Experimental Asymptotics: How Much Experimentation is Enough?

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Maximum Independent Set

- Random Graphs $G(n, p)$

  $2log_{\frac{1}{1-p}} n$ [Bollobás and Erdös]

- Trivial sequential algorithm ($Random$)

  $log_{\frac{1}{1-p}} n$

Question: Is there any polynomial algorithm better than $Random$?

$(1+\epsilon)log_{\frac{1}{1-p}}n$ for any $\epsilon > 0$

For 3-regular graphs the answer is yes [ref and result]

For d-regular graphs $d > 3$ the answer is unknown.

Sequential Greedy algorithm $Greedy$

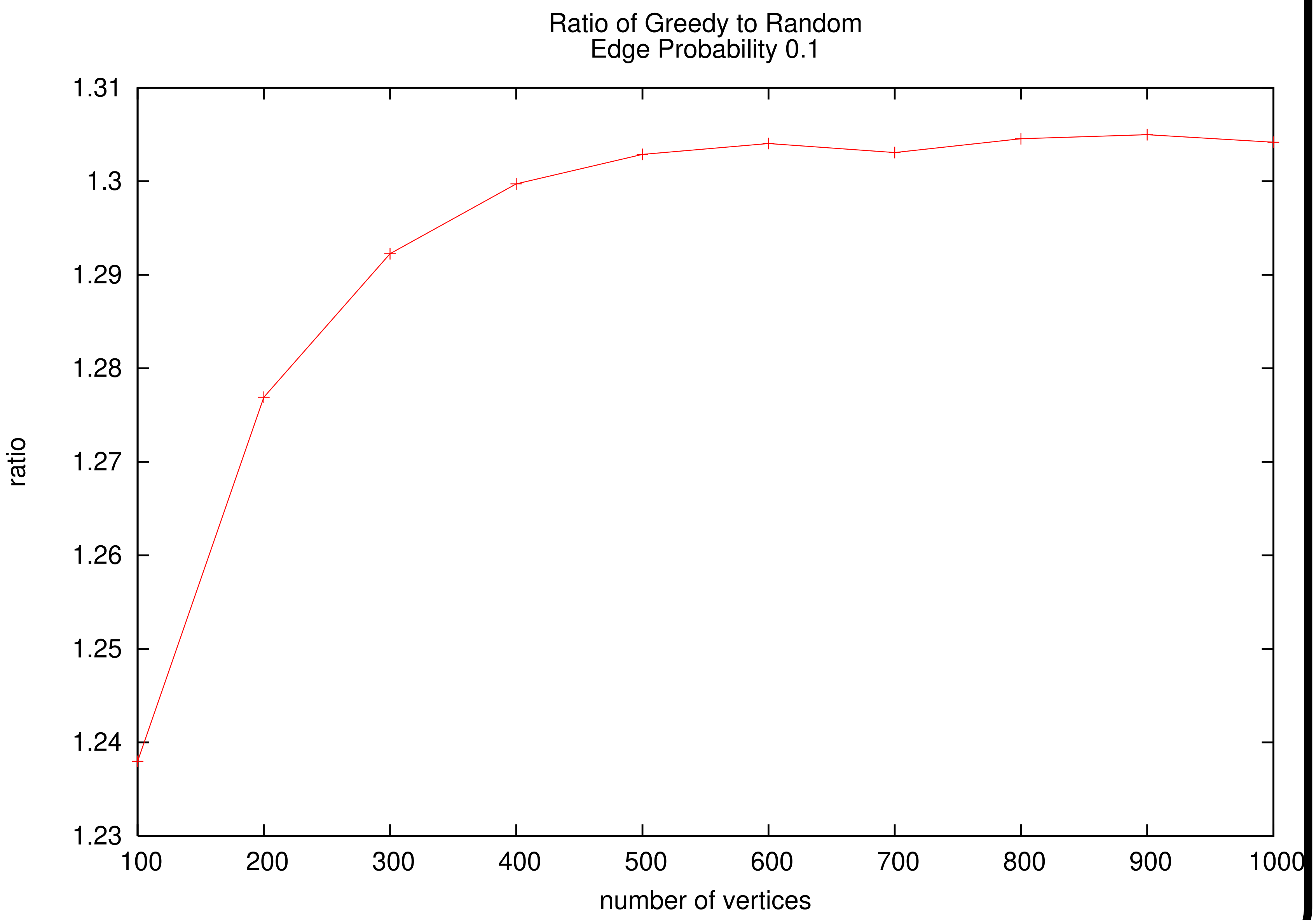
At each step select a vertex with minimum degree

Experimental evaluation should be simple

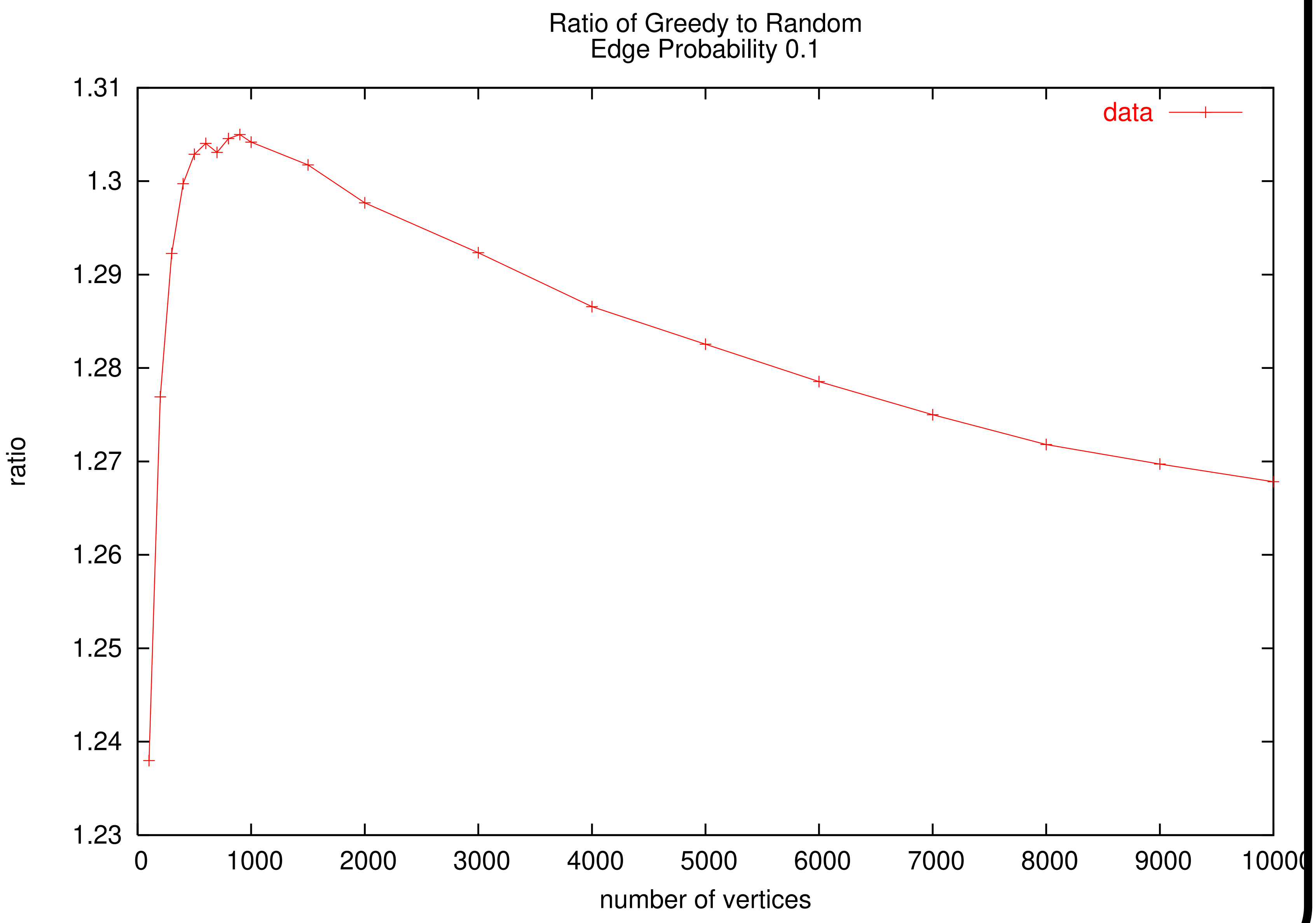
Ratio of $Greedy$ / $Random$ for fixed p

insert plot here (old stuff)

conjecture: $Greedy$ = $Random$



Ratio of Greedy to Random
Edge Probability 0.1
ratio
1.31
1.3
1.29
1.28
1.27
1.26
1.25
1.24
1.23
100
200
300
400
500
600
700
800
900
1000
number of vertices


Ratio of Greedy to Random
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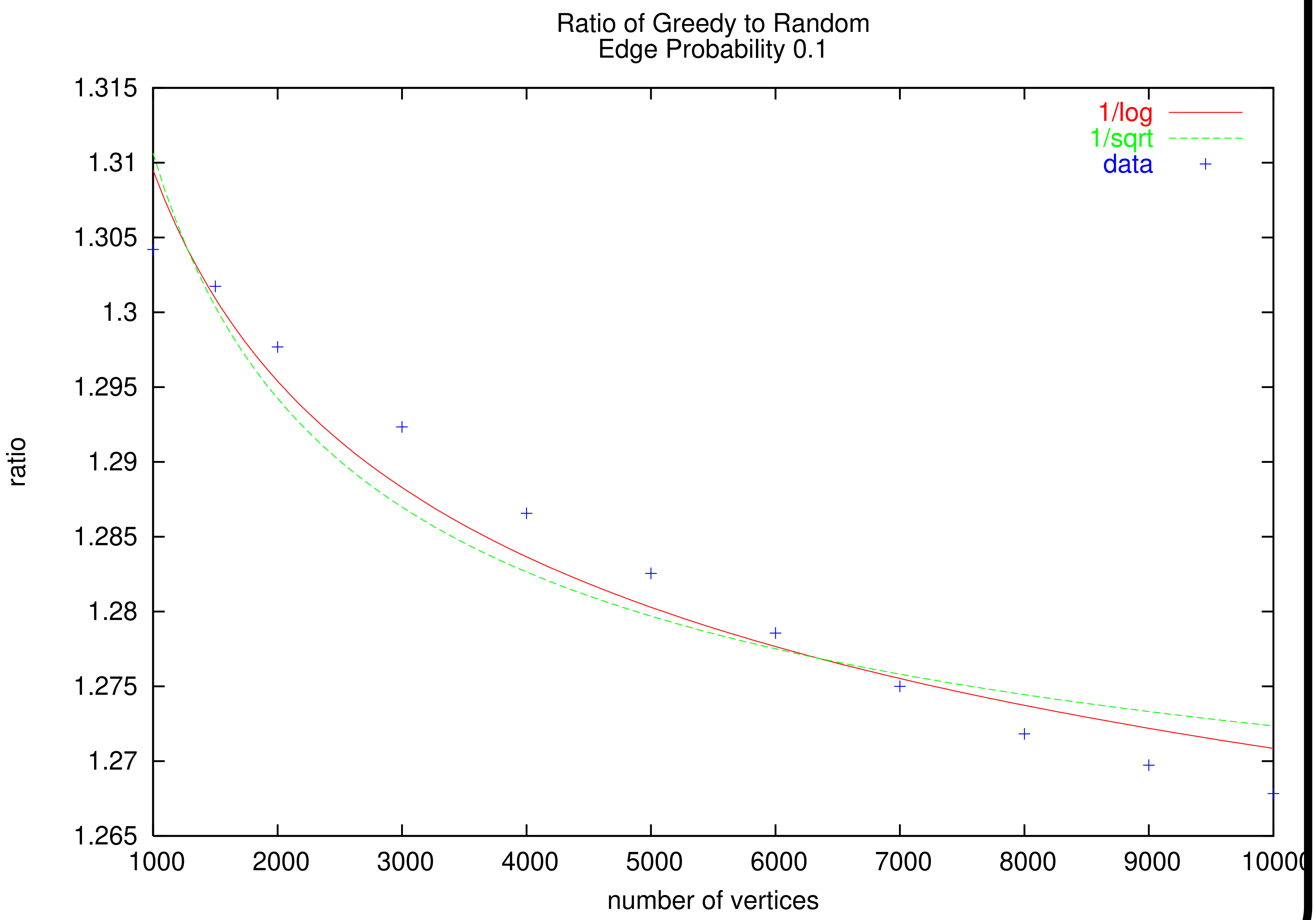
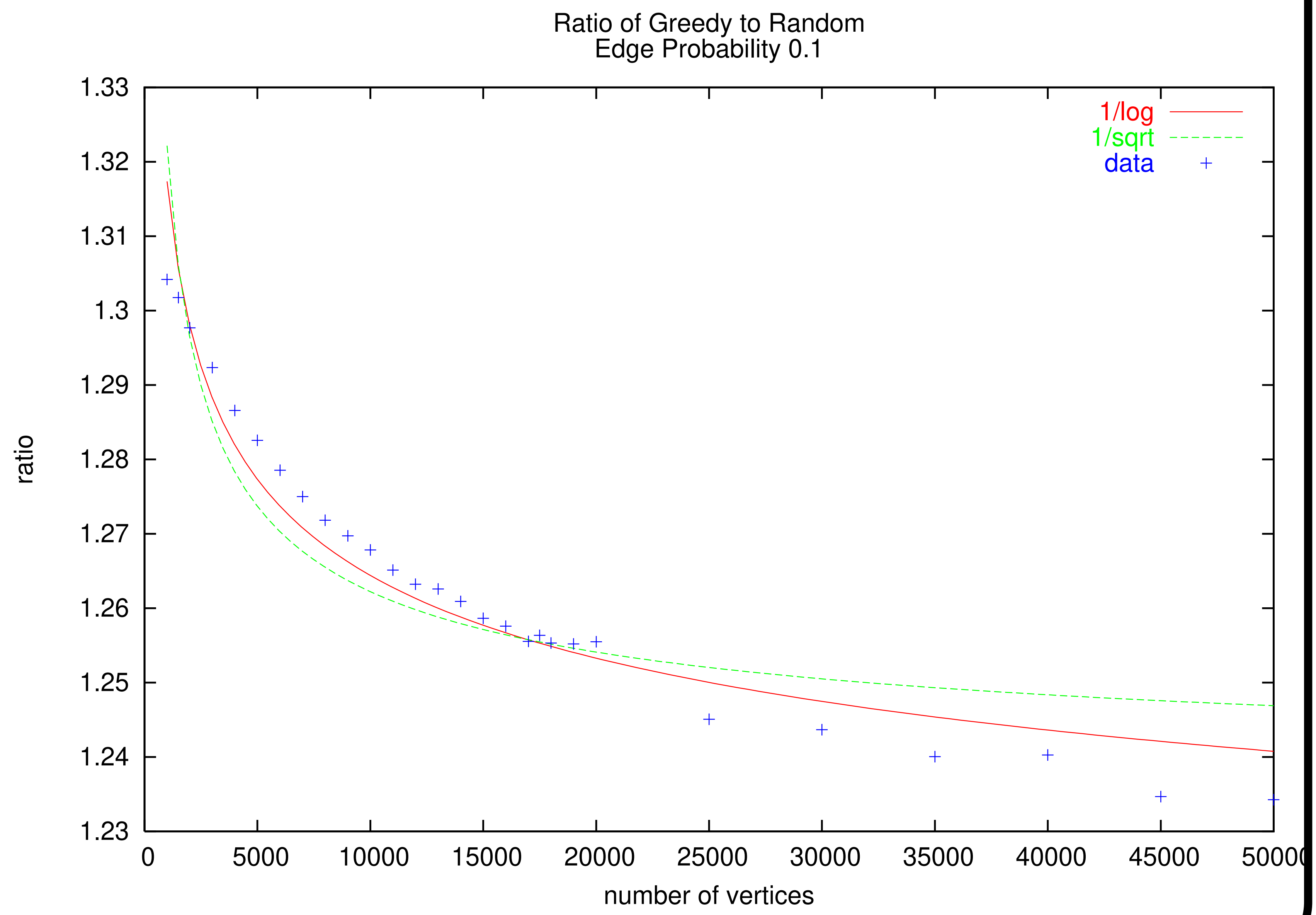
1.28
1.27
1.26
1.25
1.24
1.23
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000



Ratio of Greedy to Random
Edge Probability 0.1
1/log
1/sqrt
data
ratio
number of vertices
1.315
1.31
1.305
1.3
1.295
1.29
1.285
1.28
1.275
1.27
1.265
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000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1.27
1.26
1.25
1.24
1.23
0
5000
10000
15000
20000
25000
30000
35000
40000
45000
50000

Fixed average degree d

plot of d=3 here

Average degree $d = 3$

Discovery: $Greedy$ never selects a vertex of degree 3 (or more).

show / discuss experimental evidence

$$I(n) = I_0 + I_1 + I_2$$

$$n = I_0 + 2I_1 + 3I_2$$

- $I(n)$ is the size of independent set found by $Greedy$ on a graph with n vertices and average degree 3.
- I_k is the number of vertices with degree k when selected for inclusion in the independent set.

Additional discovery for average degree $d = 3$

$$I_2 \longrightarrow 0$$

$Greedy$ *almost never needs to select a vertex of degree 2*

$$I(n) = I_0 + \frac{(n - I_0)}{2}$$

show or discuss experimental evidence

Conjecture for $Greedy$ on average degree $d = 3$ graphs

Independent set size found by $Greedy$ on random, average degree-3 graphs with n vertices:

$$I(n) = \frac{n}{e^3} + cn + \frac{(n - \frac{n}{e^3} - cn)}{2}$$

- $\frac{n}{e^3}$ is number of vertices of degree 0 in the original graph.
- cn is the number of additional vertices of degree 0 that are selected by $Greedy$ (reduced to degree 0 by previous steps).