Quiz 3

60 Minutes

First Name: Solutions

Last Name: _____

RIN: _____

NO COLLABORATION or electronic devices.
Any violations will result in an F.
No questions allowed during the test unless you think there is a mistake.

GOOD LUCK!

Circle at most one answer per question.
10 points for each correct answer.

You **MUST** show **CORRECT** work to get credit.
Correct answers with no explanation will get a 0.

Final Score: _____ / 200

1. Consider the sets $A = \{0, 1\}$ and $B = \{3, 4, 5\}$. Which function is an injection from A to B?

   [A] $f(0) = 0, f(1) = 1$ Not $A \to B$

   [B] $f(0) = 3, f(1) = \{4.5\}$ Not $A \to B$

   [C] $f(0) = 3, f(1) = 4$

   [D] $f(0) = 3, f(1) = 3$ Not an injection

   [E] None of the above.

2. Consider the sets $A = \{0, 1\}$ and $B = \{3, 4, 5\}$. Which function is a surjection from A to B?

   [A] $f(0) = 0, f(1) = 1$

   [B] $f(0) = 3, f(1) = \{4, 5\}$

   [C] $f(0) = 3, f(1) = 4$ $|A| < |B|$. No surjection onto B.

   [D] $f(0) = 3, f(1) = 3$

   [E] None of the above.

SHOW WORK

3. Consider the real intervals $A = [0, 1]$, $B = [2, 4]$. Which function is a bijection from A to B?

   [A] $f(x) = 2x + 2$ Injection: $f(x) = f(y) \Rightarrow x = y$.

   [B] $f(x) = 2x + 1$ Surjection: Take any $y \in [2, 4)$.

   [C] $f(x) = 2x$ Then $x = \frac{y-2}{2} \in [0, 1)$,

   [D] $f(x) = 0.5x$ so that $f(x) = y$.

   [E] None of the above.

4. Consider the set $A = \mathbb{N} \times \mathbb{N}$ of pairs of natural numbers. What do we know about A?

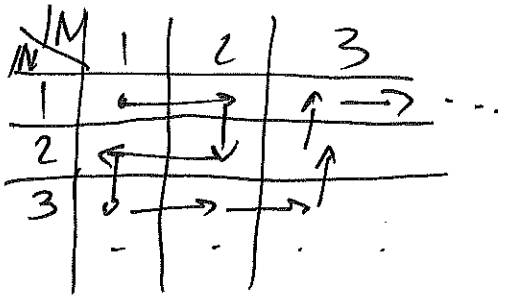
   [A] A is uncountable. Similar to $\mathbb{Q}$.

   [B] A is countable.

   [C] A is finite.

   [D] A has the same cardinality as the real interval $[0, 1]$.

   [E] None of the above.



5. Which of the following sets is not countable?

   [A] The set of all natural numbers. Definition of countable showed in class

   [B] The set of all rational numbers.

   [C] The set of all C programs. Finite binary strings

   [D] The set of all Turing Machines. finite binary strings

   [E] They are all countable.

6. How do we know there are functions we cannot compute?

infinite binary strings

- [A] The set of all functions is infinite, but countable.
- [B] The set of all programs is uncountable.
- [C] The set of all programs is countable, but the set of all functions is uncountable.
- [D] All functions can be computed.
- [E] None of the above.

finite binary strings

SHOW WORK

7. How many injections from $\{1,2\}$ to $\{1,2,3,4,5\}$ are there? Tinker!

- [A] 20
- [B] 32
- [C] 4
- [D] 30
- [E] None of the above.

Example injections: $f(1)=1, f(2)=2$
$f(1)=1, f(2)=3$
⋮
Each injection is a pair (i,j). $1 \le i,j \le 5$.
All permutations: $5 \times 4 = 20$.

8. What is the relationship between regular expressions and regular languages?

- [A] There exist regular languages which cannot be described with a regular expression.
- [B] There exist non-regular languages which can be described with a regular expression.
- [C] Regular languages is the set of languages that can be described with regular expressions.
- [D] All languages are regular.
- [E] None of the above.

9. Consider the language $\mathcal{L} = \{0, 001\}^*$. Which string is not in $\mathcal{L}$?

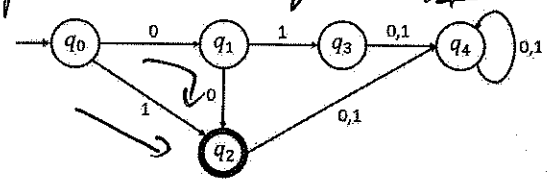
- [A] 00 $0 \cdot 0$
- [B] 01
- [C] 0000001 $0^4 \cdot 001$
- [D] 000000000001 $0^9 \cdot 001$
- [E] They are all in $\mathcal{L}$.

10. Consider the language $\mathcal{L} = \{01, 10\}^*$. What do we know about $\mathcal{L}$?

- [A] $\mathcal{L}$ contains all strings with the same number of 0s and 1s. No 01000111.
- [B] $\mathcal{L}$ contains all strings that start with a 0 or a 1. No, same example.
- [C] $\mathcal{L}$ contains all even-length strings. No, same example
- [D] $\mathcal{L}$ is finite. No, has *
- [E] None of the above.

11. Consider the DFA on the right. What is the language decided by this machine?

Two paths to q_2: 1 and 00



- [A] $\mathcal{L} = \{00\}$
- [B] $\mathcal{L} = \{00\}^*$
- [C] $\mathcal{L} = \{1\}$
- [D] $\mathcal{L} = \{1, 00\}$
- [E] None of the above.

SHOW WORK

12. Consider the DFA in Question 11. Which state does the DFA terminate in for the input string 0010?

$q_0 \xrightarrow{0} q_1 \xrightarrow{0} q_2 \xrightarrow{1} q_4 \xrightarrow{0} q_4$

- [A] q_1
- [B] q_2
- [C] q_3
- [D] q_4
- [E] None of the above

13. Consider the language $\mathcal{L} = \{0\}^*\{1\}^*\{10\}^*$. Which word is not in $\mathcal{L}$?

- [A] 0110 $0 \cdot 1 \cdot 10$
- [B] 1110 $\varepsilon \cdot 1 \cdot 1 \cdot 10$
- [C] 0011 $0 \cdot 0 \cdot 1 \cdot 1$
- [D] 0100 Can't have two 0's after 1.
- [E] They are all in $\mathcal{L}$.

14. Which of the following languages cannot be solved by a DFA? .

- [A] $\mathcal{L} =$ {strings with at least five 1s}
- [B] $\mathcal{L} =$ {strings with at least five million 1s}
- [C] $\mathcal{L} =$ {strings with no 1s}
- [D] $\mathcal{L} =$ {strings with same number of "10" and "01" substrings} Needs memory.
- [E] They are all regular languages.

15. Which of the following strings *cannot* be generated by the CFG: $S \to \varepsilon \mid 1S0 \mid 0S1$?

- [A] 1001 S only generates string w/ different 1st and last bit
- [B] 0011 $S \to 0S1 \to 00S11 \to 0011$
- [C] 1100 $S \to 1S0 \to 11S00 \to 1100$
- [D] 1010 $S \to 1S0 \to 10S10 \to 1010$
- [E] None of the above.

SHOW WORK.

16. Which CFG generates all strings of even length?

- [A] $S \to \varepsilon \mid 1S0 \mid 0S1$ Can't do 11
- [B] $S \to \varepsilon \mid 1S0S \mid 0S1S$ Can't do 11.
- [C] $S \to \varepsilon \mid 1S0 \mid 0S1 \mid 0S0 \mid 1S1$
- [D] $S \to 1S0S \mid 0S1S \mid 0S0S \mid 1S1S$ No empty string.
- [E] None of the above.

17. Compare the two CFGs G_1 and G_2 below. What do we know about them?

- [A] They generate the same languages.
- [B] They can both generate the word 1010.
- [C] They may produce different parse trees for the same word.
- [D] They generate regular languages.
- [E] All of the above.

G_2 can produce the same string in multiple ways

$G_1 : A \to \varepsilon \mid 1A0 \mid 0A1 \mid 0A0 \mid 1A1$

$G_2 : B \to \varepsilon \mid 1B0B \mid 0B1B \mid 0B0B \mid 1B1B$

All even length strings

18. What is the relationship between DFAs and pushdown automata (PDAs)?

- [A] They can decide the same set of languages.
- [B] DFAs can decide some languages that no PDA can decide.
- [C] PDAs can decide some languages that no DFA can decide.
- [D] They are the same model.
- [E] None of the above.

PDAs are more expressive than DFAs.

19. Why are Turing Machines (TMs) more expressive than pushdown automata (PDAs)?

- [A] TMs can have more discrete states.
- [B] There are uncountably many TMs.
- [C] TMs model the human brain.
- [D] TMs have random memory access, which allows them to decide more computing problems.
- [E] They are equally expressive.

20. Which statement is true about the language $\mathcal{L} = \{ww \mid w \in \{0,1\}^*\}$? ← Not regular, but decidable.

- [A] A DFA can solve this language. A TM can solve this language.
- [B] A DFA cannot solve this language. A TM can solve this language.
- [C] A DFA can solve this language. A TM cannot solve this language.
- [D] A DFA cannot solve this language. A TM cannot solve this language.
- [E] None of the above.

Scratch