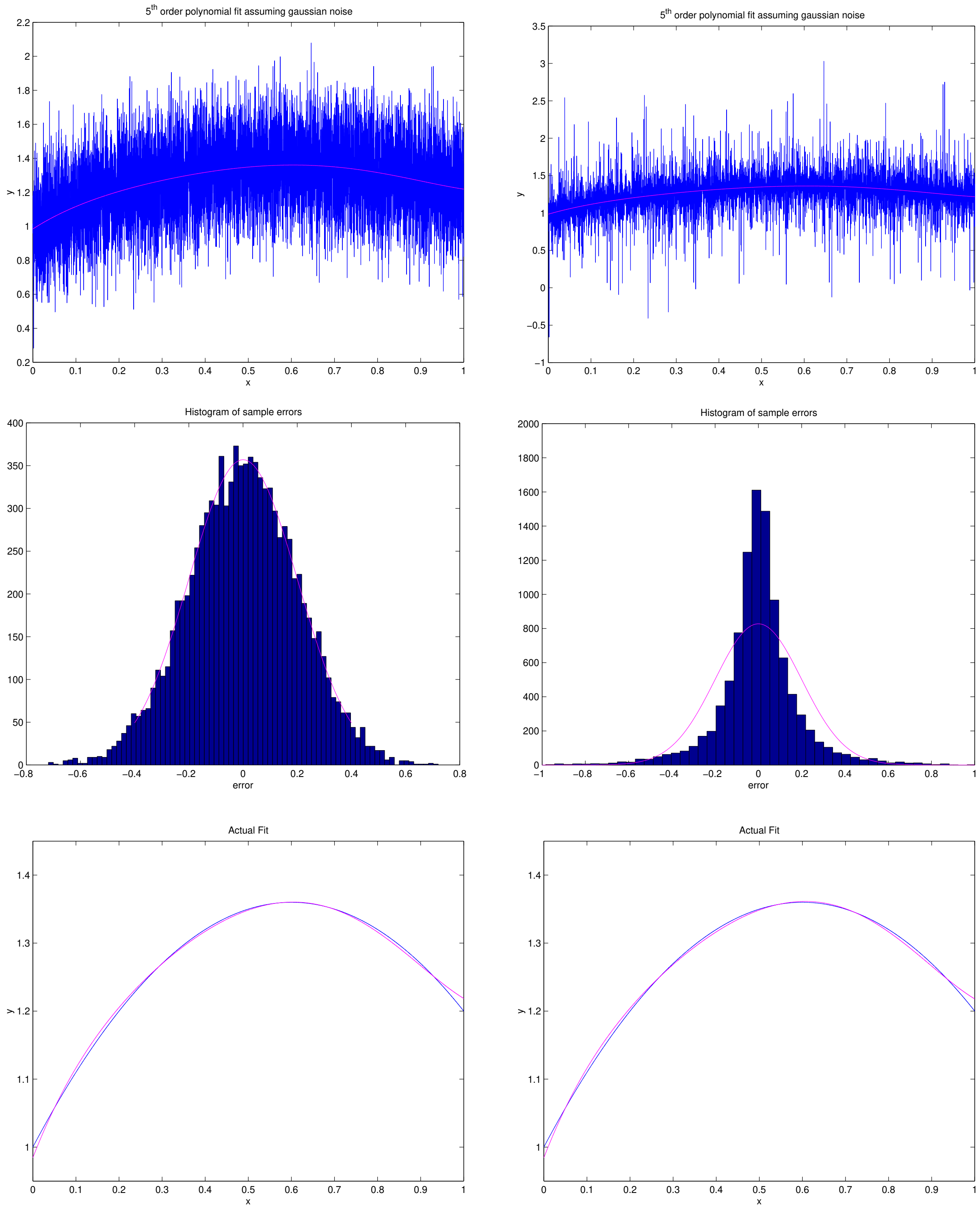
But, . . . , suppose that we want the volatility term structure as well.

HIGHER ORDER STRUCTURE



Severe mismatch between model and historical!

WHAT HAPPENED



CORRECTING THE ERROR

1. Change model: assume different noise model, start over (eg. fat tails).

2. Control the way we search the space: regularize.

   - Callibrate

   - Make sure that model assumptions are satisfied.

Ensure that Final Model is **CONSISTENT** with **initial** assumptions

CONSISTENCY HINTS

1. **Consistency** of states with parameters.

2. **Plausibility** of the parameters.

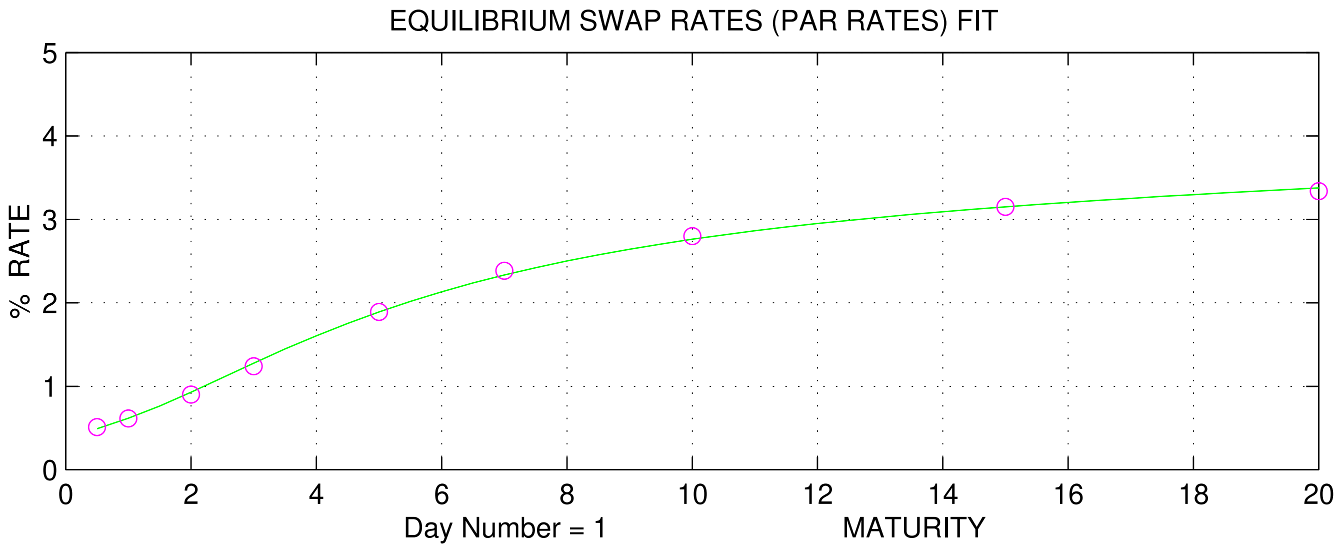
3. **Validity** of the driver as a Wiener Process.

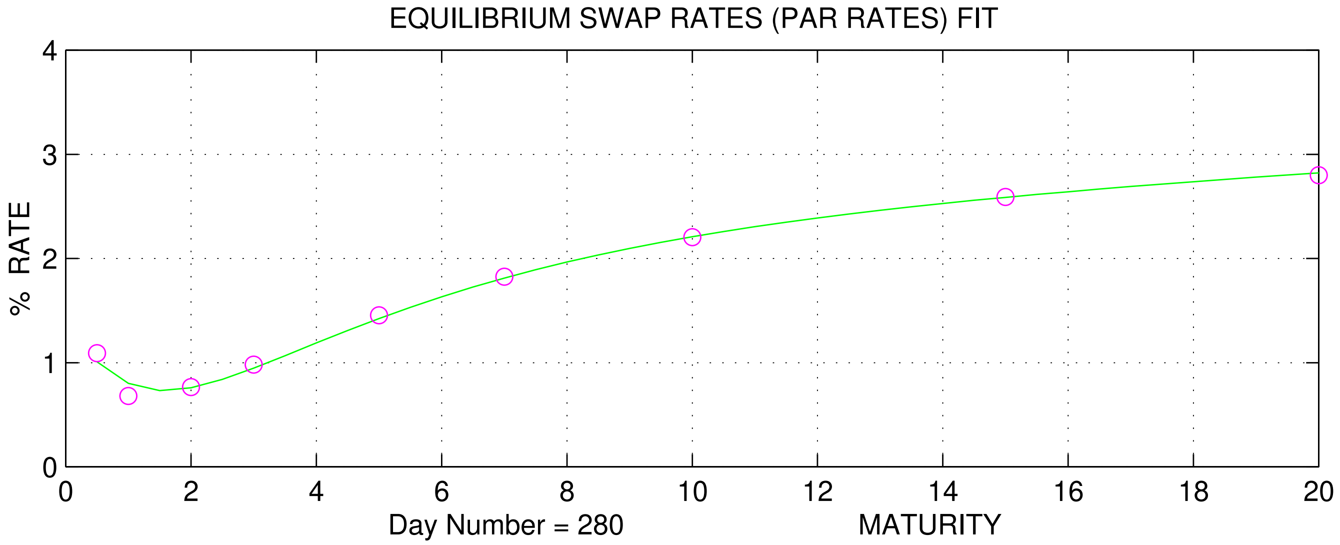
How does one get these?

- Probabilistic approach: Expand $Prob[(\cdot)|Data]$ in terms of conditionals. How to get these probabilities?

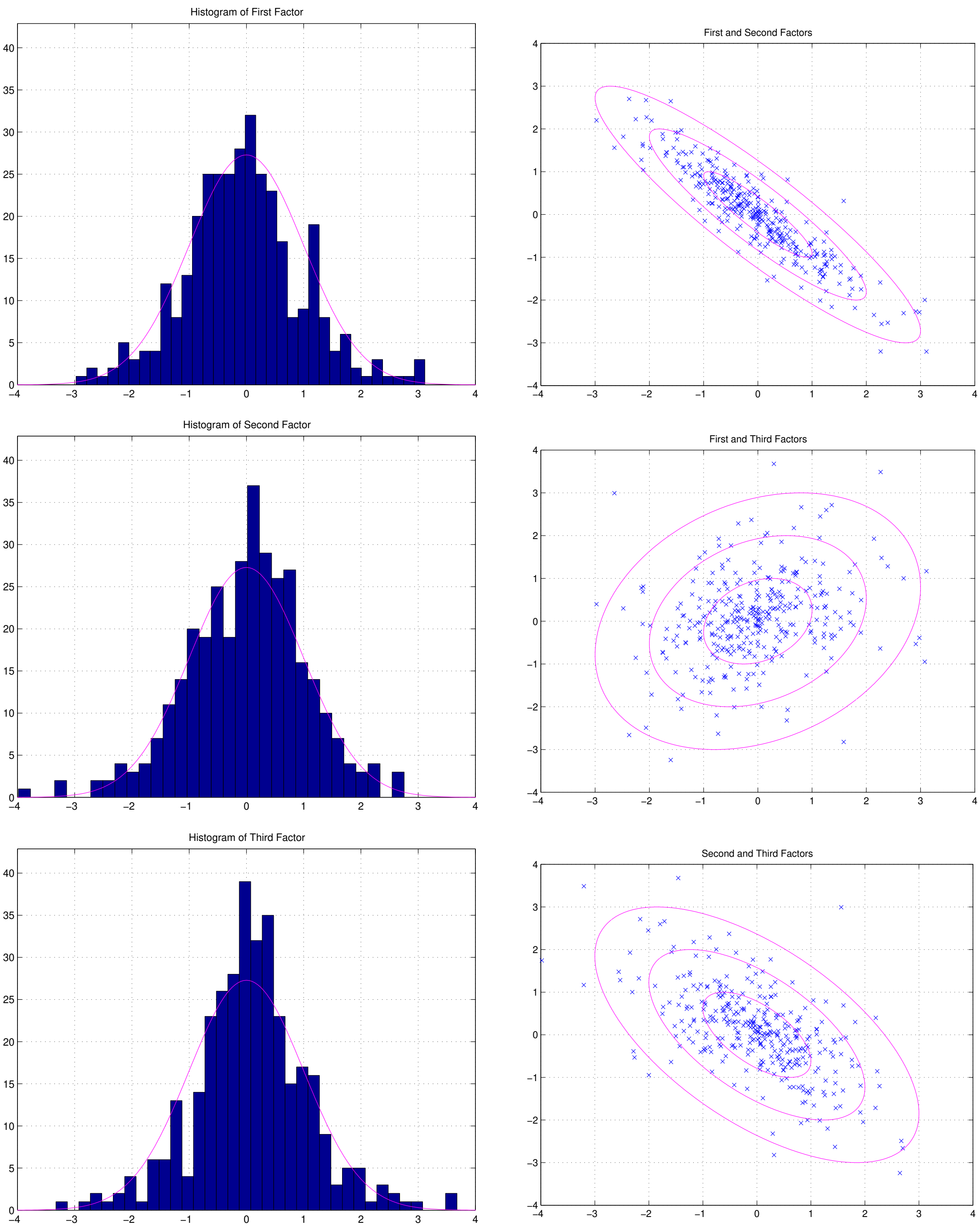
- Add error terms that enforce these "initial" assumptions with corresponding regularization parameters. How to get the regularization parameters?

FITS TO SWAP DATA – Part II

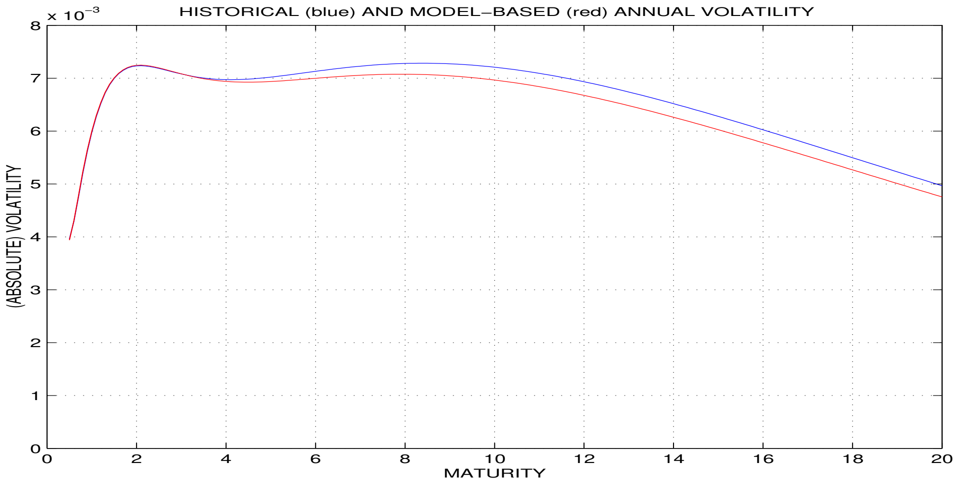


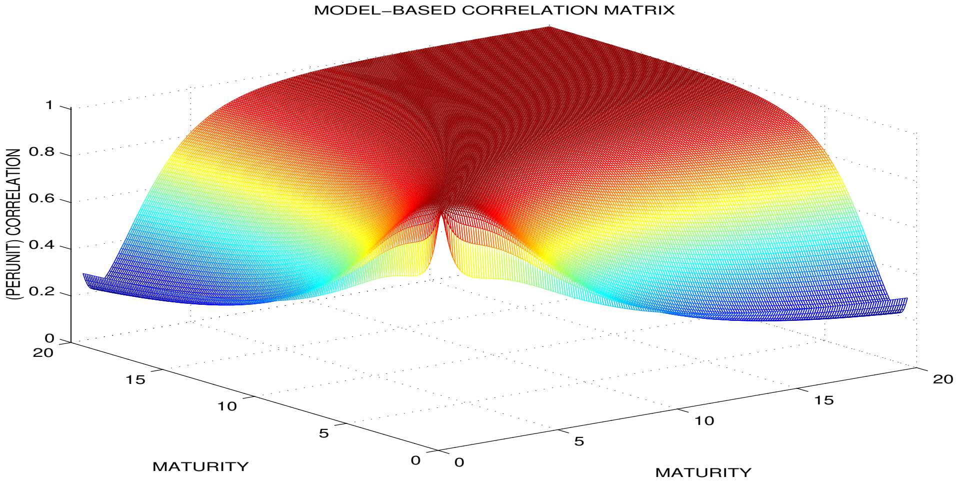


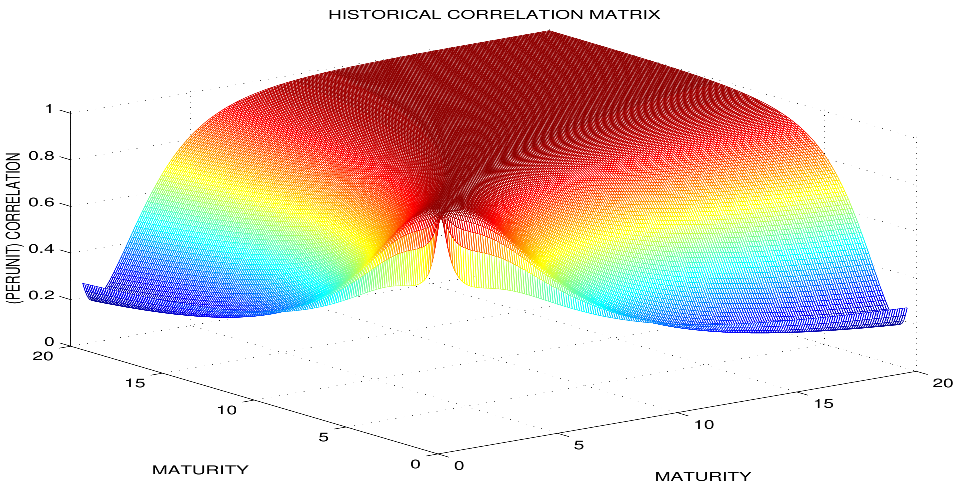
HIGHER ORDER STRUCTURE



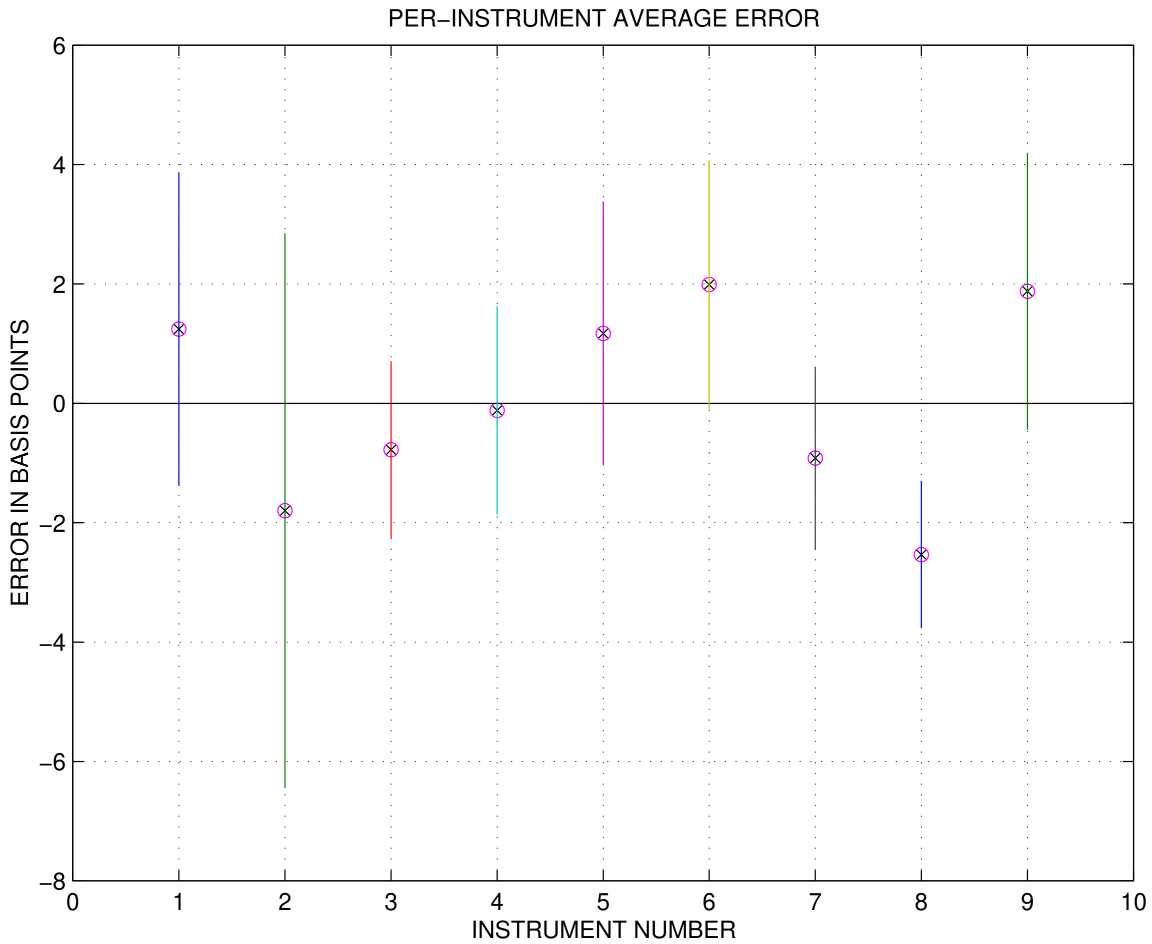
VOLATILITY TERM STRUCTURE







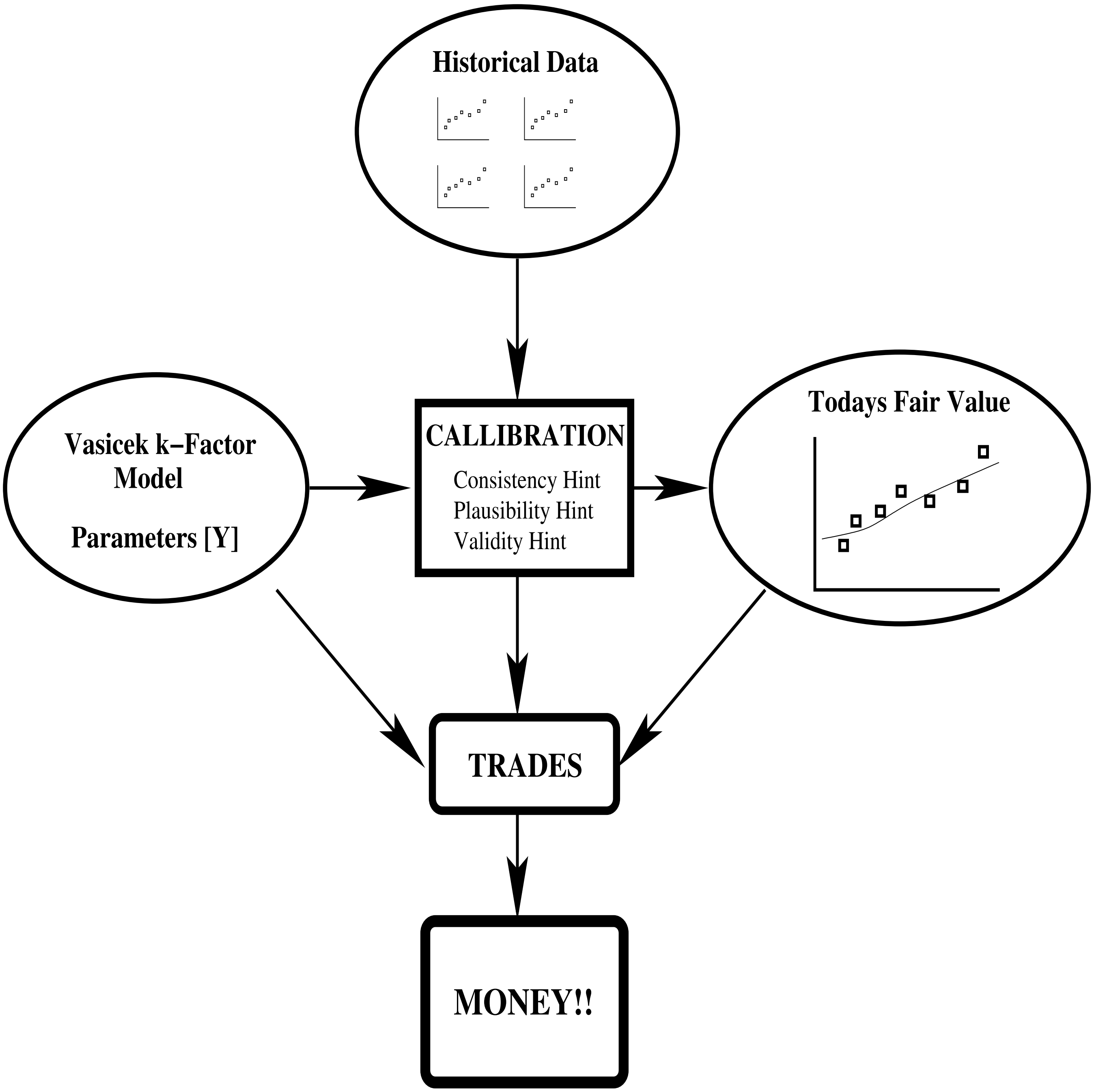
STATISTICS FOR MISPRICINGS



- Errors are distributed according to our assumptions?

- Errors are mis-pricings?

OVERVIEW


Historical Data
Vasicek k−Factor Model
Parameters [Y]
CALLIBRATION
Consistency Hint
Plausibility Hint
Validity Hint
Todays Fair Value
TRADES
MONEY!!