Power Optimal Connectivity and Coverage in Wireless Sensor Networks

Metin Inanc, Malik Magdon-Ismail and Bulent Yener Department of Computer Science, Rensselaer Polytechnic Institute
110 8th Street, Troy, New York 12180, Tel: (518) 276 8326
Email: {inancm,magdon,yener}@cs.rpi.edu

Abstract— **This work addresses the problem of minimizing power consumption in each sensor node locally while ensuring two global (i.e., network wide) properties: (i) communication *connectivity*, and (ii) sensing *coverage*. A sensor node saves energy by suspending its sensing and communication activities according to a Markovian stochastic process. It is shown that a power level to induce a coverage radius $\frac{w(n)}{n}$ is sufficient for connectivity provided that $w(n)$ is a function approaching to infinity. The paper presents a Markov model and its solution for steady state distributions to determine the operation of a single node. Given the steady state probabilities, we construct a non-linear optimization problem to minimize the power consumption. Simulation studies to examine the collective behavior of large number of sensor nodes produce results that are predicted by the analytical model.**

Keywords: ***Simulations, Stochastic processes/Queueing theory, Mathematical programming/Optimization***

I. Introduction

This work considers a sensor network which is comprised of a large number of sensor nodes communicating with RF links. We assume that sensor nodes are deployed in an ad-hoc fashion so as to *cover* a specified area with their sensing capabilities. Sensors monitor, sense and collect data from a target domain, process it and transmit the information back to the specific sites (e.g., headquarters, disaster control centers). There are many potential applications of sensor networks including military, environmental and health related areas. Although the sensor nodes communicate using wireless links, there are fundamental differences between a sensor network and other wireless ad-hoc networks. One important property of a sensor network is *redundancy*. Sensor nodes are usually densely deployed (approximately 20 sensor/ m^3) [1], hence the underlying network has high redundancy for sensing and communications.

Increasing the lifetime of a sensor network is of primary importance. The high density can cause significant inefficiency problems leading to excessive power wastage. Sensor nodes may sense the same event and try to report it, increasing collisions by transmiting redundant data. Collisions require re–transmissions and increase the energy consumption. Although data aggregation techniques [2] can help to reduce the traffic that propagates to the control centers, they do not provide a complete solution to the problem. Coordination among sensor nodes requires synchronization based on either a global time reference (e.g., GPS) or clock synchronization algorithms. While equipping each sensor node with a GPS is a possibility for the future, current solutions cannot assume a global time reference. The clock synchronization protocols are based on message (e.g., control packets) exchange [3], [4] and they are costly for sensor networks. Thus, coordination of sensor nodes must be done with local and independent (asynchronous) decisions which motivates the deployment of randomized protocols.

In this work we propose a probabilistic scheme in which each sensor node makes an independent decision to be in the *transmit, receive/sense* or turn *off* state.

We are interested in determining the optimal parameters governing the probabilistic transitions of a sensor node so as to minimize power consumption locally while ensuring connectivity and coverage globally.

Some recent works suggest energy conservation by powering off a subset off the nodes in an ad–hoc wireless network [5], [6], [7], [8]. The common theme is to enable nodes to power off or go to low energy sleeping mode during idle time, while ensuring connectivity.

A. Our Contributions

This work is the first to provide rigorous analysis for an energy saving protocol which ensures both connectivity and coverage in sensor networks. We express the sensor network design problem as an optimization problem with the objective of minimizing the power used by the nodes. The sensor network design method we present is a general tool which can be used to optimize parameters of any existing energy saving network protocol dependent on local node decisions. We present a variety of such protocols and analyze one. For the protocol we present, the power savings over existing ad-hoc protocols is ~ 4 at low sensing event densities and $\sim 20+$ at higher densities.

Related Work

In one of the pioneering works on energy saving in wireless networks, the authors in [9] report that leaving the network interface (NI) idle consumes as much energy as reception. They argue that power aware MAC and transport level protocols should be used. Furthermore [9] reports that it is not the number of packets but the duration of the sending period that correlates with the energy usage. The authors also note that (i) most of the energy is spent while idling, and (ii) in order to decrease the energy consumption the NI should be turned off.

In [10] the authors present two routing protocols BECA and AFECA which have a Markov Model with sleeping, listening and active states. In BECA the sojourn times of the nodes are deterministic. In AFECA they are adaptive, the sleeping time being a random variable that depends on the number of neighbors the node has. The authors show (using simulations) a 50% energy saving over naive ad-hoc routing algorithms. In the simulation study (III) we compare our protocol to AFECA.

The GAF routing protocol in [6] aims to extend the lifetime of the network by minimizing the energy consumption and preserving connectivity at the same time. They present a 3-state transition diagram which is a simplified version of ours, and is confined to GAF (Geographic Adaptive Fidelity). Using GAF they discover the locations of redundant nodes. GAF simply imposes a virtual grid on the network. If in any of the grid squares there are more than one node, the redundant nodes are turned off. They also use a protocol called CEC (Cluster-based Energy Conservation) which further eliminates redundant nodes by clustering them. The authors show 40-60% energy saving over other ad-hoc routing algorithms.

While the above approaches address the power control problem at the network layer, the third class of approaches aims to enhance the MAC layer [5], [11], [8]. For example, in [11] the authors propose a modification of the 4-way handshake procedure in the IEEE802.11 protocol for power saving.

In [5] the authors present a MAC protocol PAMAS which saves energy by powering off radios that overhear transmission. PAMAS is a hybrid MAC protocol and provides for 10–50% savings.

In [12] the authors propose a MAC protocol for sensor networks in which nodes go into periodic listen and sleep cycles so as to reduce the energy consumption. The sleep and listen periods are implemented using timers. Neighboring nodes listen and go to sleep at the same time thus the scheme requires synchronization among the neighbors. The authors show that the proposed MAC protocol consumes 2-6 times less energy than IEEE 802.11.

In [7] the authors present a distributed randomized algorithm SPAN where each node makes a decision on its own, based on the amount of energy its number of neighbors. Each node either sleeps (802.11 Power Saving mode) or becomes a coordinator (part of the networking backbone). Coordinators forward the messages they receive from the other nodes. A node which has a message to send automatically becomes a coordinator. SPAN is built on the top of 802.11 and it uses both MAC and routing layer protocols to make decisions.

While GAF [6] and SPAN [7] are distributed approaches with coordination among neighbors, in ASCENT a node decides locally whether to be on or off [8].

The pioneering work in [13] provided the first asymptotic results relating the power level to the connectivity. The authors showed, using percolation theory, that in order to have connectivity in a network with randomly placed nodes, the ratio of the number of neighbors to the total number of nodes should be $(\log n + c)/n$ where c should go to infinity asymptotically.

In [14] the authors propose an algorithm to adjust the power level in order to ensure a minimum degree constraint on each node. In [15] a similar degree constraint is enforced to ensure a bound on the end-to-end throughput. In [16] COMPOW protocol and its architecture are discussed.

In [17] the authors consider the coverage problem and use Voronoi diagrams generated with delaunay triangulation to calculate the coverage of the network.

Recently, in [18], the joint problem of coverage and connectivity is considered using a grid of sensors each of which can probabilistically fail. The authors find the necessary and sufficient conditions for connectivity and coverage in this type of a sensor network. The main result in [18] is that within the transmission radius the number of active nodes should be a logarithm of the total number of nodes, for the network to have connectivity and coverage. They also show that the diameter of the network is of order $\sqrt{n/logn}$. They cover the network area with disks and use the argument that each disk should contain at least one active node for coverage and connectivity.

Organization of the Paper

In the next section we present the model of the sensor network and analyze its steady state behavior including its global connectivity and coverage properties. We formulate power conservation as an optimization problem. In Section III we compare the theoretical analysis with a simulation of a sensor network as well as with AFECA. We end with some concluding remarks in Section IV.

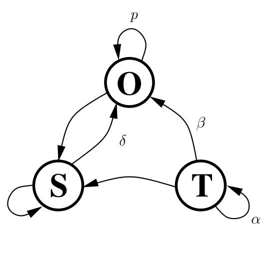
II. Analysis

There are three components to this section. First we discuss the Markov chain that governs the behavior of an individual node. Then we discuss how the properties of this Markov chain affect the connectivity and coverage of the sensor network system. Finally we discuss optimizing with respect to the parameters of the Markov chain so as to maximize the life time of the sensor network system, or other parameters. Extensions and heuristics will be investigated in more detail in the simulation section.

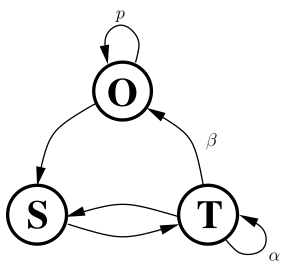
A. The Markov Model

Each node is a three state Markov chain. The three states are the *off*, $\mathbf{O}$, the *sense/receive*, $\mathbf{S}$, and the *transmit*, $\mathbf{T}$, states. Consider a node. Its transition matrix depends on the state of its environment. The environment of a node can be in one of two states: either a sense/receive event is occurring or no such event is occurring. The Markov state diagram in each of these cases is given below, along with the Markov transition probability matrices, $\mathbf{M}$ when there is an event and $\bar{\mathbf{M}}$ when there is no event.

No Event



Event



$$
\mathbf{M}:\quad
\begin{array}{c}
 \\ \mathbf{O} \\ \mathbf{S} \\ \mathbf{T}
\end{array}
\begin{array}{c}
\begin{array}{ccc} \mathbf{O} & \mathbf{S} & \mathbf{T} \end{array} \\
\begin{bmatrix} p & 1-p & 0 \\ \delta & 1-\delta & 0 \\ \beta & \gamma & \alpha \end{bmatrix}
\end{array}
\qquad
\bar{\mathbf{M}}:\quad
\begin{array}{c}
 \\ \mathbf{O} \\ \mathbf{S} \\ \mathbf{T}
\end{array}
\begin{array}{c}
\begin{array}{ccc} \mathbf{O} & \mathbf{S} & \mathbf{T} \end{array} \\
\begin{bmatrix} p & 1-p & 0 \\ 0 & 0 & 1 \\ \beta & \gamma & \alpha \end{bmatrix}
\end{array}
$$

$$\alpha + \beta + \gamma = 1$$
$$0 \le \alpha, \beta, \gamma, \delta, p \le 1$$

Notice that when a sensing event occurs, the node will always transition to the transmit state. This requirement can be relaxed. There is also an ambiguity if both a sensing and receiving event occur. In this case, we can require that the node always attempts to transmit the sensed event rather than the received event. At time t, there is some probability that the node is in each of its three states. Denote $p_{\text{O}}, p_{\text{S}}, p_{\text{T}}$ as the respective probabilities of finding the node in the off, sensing/receiving and transmit states, and collect these three probabilities into the vector $\mathbf{p}(t) = [p_{\text{O}}(t), p_{\text{S}}(t), p_{\text{T}}(t)]$. Let P_E be the probability that there is an event. Then the state probabilities for the node at time $t+1$ are given by

$$\mathbf{p}(t+1) = \mathbf{p}(t)[P_E\mathbf{M} + (1-P_E)\bar{\mathbf{M}}]. \qquad (1)$$

Since an event can be either sensing or receiving, the probability of an event will depend on the probability that a single neighbor is transmitting. We now suppose that the system has equilibrated to a steady state, in which $\mathbf{p}(t+1) = \mathbf{p}(t) = \mathbf{p}^s$. We also make the mean field approximation that all the neighbors of the node are in the same steady state and can be treated as independent. In which case, we can compute P_E as follows. Let q be the probability of a sensing event and let r be the probability of a receiving event. q will be related to the sensing radius and the sensing event density. r is the probability that exactly one of the nodes neighbors are transmitting. We will assume to a first order approximation that the state probabilities for the neighbors are independent. In this case, if there are K neighbors, then $r_K = Kp_{\text{T}}(1-p_{\text{T}})^{K-1}$. Note that if the transmit radius is r_{T}, then assuming that the disks are in the unit torus, the probability that a node is within transmitting range of our node is $\pi {r_{\text{T}}}^2$, and K has a Binomial distribution $\mathbf{P}[K] = B(K; n-1, \pi {r_{\text{T}}}^2)$, where $B(K;N,p) = \binom{N}{K} p^K(1-p)^{N-K}$. Multiplying r by $\mathbf{P}[K]$ and summing over K, we finally arrive at the following expression for r:

$$r = (n-1)\pi {r_{\text{T}}}^2 p_{\text{T}}(1-\pi {r_{\text{T}}}^2 p_{\text{T}})^{n-2}. \qquad (2)$$

Notice that r is a function of p_{T}. Since the sensing and receiving events are independent, $P_E = \mathbf{P}[\text{sense or receive}] = q + r - qr$. We can now use this expression for P_E to solving (1) for the steady state probabilities $\mathbf{p}^s$, which leads to the following set of *non-linear* equations.

$$
\begin{aligned}
\mathbf{p}^s &= \mathbf{p}^s[P_E\mathbf{M} + (1-P_E)\bar{\mathbf{M}}], \\
P_E &= q + (n-1)(1-q)c(1-c)^{n-2}, \\
1 &= \mathbf{p}\cdot\mathbf{1} = p_{\text{O}} + p_{\text{S}} + p_{\text{T}}.
\end{aligned} \qquad (3)
$$

where $c = \pi {r_{\text{T}}}^2 p_{\text{T}}$ and $\mathbf{1}$ is a vector of ones. Had P_E been a constant independent of $\mathbf{p}$, it is well known from the theory of finite state Markov chains that a steady state set of probabilities exists, [22]. It turns out that the introduction of this non-linearity does not change the situation.

Theorem 1: The set of non-linear steady state equations for $\mathbf{p}^s$ given in (3) has at least one solution.

Proof: Let $\mathbf{Q}(\mathbf{p}) = [P_E(\mathbf{p})\mathbf{M} + (1 - P_E(\mathbf{p}))\bar{\mathbf{M}}]$ as defined in (3). $\mathbf{Q}(\mathbf{p})$ is a transition matrix, i.e., $Q_{ij} \geq 0$ and $\sum_j Q_{ij} = 1$ for all i. Let $\mathcal{X}$ be the m-dimensional probability simplex,

$$\mathcal{X} = \{\mathbf{x} : x_i \geq 0, \sum_i x_i = 1\}.$$

$\mathcal{X}$ is compact, and $f(\mathbf{p}) = \mathbf{Q}(\mathbf{p})^T \mathbf{p}$ maps $\mathcal{X}$ into itself. $P_E(\mathbf{p})$ is a polynomial in $\mathbf{p}$, and hence is continuous. Thus, $f(\mathbf{p})$ is a continuous mapping. Thus the conditions to apply the Brower fixed point theorem are satisfied for $f(\mathbf{p})$ [23], and so $f(\mathbf{p})$ has a fixed point. ■

While we have hidden the dependence up to now, we explicitly note here that $\mathbf{p}^s$ is a function of α, β, δ, p and continue with this dependence being understood.

B. Connectivity and Coverage

Here we will discuss the coverage and connectivity properties of the system of sensors. There are already some results regarding these issues in the literature, and we add one more that is appealing on account of its elementary probabilistic derivation. Existing results for coverage and connectivity have also dealt with various forms of random graphs ranging from various types of disk graphs, [13], [18], [20], to Bernoulli graphs, [19], to percolation processes, [21].

We assume that the n sensors are well approximated by points independently and uniformly distributed in the unit torus, $T = [0,1] \times [0,1]$, where the opposite edges are identified. We use a torus to avoid unnecessary complications due to edge effects. Similar results would hold for the square, with only minor additional technicalities. Let r_{S} be the sensing radius and let r_{T} be the transmitting radius.

1) Coverage: We first consider coverage. We assume that the system has equilibrated to its steady state, and that every node can be treated as independent to first order, with state probabilities given by $\mathbf{p}^s$. A point $\mathbf{x} \in T$ is covered if there is a node in the sensing state within r_{S} of $\mathbf{x}$. In this case, an event that occurs at $\mathbf{x}$ will be detected. Thus, the probability that a given node is sensing and within r_{S} of $\mathbf{x}$ is $\pi {r_{\text{S}}}^2 p_{\text{S}}$. Under the independence assumption, the probability that no node can sense an event at $\mathbf{x}$ is then given by $(1 - \pi {r_{\text{S}}}^2 p_{\text{S}})^n$, which is the probability that $\mathbf{x}$ is not covered. Define the coverage function by,

$$f(\mathbf{x}) = \begin{cases} 1 & \mathbf{x} \text{ is not covered,} \\ 0 & \mathbf{x} \text{ is covered.} \end{cases} \tag{4}$$

Then, $\mathbf{P}[f(\mathbf{x}) = 1] = (1 - \pi {r_{\text{S}}}^2 p_{\text{S}})^n$. Let A be the area that is not covered, then

$$A = \int d\mathbf{x}\ f(\mathbf{x}) \tag{5}$$

and so $E[A] = \int d\mathbf{x}\ \mathbf{P}[f(\mathbf{x}) = 1] = (1 - \pi {r_{\text{S}}}^2 p_{\text{S}})^n$. Thus we see that the expected area covered is $1 - E[A] = 1 - (1 - \pi {r_{\text{S}}}^2 p_{\text{S}})^n$, which, after using the fact that $\log(1-x) \leq -x$ for $x < 1$, leads to the following proposition:

Proposition 2: Let $\pi {r_{\text{S}}}^2 p_{\text{S}} = \omega(n)/n$. Then, the expected coverage is given by

$$1 - \left(1 - \frac{\omega(n)}{n}\right)^n \geq 1 - e^{-\omega(n)},$$

(Note: $\omega(n)/n \leq 1$.)

Thus, as long as $\omega(n) \to \infty$, the expected coverage approaches 1. $\omega(n)$ can be interpreted as the expected power used by the sensing nodes. In order to get a concentration result on the coverage, we will use a second moment method, and compute $var(A)$, to which end we would need $E[A^2]$. We use the mean field approximation that our nodes are acting independently in the mean field environment of the neighbors. Then, using a second moment method, we have that

Theorem 3: Let $r_{\text{S}} \leq \frac{1}{2\sqrt{2}}$. Then, for any $\epsilon > 0$.

$$\mathbf{P}[A \geq 2e^{-\frac{(1-\epsilon)}{10\pi}\omega(n)}] < \frac{2\pi \exp\left(-\frac{\epsilon\omega(n)}{5\pi} + \Theta(\frac{1}{\omega(n)})\right)}{\omega(n)\left(1 + \Theta(\frac{1}{\omega(n)})\right)},$$

where $\omega(n) = n\pi {r_{\text{S}}}^2 p_{\text{S}}$.

Proof: Since the proof, though elementary, is long and tedious, we first provide a proof sketch. First we observe that the coverage by squares inscribed in the disks cannot be more than the coverage by the disks. Thus it will suffice to show that the coverage by these inscribed squares is large. Proposition 2 gives the expected coverage. We will show that the variance of this coverage goes to zero sufficiently fast so that the actual coverage will not deviate too much from the expectation. The variance is given by a double integral over two two dimensional variables. We compute this integral as a finite summation, and then bound the variance by bounding this summation. Once we have bound the variance, we can use the Markov inequality to bound the probability of a large deviation from the expected value, and this leads to the result claimed.

We can inscribe a square of side $\Delta = r_{\text{S}}\sqrt{2}$ in a circle of radius r_{S}. The coverage by the disks will then be no less than the coverage by the squares. Let S be the area not covered by the squares, then $A \leq S$. Defining the coverage function $f_S(\mathbf{x})$ for the squares analogously to (4), we find that $E[S] = (1 - \Delta^2 p_{\text{S}})^n$.

$E[S^2] = \int d\mathbf{x} \int d\mathbf{y}\ f_S(\mathbf{x})f_S(\mathbf{y})$. The $f_S(\mathbf{x})f_S(\mathbf{y})$ term in the integrand is the probability that both the points $\mathbf{x}$ and $\mathbf{y}$ are not covered. Let $S_\mathbf{z}$ denote the square centered at the point $\mathbf{z} \in T$. Then the probability that both points $\mathbf{x}$ and $\mathbf{y}$ (in the integrand) are not covered is given by the probability that all sensing squares are outside $S_\mathbf{x} \cup S_\mathbf{y}$, so $E[S^2] = \int d\mathbf{x} \int d\mathbf{y}\ (1 - p_\text{s}|S_\mathbf{x} \cup S_\mathbf{y}|)^n$. In the integral, let $\mathbf{x} = (x_1, x_2)$ and $\mathbf{y} = (y_1, y_2)$. If $|x_1 - y_1| \geq \Delta$ or $|x_2 - y_2| \geq \Delta$ then $|S_\mathbf{x} \cup S_\mathbf{y}| = 2\Delta^2$. Otherwise, $|S_\mathbf{x} \cup S_\mathbf{y}| = 2\Delta^2 - (\Delta - |x_1 - y_1|)(\Delta - |x_2 - y_2|)$. Fix $\mathbf{x}$ in the $\mathbf{y}$ integral. The area over which $\mathbf{y}$ can range with $S_\mathbf{y}$ disjoint from $S_\mathbf{x}$ is $1 - 4\Delta^2$ This area thus contributes $(1-4\Delta^2)(1-2\Delta^2)^n$ to the integral. Over the remaining area, changing coordinate in the $\mathbf{y}$ integral so that its origin lies at $\mathbf{x}$, this contribution to the integral (over the area when the two squares overlap) becomes

$$I = 4 \int d\mathbf{x} \int_{0 \leq y_1, y_2 \leq \Delta} d\mathbf{y} (1 - 2p_\text{s}\Delta^2 + p_\text{s}(\Delta - y_1)(\Delta - y_2))^n.$$

A tedious but elementary computation to perform these integrals then leads to the following result, after adding the contribution from the part of the integral over the region where $S_\mathbf{x}$ and $S_\mathbf{y}$ are disjoint.

$$E[S^2] = (1-2p_\text{s}\Delta^2)^n \left(1 + 4p_\text{s}\Delta^2 \sum_{i=1}^{n} \binom{n}{i} \frac{\lambda^i}{(i+1)^2}\right),$$

where $\lambda = p_\text{s}\Delta^2/(1 - 2p_\text{s}\Delta^2)$. Using the facts that $var(S) = E[S^2] - E[S]^2$ and $E[S]^2 = (1-2p_\text{s}\Delta^2)^n(1 + p_\text{s}\Delta^2\lambda)^n$, we arrive at

$$\begin{aligned} var(A) &= (1 - 2p_\text{s}\Delta^2)^n \sum_{i=1}^{n} \binom{n}{i} \lambda^i \times \\ &\quad \left(\frac{4p_\text{s}\Delta^2}{(i+1)^2} - (p_\text{s}\Delta^2)^i\right), \\ &\leq 4p_\text{s}\Delta^2(1 - 2p_\text{s}\Delta^2)^n \sum_{i=1}^{n} \binom{n}{i} \frac{\lambda^i}{(i+1)^2}, \\ &\leq 4p_\text{s}\Delta^2 e^{-2np_\text{s}\Delta^2} \sum_{i=1}^{n} \binom{n}{i} \frac{\lambda^i}{(i+1)^2}. \end{aligned}$$

Let $F(i) = \binom{n}{i} \frac{\lambda^i}{(i+1)^2}$, then we can bound the sum by $n \max_i F(i)$, so we bound $F(i)$. $F(i)$ is a very sharply peaked function of i. Its maximum occurs at i^* for which $F(i^*)/F(i^* - 1) \geq 1$ and $F(i^* + 1)/F(i^*) < 1$. Since $F(i+1)/F(i) = \lambda(i+1)(n-i)/(i+1)^2$, this condition can be solved for i^* to give $i^* = n\lambda/(1+\lambda) + \Theta(1/n\lambda)$. Using the fact that $\binom{n}{i^*} \leq (en/i^*)^{i^*}$, we get the following bound,

$$\frac{4p_\text{s}\Delta^2 \exp\left(-2n\Delta^2 p_\text{s} + \frac{n\lambda(1+\log(1+\lambda))}{1+\lambda} + \Theta(\frac{1}{n\lambda})\right)}{\frac{n\lambda}{1+\lambda}\left(1 + \Theta(\frac{1}{n\lambda})\right)}$$

Noting that for $r \leq 1/2\sqrt{2}$, $\Delta \leq \frac{1}{2}$, hence, $(1 + \log(1 + \lambda))/(1 - p_\text{s}\Delta^2) \leq 9/10$, we get that

$$var(S) \leq \frac{4\exp\left(-\frac{n\Delta^2 p_\text{S}}{10} + \Theta(\frac{1}{n\Delta^2 p_\text{S}})\right)}{n\Delta^2 p_\text{s}\left(1 + \Theta(\frac{1}{n\Delta^2 p_\text{S}})\right)}$$

Since $n\Delta^2 p_\text{s} = \frac{2}{\pi}\omega(n)$, we have that

$$var(S) \leq \frac{2\pi\exp\left(-\frac{\omega(n)}{5\pi} + \Theta(\frac{1}{\omega(n)})\right)}{\omega(n)\left(1 + \Theta(\frac{1}{\omega(n)})\right)}$$

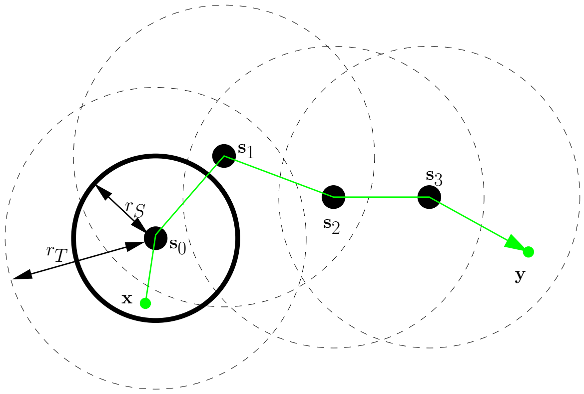
Since $E[S] \leq e^{-\frac{2}{\pi}\omega(n)} \leq e^{-\frac{(1-\epsilon)}{10\pi}\omega(n)}$, we can now apply the Markov inequality to S to get

$$\mathbf{P}[S \geq 2e^{-\frac{(1-\epsilon)}{10\pi}\omega(n)}] < \frac{2\pi\exp\left(-\frac{\epsilon\omega(n)}{5\pi} + \Theta(\frac{1}{\omega(n)})\right)}{\omega(n)\left(1 + \Theta(\frac{1}{\omega(n)})\right)}$$

Noting that $P[A \geq z] \leq P[S \geq z]$ for any z, we get the required bound. ■

$\omega(n)$ can be interpreted as the expected total power expended by the sensing nodes. It should be no surprise that as the total sensing power approaches infinity, the coverage approaches 1 not only in expected value, but also with high probability. Theorem 3 also gives a lower bound on the rate at which it approaches one. If $\omega(n) = \log n + \log\log n + \omega'(n)$ where $\omega'(n) \to \infty$, then it is also the case that $\mathbf{P}[A = 0] \to 1$, [20]. The faster that $\omega(n)$ grows, the faster the convergence to complete coverage. However, this also means that the power consumption by all the nodes will be large.

2) Connectivity: We present here two possible notions of connectivity for a sensor network. The first considers only the topology of the connectivity graph that can be derived from the sensor network. The second is a more stringent condition that also considers contention issues in the network. The existing results use the first definition, which is the tradition we will continue with for the most part, however we will present some heuristics for addressing the second requirement of connectivity. The goal of connectivity can be summarized as follows. Suppose a sensing event fires at some position $\mathbf{x} \in T$, and we wish to transmit this occurrence of this event to $\mathbf{y} \in T$. We would like to be able to successfully transmit this occurrence with high probability for any $\mathbf{x}, \mathbf{y}$. The situation is illustrated below.



A *path* exists from $\mathbf{x}$ to $\mathbf{y}$ if there is a sequence of nodes in the receiving state (which is the same as the sensing state for us) at locations $\mathbf{s}_0, \mathbf{s}_1, \ldots, \mathbf{s}_K$ such that

P1: $\| \mathbf{x} - \mathbf{s}_0 \| \leq r_{\mathrm{S}}$ ($\mathbf{x}$ can be sensed);

P2: $\| \mathbf{s}_i - \mathbf{s}_{i-1} \| \leq r_{\mathrm{T}}$ for $i = 1 \ldots K$, hence the event can be transmitted from $\mathbf{s}_{i-1}$ to $\mathbf{s}_i$, and it will be received since $\mathbf{s}_i$ is in the receiving state); and

P3: $\| \mathbf{s}_K - \mathbf{y} \| \leq r_{\mathrm{T}}$ ($\mathbf{s}_K$ can transmit to $\mathbf{y}$).

We will say that the path above is a K-hop path. The network is *path connected* if for any $\mathbf{x}, \mathbf{y}$, there exists such a path connecting $\mathbf{x}$ to $\mathbf{y}$. Notice that while we have required the existence of this path, we have not required that the path be contention free. In other words, when $\mathbf{s}_0$ transmits to $\mathbf{s}_1$, it must be that $\mathbf{s}_1$ is in the sensing state *and* no other node that is within transmission range of $\mathbf{s}_1$ is attempting to transmit, and similarly for every link $\mathbf{s}_{i-1}, \mathbf{s}_i$ in the path. If there exists such a contention free path for any $\mathbf{x}, \mathbf{y}$, then we say that the sensor network is *transmission connected*. Note that our notions of connectivity implicitly embed the fact that the network covers the area as well. We will focus mostly on path connectivity.

We see that in order to have $\mathbf{x}$ covered, the sensing nodes need to cover the area with respect to r_{S}. However, to guarantee that $\mathbf{y}$ can be transmitted to, it is necessary that the sensing nodes cover the area with respect to r_{T} as well. Thus it suffices to apply the results of the previous section on coverage with r_{S} replaced by $r = \min\{r_{\mathrm{S}}, r_{\mathrm{T}}\}$. This leads to the following result.

Proposition 4: Let A' be the area that cannot be transmitted to and let A be the area not covered. Then, for any $\epsilon > 0$.

$$\mathbf{P}[A \cup A' \leq 2e^{-\frac{(1-\epsilon)}{10\pi}\omega'(n)}] \geq 1 - O\left(\frac{e^{-\frac{\epsilon}{5\pi}\omega'(n)}}{\omega'(n)}\right),$$

where $\omega'(n) = n\pi r^2 p_{\mathrm{S}}$.

Proof: The claim follows from Theorem 3 and the observation that if $r_{\mathrm{S}} \leq r_{\mathrm{T}}$, then $A \cup A' \subseteq A$, otherwise $A \cup A' \subseteq A'$. ∎

Thus, we see that the coverage results should imply conditions P1 and P3 of path connectivity. We now consider requirement P2. For this requirement, it is sufficient that the disk graph obtained by taking disks with radii r_{T} centered at the sensing nodes be connected. Such results were developed in [13] for the case where n nodes are uniformly scattered on T, each having radius $r(n)$. The minor complication here is that while n nodes are scattered in our situation, only about np_{S} of them are sensing. In [13], the following result is proved.

Theorem 5 ([13]): The probability that the random disk graph is connected asymptotically approaches 1 if and only if $\pi r^2(n) = (\log n + c(n))/n$ where $c(n) \to \infty$.

It is also known that in grid-disk graphs, with unreliable nodes, the results are very similar to the random node placement, [18], and in this case it is known that the number of hops required (or the diameter of the graph) is of order $\sqrt{n/\log n}$. We expect that such results should hold in our case as well. For our case, the intuition is that we can apply these results with n replaced by the number of sensing nodes, n_{S}. Thus we have the following theorem,

Theorem 6: Let $r(n) = \min\{r_{\mathrm{S}}(n), r_{\mathrm{T}}(n)\}$, and for any $0 < \epsilon < 1$, let $n_{\mathrm{S}}(\epsilon) = (1-\epsilon)np_{\mathrm{S}}$. Let C be the area that is path connected. If

(i) $\pi r^2(n) np_{\mathrm{S}} \to \infty$, and

(ii) $\pi r^2(n) n_{\mathrm{S}}(\epsilon) = \log(n_{\mathrm{S}}(\epsilon)) + c(n_{\mathrm{S}}(\epsilon))$;

$\lim_{m\to\infty} c(m) = \infty$,

then for any $\eta > 0$, $\lim_{n\to\infty} \mathbf{P}[|C| \geq 1 - \eta] = 1$.

(Note: (i) implies that $np_{\mathrm{S}} \to \infty$.)

Proof: Conditions P1 and P3 of path connectivity for a large enough area (of size $\geq 1 - \eta$) are implied by condition (i) in the theorem and Proposition 4. It remains to show that the disk graph obtained from nodes in the sensing state is connected with probability 1 in the limit. Let n_{S} be the number of sensing nodes (randomly scattered). Then, on account of the independence assumption, n_{S} is a binomial random variable, $B(n_{\mathrm{S}}; n, p_{\mathrm{S}})$. $E[n_{\mathrm{S}}] = np_{\mathrm{S}}$, and so the Chernoff bound, [24], gives $\mathbf{P}[n_{\mathrm{S}} < (1-\epsilon)np_{\mathrm{S}}] < exp(-np_{\mathrm{S}}\epsilon^2/2)$. Since $np_{\mathrm{S}} \to \infty$, we have that $\mathbf{P}[n_{\mathrm{S}} \geq (1-\epsilon)\mu] \to 1$. Let $\mathbf{P}[P2]$ be the probability that condition P2 holds, and let $n_{\mathrm{S}}(\epsilon) = (1-\epsilon)np_{\mathrm{S}}$. Then,

$$\mathbf{P}[P2] \geq \mathbf{P}[P2 | n_{\mathrm{S}} \geq n_{\mathrm{S}}(\epsilon)]\mathbf{P}[n_{\mathrm{S}} \geq n_{\mathrm{S}}(\epsilon)],$$

$c(n_{\mathrm{S}}) = \pi r^2 n_{\mathrm{S}} - \log n_{\mathrm{S}} \to \infty$, because $n_{\mathrm{S}} \geq (1-\epsilon)np_{\mathrm{S}}$ and $np_{\mathrm{S}} \to \infty$, and so from Theorem 5, we have that $\mathbf{P}[P2 | n_{\mathrm{S}} \geq n_{\mathrm{S}}(\epsilon)] \to 1$. Since we also have that $\mathbf{P}[n_{\mathrm{S}} \geq n_{\mathrm{S}}(\epsilon)] \to 1$, we then have that $\mathbf{P}[P2] \to 1$. So there is a sufficiently large area for which we have that $\mathbf{P}[P1] = 1 - e_1(n)$ for that area, $\mathbf{P}[P2] = 1 - e_2(n)$ and $\mathbf{P}[P3] = 1 - e_3(n)$ for that area, where $e_i(n) \to 0$.

By the union bound, $\mathbf{P}[\sim P1 \vee \sim P2 \vee \sim P3] \leq e_1(n) + e_2(n) + e_3(n) \to 0$, hence we conclude that $\mathbf{P}[P1 \wedge P2 \wedge P3] \to 1$ for a sufficiently large area, proving that the network is path connected on a sufficiently large area, with probability 1 in the asymptotic limit. ■

While we can provide sufficient conditions under which the graph is path connected, let us note here some of the limitations of this result. The first is the assumption of independence of the nodes (the mean field theory approximation). This is not strictly true, since the probability that a node is in the transmit state (say) will be dependent on whether one of its neighbors was in the transmit state one time step earlier, and so the current state of neighboring nodes will exhibit a weak dependence which we have ignored. The extent to which this dependence will affect the analysis will be investigated in the simulations. The second limitation is of course that while there may exist a path, it may not be usable due to contention.

To address the contention, we need to look at the transmission connectivity of the network. However, introducing the constraint that there is no contention along the path introduces significant dependence among the nodes. As a result, analysis is difficult, and we present a heuristic which we refer to as *ρ-flooding*. We require that in the event that a node needs to transmit a message, the expected number of recipients will be given by $\rho > 1$.

In such a scenario, it is easy to see that the particular message will rapidly flood through the network. In fact, we can expect the message to spread exponentially fast. Since there are n_{S} nodes, we can expect that in order of $\log n_{\text{S}} / \log \rho$ time steps, every member in the network will have received the message. If we simply use ρ-flooding, the contention in the network will become uncontrollable. To alleviate this problem, we would need to also implement a safety mechanism to prevent such over flooding – one approach might be to bound the maximum number of hops a packet is allowed to make. This can be implemented in practice by adding to each packet a hop counter, and setting its maximum allowed value appropriately. Two possibilities are $\log E[n_{\text{S}}] / \log \rho$, the time we expect it takes to flood the whole network, or $\sqrt{n_{\text{S}} / \log n_{\text{S}}}$, the expected diameter of the network, [18]. The requirement of ρ-flooding sets constraints on the allowable parameters in the Markov model, which is what we derive here.

Let's consider the situation when a node is in the transmission state, and let σ be any one of the other $n-1$ nodes. Let Q be the probability that you successfully transmit the packet to σ given that σ is within transmission range. Let P_{suc} be the probability to successfully transmit the packet to σ, then $P_{suc} = \pi {r_{\text{T}}}^2 Q$. To achieve successful transmission given that σ is within transmission range, either the first try was successful, or the first try was not successful, and some try after the first try was successful. Since the process is Markov and since the nodes are independent, the probability that some try after the first one is successful (given that you remain in the transmit state) is also Q. Let Q_1 be the probability that you were successful on the first try given that σ is within transmission range. Since the probability to remain transmitting is α, we have that $Q = Q_1 + (1 - Q_1)\alpha Q$, or that

$$Q = \frac{Q_1}{1 - \alpha + \alpha Q_1}. \tag{6}$$

Suppose that σ has K neighbors. Then you are successful on the first try if σ is in the sensing state and no other neighbor of σ is transmitting, which occurs with probability $p_{\text{S}}(1-p_{\text{T}})^K$. Multiplying by $P(K)$, summing over K using the fact that K has a Binomial distribution $B(K; n-2, \pi {r_{\text{T}}}^2)$, we arrive at $Q_1 = p_{\text{S}}(1 - \pi {r_{\text{T}}}^2 p_{\text{T}})^{n-2}$. Since there are $n-1$ packets to whom you could transmit, the expected number of successful transmissions is given by $(n-1)P_{suc}$. Requiring that the expected number of successful transmissions is ρ then leads to the following constraint,

Proposition 7: In order to achieve ρ-flooding, the following condition must be satisfied,

$$\rho = \frac{(n-1)\pi {r_{\text{T}}}^2 p_{\text{S}}(1 - \pi {r_{\text{T}}}^2 p_{\text{T}})^{n-2}}{1 - \alpha + \alpha p_{\text{S}}(1 - \pi {r_{\text{T}}}^2 p_{\text{T}})^{n-2}} \tag{7}$$

C. Optimizing The Power Consumption

The main goal of this paper is to develop a systematic approach for power conservation in sensor networks. The idea is to select the available parameters in the Markov model so as to minimize the power consumption, while at the same time guaranteeing coverage and connectivity. Accomplishing this involves solving a constrained optimization problem, which we effect numerically, the details being given in the Simulation section.

We assume that the power consumption in each of the three states is given by $\lambda_{\text{O}}, \lambda_{\text{S}}, \lambda_{\text{T}}$. Suggested values for these parameters have been given in the literature, [9]. For our purposes, we assume that these are externally supplied parameters, or functional forms that may depend on $r_{\text{S}}, r_{\text{T}}$. The expected power consumption per node in steady state is then given by $E = \lambda_{\text{O}} p_{\text{O}} + \lambda_{\text{S}} p_{\text{S}} + \lambda_{\text{T}} p_{\text{T}}$. In order to guarantee path connectivity and coverage, it is sufficient to enforce the conditions in Theorem 6. We are thus led to the following optimization problem:

OPT1: Let $f_1(n), f_2(n)$ be any two functions that approach infinity in the asymptotic limit, for example $\log n$ or n^a. Let $0 < \epsilon < 1$.

$$\underset{\alpha,\beta,p,\delta}{\text{minimize}}\ \lambda_{\text{O}} p_{\text{O}} + \lambda_{\text{S}} p_{\text{S}} + \lambda_{\text{T}} p_{\text{T}},$$

subject to the constraints

$$0 \le \alpha, \beta, p, \delta \le 1$$
$$\alpha + \beta \le 1$$
$$\pi r^2 n p_{\text{S}} \ge f_1(n)$$
$$\pi r^2 n_{\text{S}}(\epsilon) = \log(n_{\text{S}}(\epsilon)) + f_2(n_{\text{S}}(\epsilon))$$

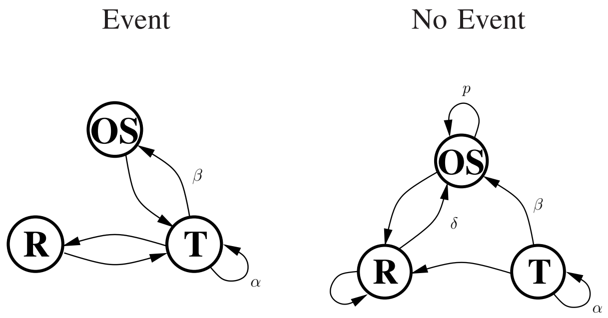
where $r = \min\{r_{\text{S}}, r_{\text{T}}\}$ and $n_{\text{S}}(\epsilon) = (1-\epsilon) n p_{\text{S}}$. Here n and the sensing event density are given, from which q, the probability of sensing an event can be calculated. $p_{\text{O}}, p_{\text{S}}, p_{\text{T}}$ are the solutions to the steady state equations, (3), which depend on the parameters.

$f_1(n)$ and $f_2(n)$ can chosen so that the connectivity and coverage converge to 1 at the desired rate. In order to enforce transmission connectivity, one can incorporate the additional constraint given in Proposition 7. After this constraint has been incorporated, and the power consumption minimized, one can use the additional heuristic of a maximum number of hops to avoid over-flooding the sensor network.

D. Extensions

There are a number of ways in which the general methodology we have presented may be extended, the most immediate is to consider different Markov models. We have presented a relatively simple Markov model for the state diagram of a single sensor node. We list below several other interesting models. The analysis of these models follows virtually identical lines to the model we have presented, the main difference being the the introduction of additional parameters and/or states in the Markov chain of a sensor. The only change in the form of the steady state equations, (3) may be a change in the dimensionality of the system and the constant matrices $\mathbf{M}$ and $\bar{\mathbf{M}}$. Otherwise, the entire methodology remains intact, including the constraints for connectivity and coverage. Thus we will not follow through on most of the details, and we leave the further theoretical development and experimental investigation of these models as avenues for future work.

a) Off/Sensing–Receive–Transmit: In the state diagram for this Markov model, we combine the off state with the sensing state, and receiving occurs in a separate state. Otherwise, it is very similar to the model we have been describing. This model is basically the model that was used in [10]. We mention it here to demonstrate how their model fits within the general methodology we have developed here. While in [10], the authors develop some reasonably good parameters for the latency times in each state, in the present framework, one can optimize these parameters while at the same time enforcing connectivity and coverage. Figure 1 illustrates the model.



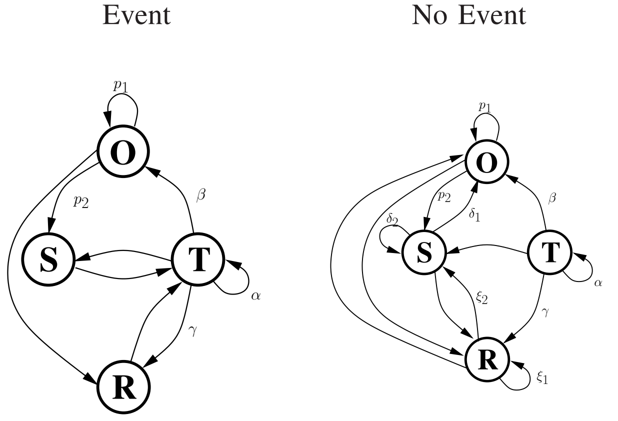
$$\begin{array}{c|ccc} \mathbf{M}: & \mathbf{OS} & \mathbf{R} & \mathbf{T} \\ \hline \mathbf{OS} & 0 & 0 & 1 \\ \mathbf{R} & 0 & 0 & 1 \\ \mathbf{T} & \beta & \gamma & \alpha \end{array} \qquad \begin{array}{c|ccc} \bar{\mathbf{M}}: & \mathbf{OS} & \mathbf{R} & \mathbf{T} \\ \hline \mathbf{OS} & p & 1-p & 0 \\ \mathbf{R} & 0 & 0 & 1 \\ \mathbf{T} & \beta & \gamma & \alpha \end{array}$$

$$\alpha + \beta + \gamma = 1$$
$$0 \le \alpha, \beta, \gamma, p \le 1$$

Fig. 1. Off/Sensing–Receive–Transmit

b) Off–Sensing–Receive–Transmit: Here, we have a separate state for each of the four possible activities. One possible advantage of this setup is that the asymmetry between sensing and receiving may allow one to preferentially treat one of these events and pay less attention to the other. In fact one could have two classes of nodes, those with a preference for $\mathbf{S}$ over $\mathbf{R}$ and those with a preference for $\mathbf{R}$ over $\mathbf{S}$. In this way, one could have "separately" functioning sensing and listening networks. While the analysis to take into account two types of nodes in the ensemble of nodes is slightly more complex, it follows the same general approach. The main difference is that the connectivity would be defined with respect to the "listening" network, and the coverage would be defined with respect to the "sensing" network. Figure 2 illustrates the model.

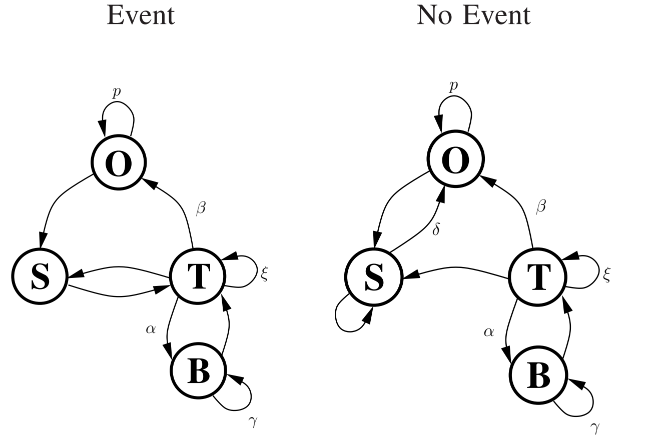
c) Back-off: This is a technique that can be used with any of the previous models and we illustrate this concept here with our original model. The idea is to allow the transmit state one more alternative rather than simply continue transmitting or exit transmitting. One also allows transmition to "pause" or back-off into the back-off state where the node holds the item to be transmitted, but is not creating contention. Such a model may allow for better contention management. Figure 3 illustrates the model.



$$
\begin{array}{c|cccc}
\mathbf{M}: & \mathbf{O} & \mathbf{S} & \mathbf{R} & \mathbf{T} \\
\hline
\mathbf{O} & p_1 & p_2 & p_3 & 0 \\
\mathbf{S} & 0 & 0 & 0 & 1 \\
\mathbf{R} & 0 & 0 & 0 & 1 \\
\mathbf{T} & \beta & \kappa & \gamma & \alpha
\end{array}
\qquad
\begin{array}{c|cccc}
\bar{\mathbf{M}}: & \mathbf{O} & \mathbf{S} & \mathbf{R} & \mathbf{T} \\
\hline
\mathbf{O} & p_1 & p_2 & p_3 & 0 \\
\mathbf{S} & \delta_2 & \delta_1 & \delta_3 & 0 \\
\mathbf{R} & \xi_3 & \xi_2 & \xi_1 & 0 \\
\mathbf{T} & \beta & \kappa & \gamma & \alpha
\end{array}
$$

$$
\begin{gathered}
p_1 + p_2 + p_3 = 1 \\
\delta_1 + \delta_2 + \delta_3 = 1 \\
xi_1 + \xi_2 + \xi_3 = 1 \\
\alpha + \beta + \kappa + \gamma = 1 \\
0 \leq \alpha, \beta, \gamma, \kappa, \delta_i, p_i, \xi_i \leq 1
\end{gathered}
$$

Fig. 2. Off–Sensing–Receive–Transmit.



$$
\begin{array}{c|cccc}
\mathbf{M}: & \mathbf{O} & \mathbf{S} & \mathbf{T} & \mathbf{B} \\
\hline
\mathbf{O} & p & \bar{p} & 0 & 0 \\
\mathbf{S} & 0 & 0 & 1 & 0 \\
\mathbf{T} & \beta & \kappa & \xi & \alpha \\
\mathbf{B} & 0 & 0 & \bar{\gamma} & \gamma
\end{array}
\qquad
\begin{array}{c|cccc}
\bar{\mathbf{M}}: & \mathbf{O} & \mathbf{S} & \mathbf{T} & \mathbf{B} \\
\hline
\mathbf{O} & p & \bar{p} & 0 & 0 \\
\mathbf{S} & \delta & \bar{\delta} & 0 & 0 \\
\mathbf{T} & \beta & \kappa & \xi & \alpha \\
\mathbf{B} & 0 & 0 & \bar{\gamma} & \gamma
\end{array}
$$

$$
\begin{gathered}
p + \bar{p} = 1 \\
\delta + \bar{\delta} = 1 \\
\gamma + \bar{\gamma} = 1 \\
\alpha + \beta + \kappa + \xi = 1 \\
0 \leq \alpha, \beta, \kappa, \xi, \delta, \bar{\delta}, p, \bar{p}, \gamma, \bar{\gamma} \leq 1
\end{gathered}
$$

Fig. 3. Back-off

III. Simulation Study

The goal in this section is to investigate how closely the theory agrees with a simulation of real packet routing. In particular we wish to test the mean field theory assumption of independence. We also compare the power used by our protocol to that of AFECA [10].

This section consists of three parts. In the first part we describe the numerical solution of the optimization problem, in the second part of this section we present the simulation algorithm and its implementation and finally in the third part we presents the results of the simulation.

A. Optimization

Solving the optimization problem described in Section II-C takes several steps. (i) The first step is solving the steady state problem of our Markov model formulation. Observe that due to the nonlinearity, it is not easy to get an exact algebraic solution, thus a numerical technique must be employed. (ii) The second step is finding an initial feasible point for the optimization. (iii) The third step is running the optimizer with the objective function in Section II-C, which is a linear function of the steady state probabilities.

First we implement a fixed point iteration algorithm to find the steady state probabilities given α, β, δ, p (these are also our decision variables). In addition, the steady state probabilities will depend on n, the number of nodes in the network, r, the transmission and sensing radius and q, the probability of sensing an event, which is an external parameter that depends on the event density. We obtain the steady state by iteratively applying equation (3).

Second, we need a feasible solution to start optimization. We implement a brute–force random search algorithm, which usually succeeds after a couple of iterations.

The third and last step is to invoke the optimizer. We use Matlab's nonlinear constrained optimizer fmincon, which is available in the optimization toolbox [25]. We run the optimizer a number of times, each time, from a different initial feasible point. This allows us to find better solutions by combatting the local minima problem. We consider several different n, r and q. Given an instantiation of these parameters, the optimizer finds the Markov model design which minimizes the per node power consumption. Note that the objective function minimized is exactly that quantity.

B. Simulator Implementation

We implement a discrete-time simulator in C++. Simulation is performed on a unit torus. n network nodes are randomly distributed on the surface, see Table I. n_e event nodes are also randomly dispersed on the same surface. Each of the event nodes can generate an event with a certain firing probability. The firing probability is related to the user input sensing probability.

The nodes within a distance of r of each other are adjacent and can communicate. Nodes within a distance of r to an event node will sense the event, provided that they are in the *sense/receive* state and the event node fires. Each of the network nodes is started in a random state. At each time step, nodes undergo a state transition according to the Markov model in Section II-A. To be able to investigate the node adjacency effect on the steady state probabilities, each node in the *transmit* state sends a message to its neighbors. Each node within r can receive the message provided that it is in *sense/receive* state and there is no collision. A message that is received is forwarded to the neighbors of the receiving node.

The states of a sample of five randomly chosen nodes are recorded for up to 50000 discrete time steps. To determine the steady state probabilities for each of the nodes in the sample, the fraction of each of the three states (*off*, *sense/receive* and *transmit*) is calculated by averaging over a moving window of size 5000.

TABLE I
SIMULATION PARAMETERS

Simulation Parameter	Value
n	1000–10000
n_e	10000
q	0.0–1.0
r	0.01–0.5
T	50000

We also implement the AFECA protocol [10]. AFECA was implemented with the optimal parameters given in [10]. The goal is to compare our protocol with that of our competitors. Since the energy model used by the two protocols is the same, the comparison will be a fair one.

In [10] the authors use the power consumption figures: 1.6W for radio transmission, 1.2W for reception and 1W for idle listening. They also have 0.025W consumed in the off state in [6]. In the simulations and optimization, we made the same assumptions. For our *sense/receive* state we average the idle listening and reception figures, which are already pretty close to 1.1W.

AFECA has a *sleep* state which does not turn off the sensors. That is, during a sleep state a node can sense and upon a sensing event, it will move to a state called *active* to transmit the message. If no sensing event arrives it will transfer to the *listen* state after time $T_{sa} = K \times 10sec$, where K is a random integer between 1 and its number of neighbors. In the *listen* state it would wait T_l seconds (10sec). In the case of traffic or an event, the node moves to the *active* state. The authors give the optimal time to stay in T_a as 60sec, see Figure 4. All the times T_a, T_{sa} and T_l are values chosen by the AFECA's authors.

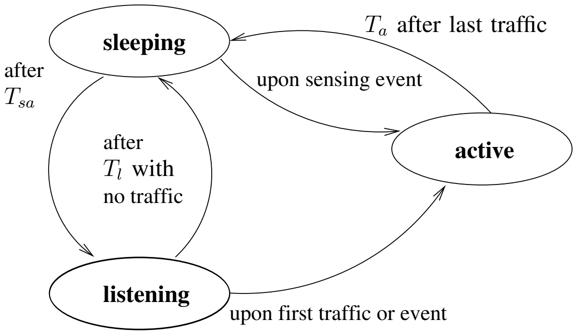


Fig. 4. States and Transitions in AFECA

C. Optimization and Simulation Results

To find out whether the weak dependence between neighboring node states affects the steady state probabilities we run the simulator to find out how far off the simulator steady states are from those found by the fixed point iteration. We use 10000 network nodes and we vary the probability of sensing an event, q between 0.1 and 0.7, we pick $r = 0.07$, Figure 5. As can be observed, the agreement between theory and simulation is convincing.

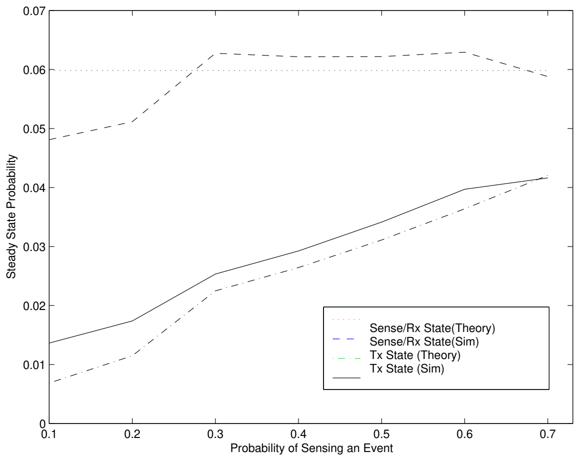


Fig. 5. Theory and Simulation, Steady State Probabilities vs. Probability of Sensing

We also investigate the behavior of the steady state probabilities as the number of nodes increases. We find a decrease in the *sense/receive* and *transition* steady state probabilities and an increase in the *off* steady state probability, as can be seen in Figure 6.

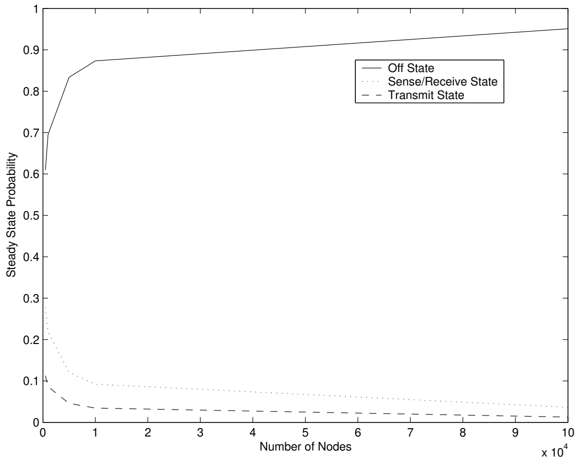


Fig. 6. Number of Nodes vs. Steady State Probabilities

This is explained by the increase in the node redundancy of the sensor network. This occurs despite the fact that we decrease r as we increase n to compensate for the increasing redundancy. $r(n) = \sqrt{1/\pi\sqrt{n}}$ is the monotonically decreasing function we use. In Figures 6 and 7, r ranges between 0.11931 and 0.031727. Figure 7 shows the average power consumption per node.

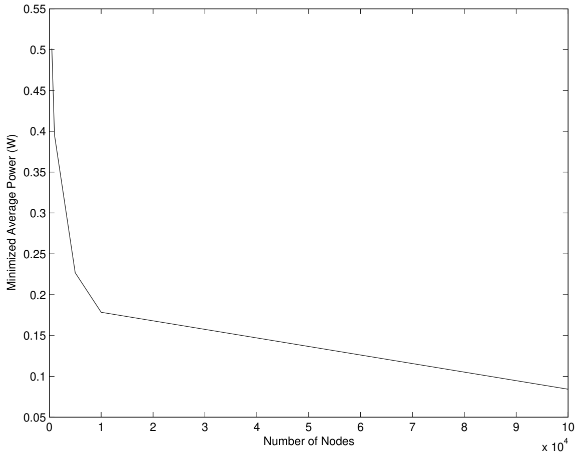


Fig. 7. Number of Nodes vs. Average Power

We compare our network design against AFECA for different event probabilities in Figure 8. The number of nodes in the network is fixed to 800. We fix the radius to 0.2 and we vary the probability of an event in the range $[0.01, 0.1]$. It can be observed that while AFECA is still far better than any protocol without energy savings, it reacts too much to small changes in the probability of an event, lagging behind our protocol in terms of the power used. This is because AFECA very quickly saturates in to a permanently active state as probability of event increases.

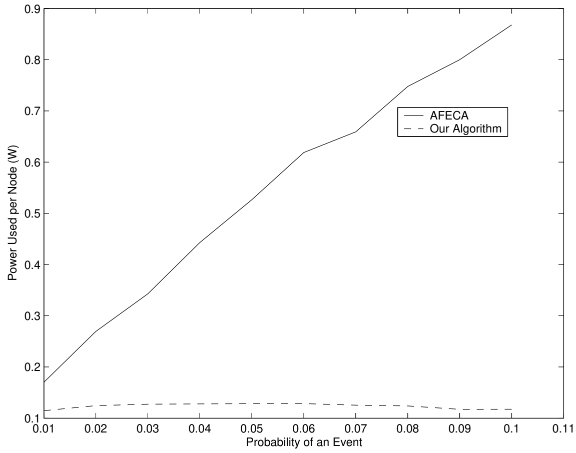


Fig. 8. AFECA vs. Our Scheme

IV. Conclusions

In this work we presented a sensor network design methodology, which conserves energy by asynchronously and probabilistically turning off redundant nodes while preserving connectivity and coverage. An important property is that the protocol is local. Each node independently determines its state transitions. The network design problem is expressed as an optimization problem which targets local node parameters. The state distribution of nodes is assumed to be independent (mean field approximation) and the simulations show that this assumption is reasonable by producing steady state distributions close to the theory. In the simulation study, we also show that the power savings of our protocol outperforms that of the existing competitors, by as much as an order of magnitude in some cases.

References

[1] I. F. Akyildiz, W. Su, and E. Cayirci Y. Sankarasubramaniam, "Wireless sensor networks: A survey," *Computer Networks*, vol. 38, no. 4, pp. 393–422, 2002.

[2] B. Krishnamachari, D. Estrin, and S. Wicker, "Impact of data aggregation in wireless sensor networks," in *Int. Workshop on DIstributed Event-Based Systems*, Vienna, Austria, July 2002.

[3] Leslie Lamport, "Time, clocks, and the ordering of events in a distributed system," *Communications of the ACM*, vol. 21, no. 7, pp. 558–565, July 1978.

[4] N. Lynch, *Distributed Algorithms*, Morgan Kaufmann, San Mateo CA, 1996.

[5] S. Singh and C. S. Raghavendra, "Power efficient mac protocol for multihop radio networks," in *Nineth IEEE ISPIMRC'98*, 1998, pp. 153–157.

[6] Y. Xu, J. Heidemann, and D. Estrin, "Geography-informed energy conservation for ad hoc networks," in *Proceedings MOBICOM'01, 2001*, 2001.

[7] B. Chen, K. Jamieson, H. Balakrishnan, and R. Morris, "Span: An energy-efficient coordination algorithm for topology maintenance in ad hoc wireless networks,," in *Proceedings of MOBICOM'01*, 2001.

[8] A. Cerpa and D. Estrin, "Ascent: Adaptive self-configuring sensor network topologies," in *Proceedings of INFOCOM'02*, 2002.
[9] M. Stemm and R. H. Katz, "Measuring and reducing energy consumption of network interfaces in hand-held devices," *IEICE Transactions on Communications*, vol. E80-B, no. 8, pp. 1125–1131, August 1997.
[10] Y. Xu, J. Heidemann, and D. Estrin, "Adaptive energy-conserving routing for multihop ad hoc networks," Tech. Rep. 527, USC/ISI, Los Angeles, CA, October 12 2000, http://www.isi.edu/ johnh/PAPERS/Xu00a.pdf.
[11] J. P. Monks, V. Bhargavan, and W. M. Hwu, "A power controlled multiple access protocol for wireless packet networks," in *Proceedings of INFOCOM'01*, 2001.
[12] Y. Wei, J. Heidemann, and D. Estrin, "An energy-efficient mac protocol for wireless sensor networks," in *Proceedings INFOCOM 2002*, 2002, vol. 3, pp. 1567 –1576.
[13] P. Gupta and P. R. Kumar, "Critical power for asymptotic connectivity in wireless networks," in *Stochastic Analysis, Control, Optimization and Applications, A Volume in Honor of W.H. Fleming. Edited by W.M. McEneany, G. Yin, and Q. Zhang*. 1998, pp. 547–566, Birkhauser.
[14] R. Ramanathan and R. Rosales-Hain, "Topology control of multihop wireless networks using transmit power adjustments," in *Proc. IEEE INFOCOM'00*, 2000.
[15] T. A. ElBatt, S. V. Krishnamurthyand D. Connors, and S. Dao, "Power management for throughput enhancement in wireless ad hoc networks," in *IEEE ICC'00*, 2000.
[16] S. Narayanaswamy, V. Kawadia, R.S. Sreenivas, and P. R. Kumar, "Power control in ad-hoc networks: Theory, architecture, algorithm and implementation of the compow protocol," in *European Wireless Conference*, 2002.
[17] S. Meguerdichian, F. Koushanfar, M. Potkonjak, and M. Srivastava, "Coverage problems in wireless ad-hoc sensor networks," in *Proceedings of IEEE INFOCOM'01*, 2001.
[18] S. Shakkottai, R. Srikant, and N. Shroff, "Unreliable sensor grids: Coverage, connectivity and diameter," in *Proceedings of IEEE INFOCOM'03*, 2003.
[19] Béla Bollobás, *Random Graphs, Second Edition*, Cambridge University Press, new york edition, 2001.
[20] P. Hall, *Introduction to the Theory of Coverage Processes*, John Wiley & Sons, New York, 1988.
[21] R. Meester and R. Roy, *Continuum Percolation*, Cambridge University Press, Cambridge, UK, 1996.
[22] Paul G. Hoel, Sifney C. Port, and Charles J. Stone, *Introduction to Stochastic Processes*, Copyright ©1972 by Houghton Mifflin Company, Reissued by Waveland Press Inc., 1987, 1987.
[23] J. Franklin, *Mathematical Methods of Economics*, Springer Verlag, New York, 1980.
[24] Rajeev Motwani and Prabhakar Raghavan, *Randomized Algorithms*, Cambridge University Press, Cambridge, UK, 2000.
[25] MathWorks, "Matlab: The language of technical computing," Available through http://www.mathworks.com/.