# Finding Communities by Clustering a Graph into Overlapping Subgraphs

Jeffrey Baumes, Mark Goldberg,
Mukkai Krishnamoorthy,
Malik Magdon-Ismail, Nathan Preston

Rensselaer Polytechnic Institute, Troy, NY, USA

# Outline

Introduction

Algorithms IS and RaRe

Experiments

Conclusions and future work

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# What is a cluster?

A *cluster* is a set of closely related objects, not as closely related to the rest

*Clustering* is the development of a collection of clusters

Traditionally, clustering is restricted to *partitioning* into non-overlapping clusters

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# Overlapping clusters

Partitioning is well researched; many algorithms and software packages exist

(Kernighan-Lin, *k*-means, hierarchical, CHACO)

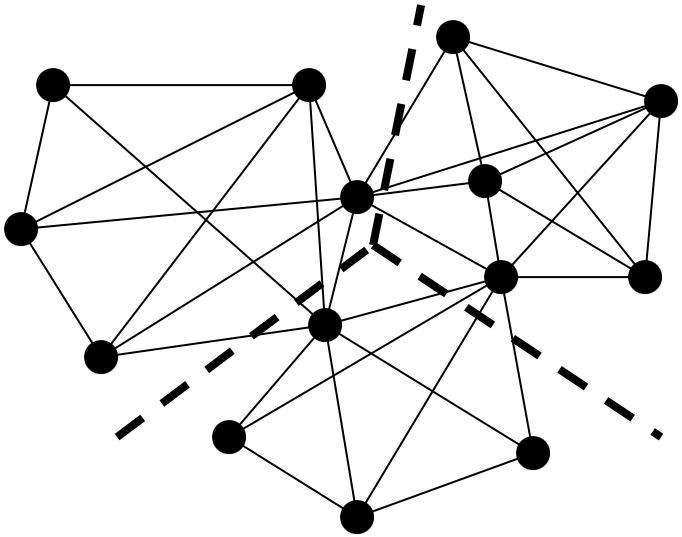
General clustering allows overlapping clusters

Such clustering is natural in social networks

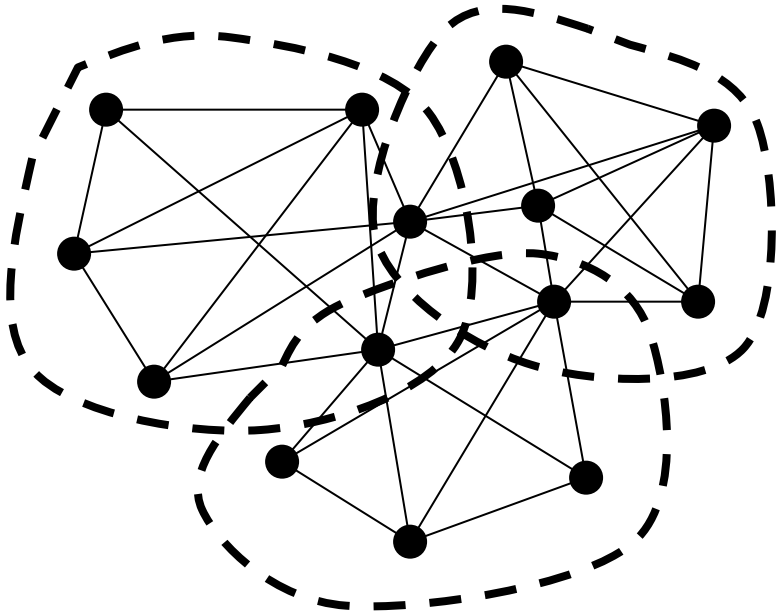
Clustering in a general sense is not well-studied

# Example

Partitioning



General Clustering



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# A new definition of a cluster

Define a *weight function* (or *density*) $W(C)$ for every subset of objects, then maximize $W$ locally

A cluster is defined as a set of objects whose weight is larger than any set close to it; the cluster is said to be *locally optimal*

A set is "close" to another if it may be derived from the other by adding or removing one object

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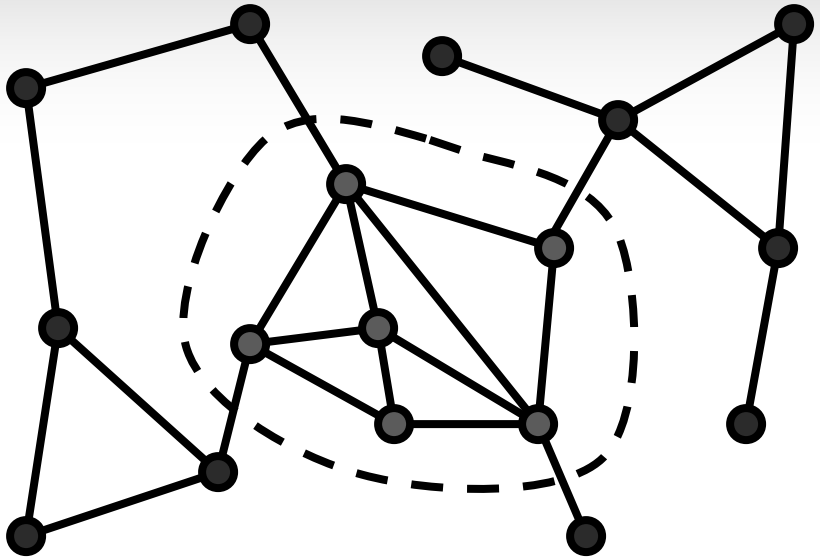
# Weighting functions



$$p_{ex} = \frac{\text{IIII}}{\text{●●● × ●●●●●}}$$

$$\frac{\text{IIII IIII}}{\frac{1}{2}(\text{●●● × ●●})} \qquad \frac{\text{IIII IIII}}{\text{IIII IIII + IIII}} \qquad \frac{p_{in}}{p_{in} + p_{ex}} \qquad \frac{\Delta\Delta\Delta\Delta\Delta}{\frac{1}{6}(\text{●●● × ●● × ●●})}$$

$W_p = p_{in}$ $\qquad$ $W_e$ $\qquad$ $W_i$ $\qquad$ $W_\Delta$

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# Iterative Scan (IS) algorithm

Begins with some cluster "seed"

Traverses nodes, adding or removing the node while the cluster weight improves

Works for *any* choice of weight metric *W*

```
procedure IS(seed,G,W)
C ← seed; w ← W(C);
increased ← TRUE;
while increased do
  for all v ∈ V(G) do
    if v ∈ C then
      C' ← C \ {v};
    else
      C' ← C ∪ {v};
    if W(C') > W(C) then
      C ← C';
  if W(C) = w then
    increased ← FALSE;
  else
    w ← W(C);
return C;
```

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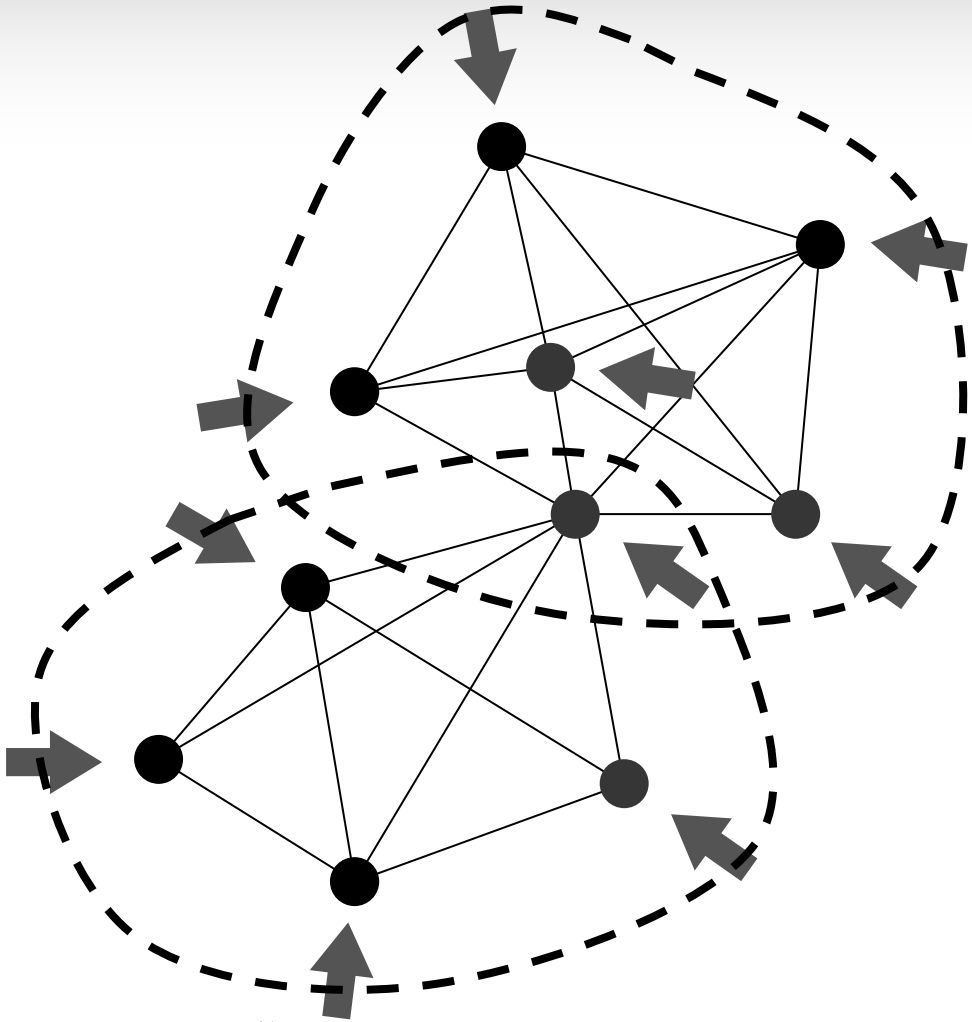
# IS algorithm



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# Seed clusters

IS depends on having good seed clusters

These clusters should represent the entire collection of objects

One option: use an existing partitioning algorithm to create the seed clusters

Another option: create a new algorithm with a global view to create seed clusters (possibly overlapping)

# Rank Removal (RaRe) algorithm

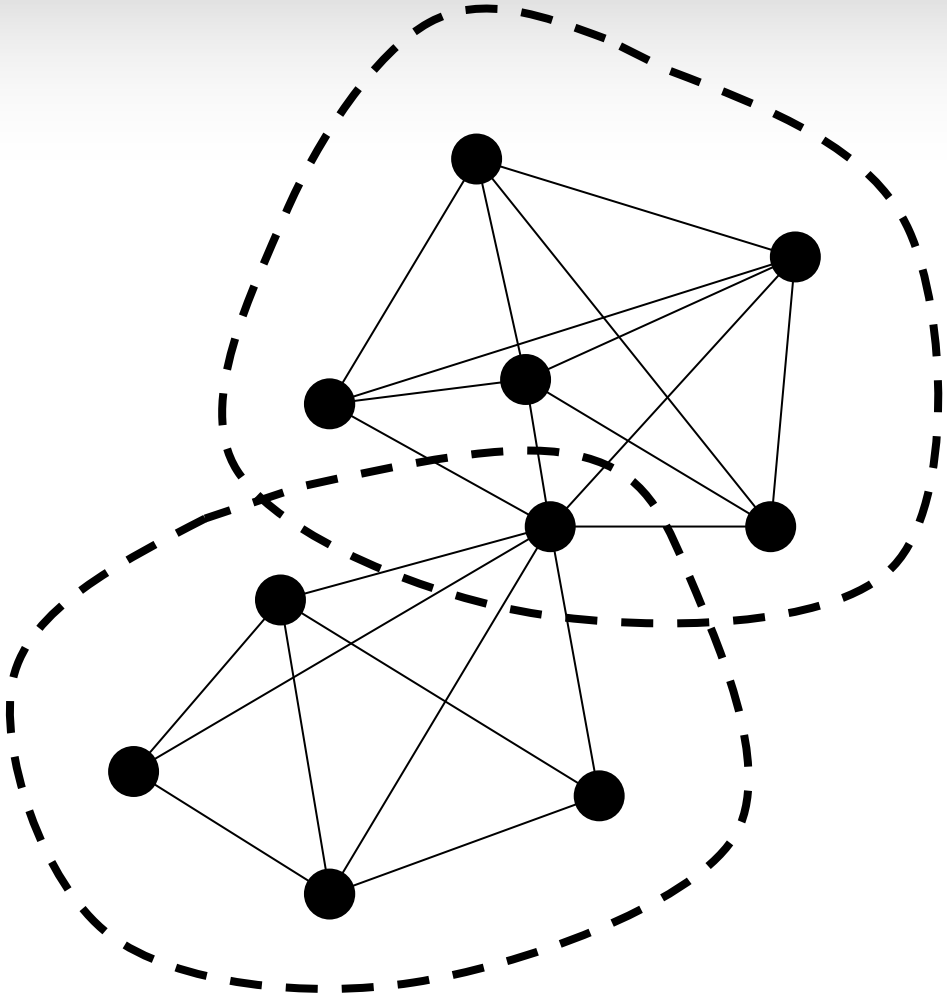
Idea: split the graph into components by removing a few key players (high-rank nodes)

Then, add the nodes back into whatever clusters they improve (possibly more than one)

**procedure** RaRe($C, W$)
global $R \leftarrow \emptyset$;
$\{H_i\}$ are connected components in $G$;
**for all** $H_i$ **do**
  ClusterComponent($H_i$);
Initial clusters $\{C_i\}$ are cluster cores;
**for all** $v \in R$ **do**
  **for all** Clusters $C_i$ **do**
    Add $v$ to cluster $C_i$ if $v$ is adjacent to $C_i$ or $W(v \cup C_i) > W(C_i)$;

**procedure** ClusterComponent($H$)
**if** $|V(H)| > max$ **then**
  $\{v_i\}$ are $t$ highest rank nodes in $H$;
  $R \leftarrow R \cup \{v_i\}$; $H \leftarrow H \setminus \{v_i\}$;
  $\{F_i\}$ are connected components in $H$;
  **for all** $F_i$ **do**
    ClusterComponent($F_i$);
**else if** $min \leq |V(H)| \leq max$ **then**
  mark $H$ as a cluster core;

# RaRe algorithm



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# *k*-Neighborhood algorithm

A naïve overlapping clustering procedure

Selects random cluster centers

Cluster contains all nodes distance at most *k* from the center

"Baseline" for comparison with IS, RaRe

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# Experiments

Runtime and quality analysis

Compare algorithms on both real-world and simulated inputs

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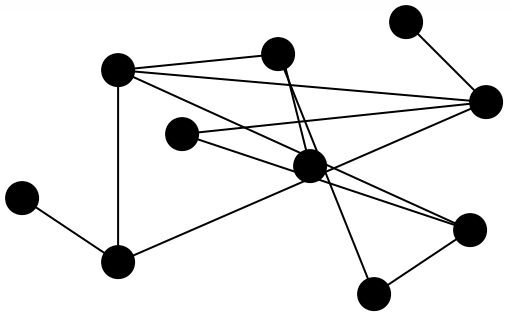
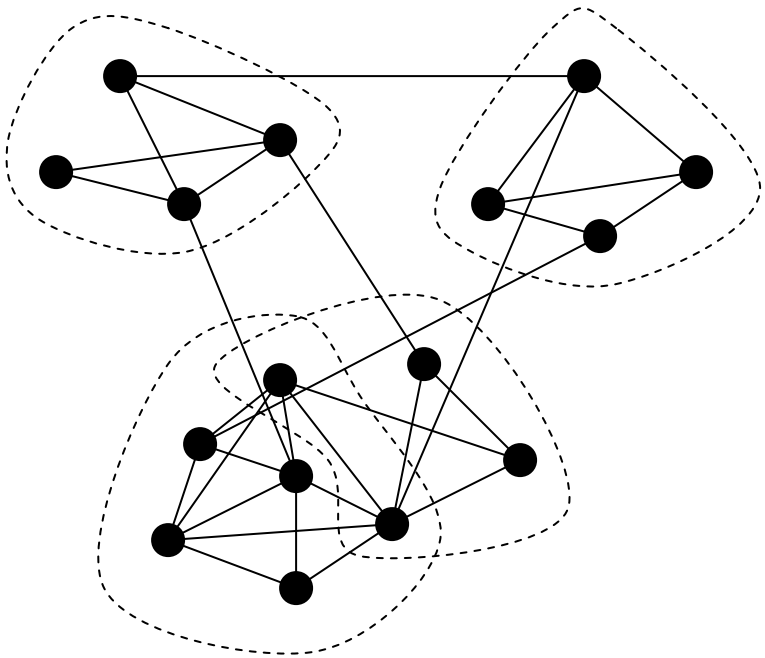
# Simulated input

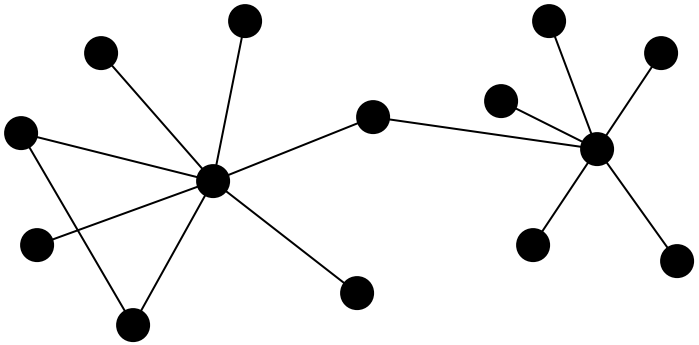
Random



Group random



Preferential attachment



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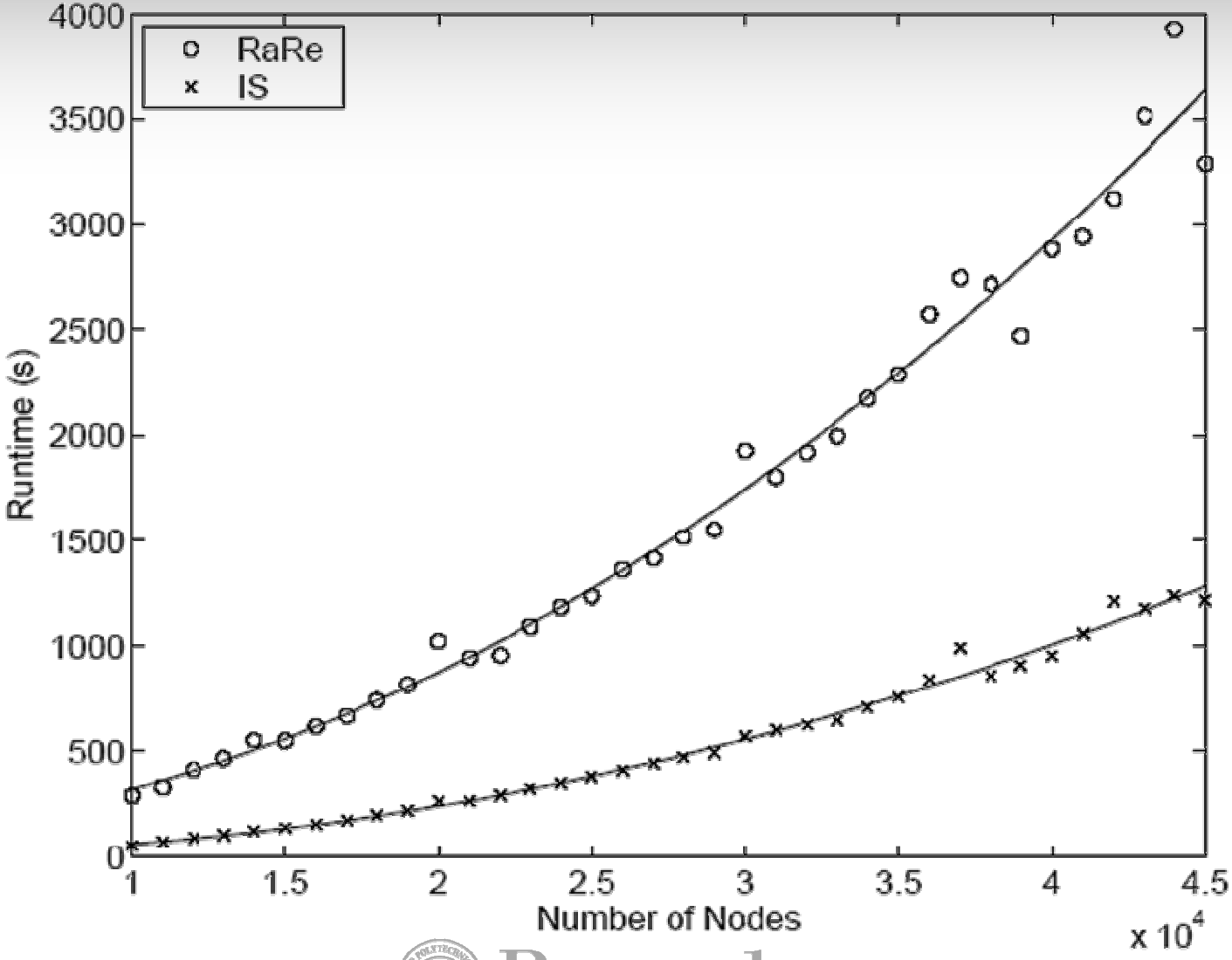
# Runtime for simulated input

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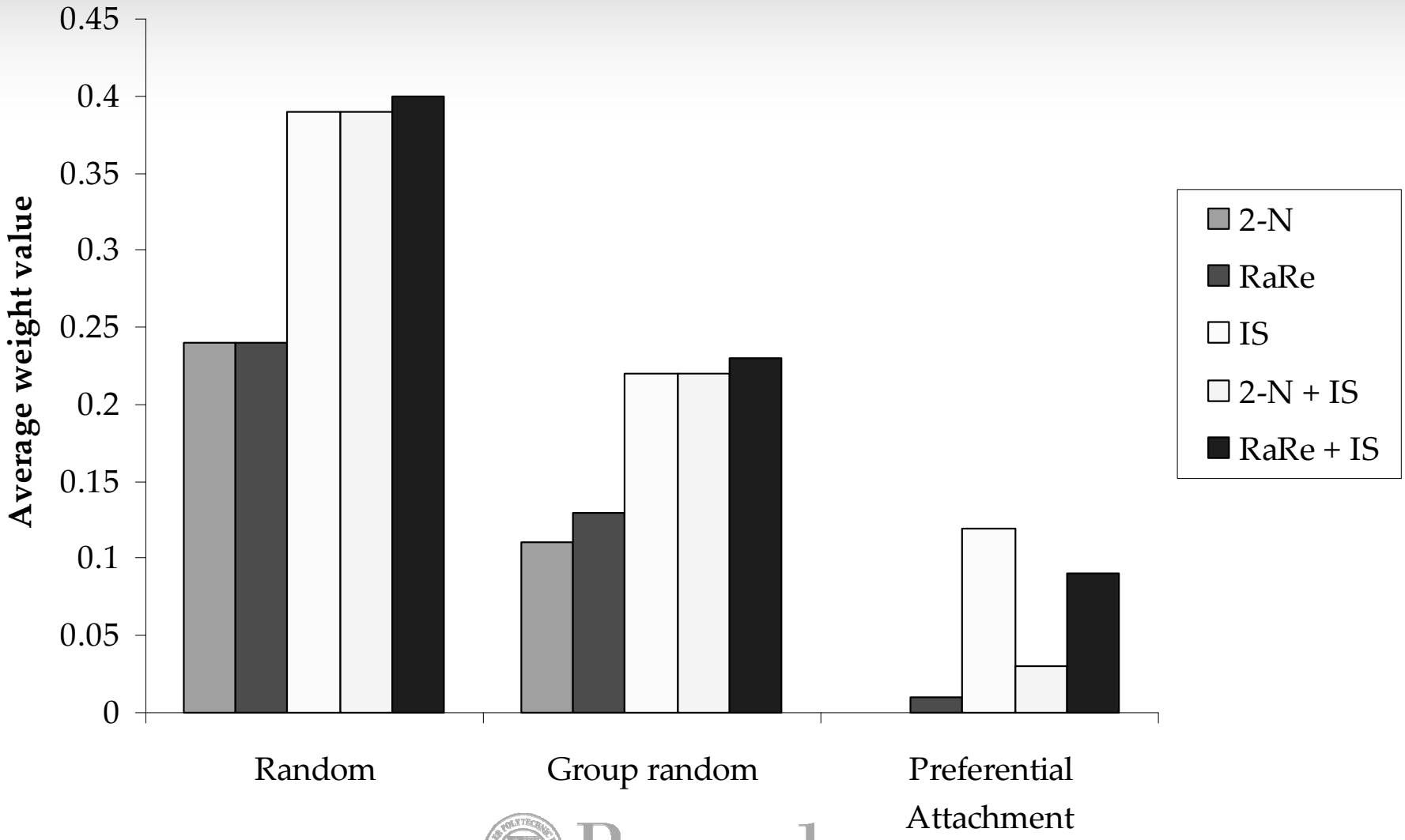
# Quality for simulated input

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# Real-world graphs

575,600 node subset of the CiteSeer database

E-mail communications among RPI community over two days

Web graph of Malik Magdon-Ismail's website (www.cs.rpi.edu/~magdon)

Newsgroup posts on alt.conspiracy

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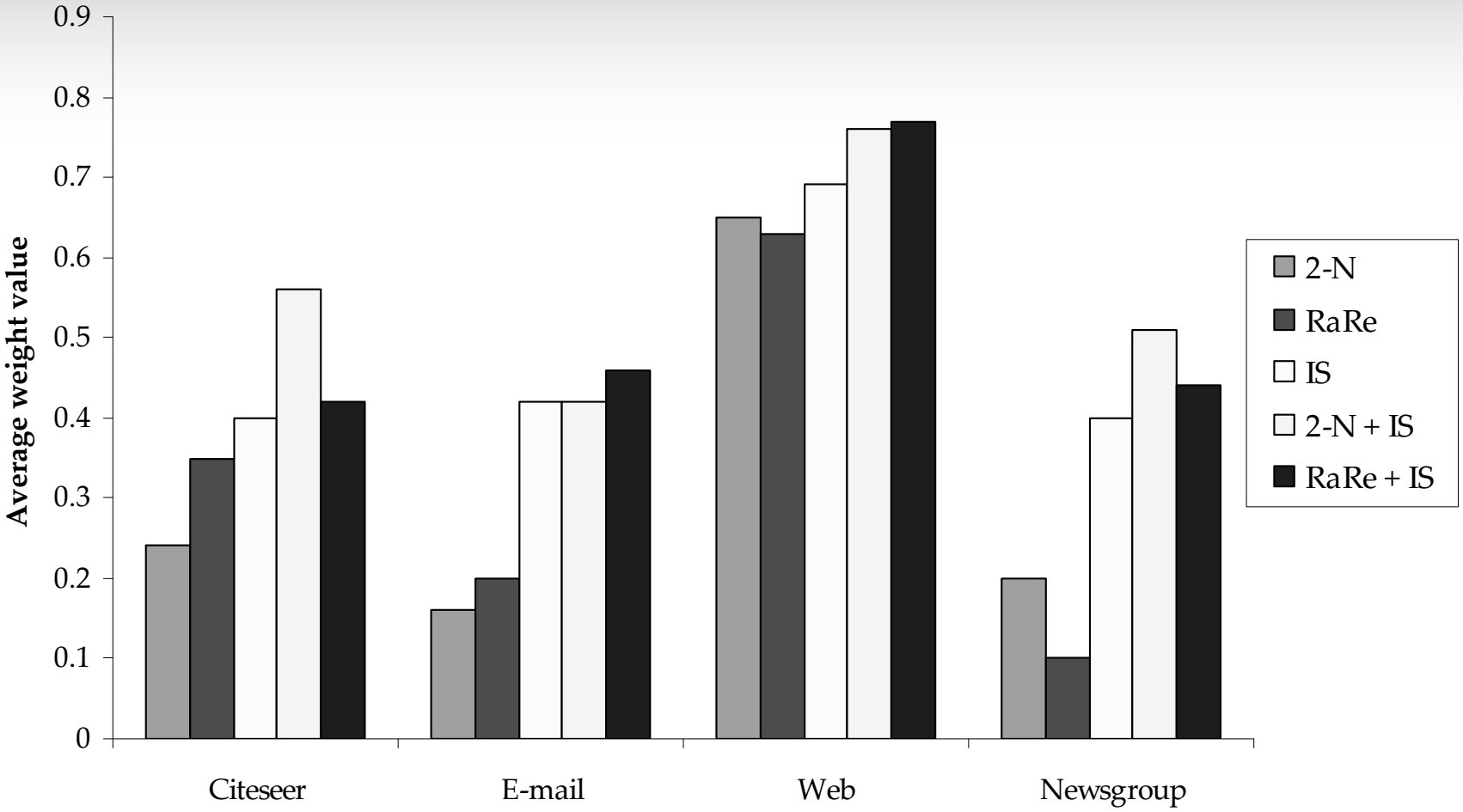
# Quality for real-world graphs

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# Conclusions

First overlapping clustering algorithms successfully developed and tested

Local optimality is an intuitive criterion for overlapping clusters

RaRe/2-N improved by IS was best overall

# Future work

More robust testing with various weight metrics, and with known clusters

Develop new, more efficient algorithms for clustering

Detect communities that evolve over time

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