Quiz 3

60 Minutes

First Name: _____

Last Name: _____

RIN: _____

NO COLLABORATION or electronic devices.
Any violations will result in an F.
No questions allowed during the test unless you think there is a mistake.

GOOD LUCK!

Circle at most one answer per question.
10 points for each correct answer.

You **MUST** show **CORRECT** work to get credit.
Correct answers with no explanation will get a 0.

Final Score: _____ / 200

1. Consider the sets $A = \{0, 1\}$ and $B = \{3, 4, 5\}$. Which function is an injection from A to B?

   A $f(0) = 0, f(1) = 1$

   B $f(0) = 3, f(1) = \{4.5\}$

   C $f(0) = 3, f(1) = 4$

   D $f(0) = 3, f(1) = 3$

   E None of the above.

2. Consider the sets $A = \{0, 1\}$ and $B = \{3, 4, 5\}$. Which function is a surjection from A to B?

   A $f(0) = 0, f(1) = 1$

   B $f(0) = 3, f(1) = \{4, 5\}$

   C $f(0) = 3, f(1) = 4$

   D $f(0) = 3, f(1) = 3$

   E None of the above.

3. Consider the real intervals $A = [0, 1]$, $B = [2, 4]$. Which function is a bijection from A to B?

   A $f(x) = 2x + 2$

   B $f(x) = 2x + 1$

   C $f(x) = 2x$

   D $f(x) = 0.5x$

   E None of the above.

4. Consider the set $A = \mathbb{N} \times \mathbb{N}$ of pairs of natural numbers. What do we know about A?

   A A is uncountable.

   B A is countable.

   C A is finite.

   D A has the same cardinality as the real interval $[0, 1]$.

   E None of the above.

5. Which of the following sets is not countable?

   A The set of all natural numbers.

   B The set of all rational numbers.

   C The set of all C programs.

   D The set of all Turing Machines.

   E They are all countable.

6. How do we know there are functions we cannot compute?

   A The set of all functions is infinite, but countable.
   B The set of all programs is uncountable.
   C The set of all programs is countable, but the set of all functions is uncountable.
   D All functions can be computed.
   E None of the above.

7. How many injections from $\{1, 2\}$ to $\{1, 2, 3, 4, 5\}$ are there?

   A 20
   B 32
   C 4
   D 30
   E None of the above.

8. What is the relationship between regular expressions and regular languages?

   A There exist regular languages which cannot be described with a regular expression.
   B There exist non-regular languages which can be described with a regular expression.
   C Regular languages is the set of languages that can be described with regular expressions.
   D All languages are regular.
   E None of the above.

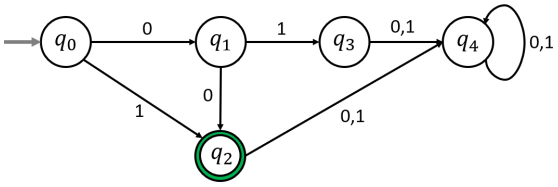
9. Consider the language $\mathcal{L} = \{0, 001\}^*$. Which string is not in $\mathcal{L}$?

   A 00
   B 01
   C 0000001
   D 000000000001
   E They are all in $\mathcal{L}$.

10. Consider the language $\mathcal{L} = \{01, 10\}^*$. What do we know about $\mathcal{L}$?

    A $\mathcal{L}$ contains all strings with the same number of 0s and 1s.
    B $\mathcal{L}$ contains all strings that start with a 0 or a 1.
    C $\mathcal{L}$ contains all even-length strings.
    D $\mathcal{L}$ is finite.
    E None of the above.

11. Consider the DFA on the right. What is the language decided by this machine?



A $\mathcal{L} = \{00\}$

B $\mathcal{L} = \{00\}^*$

C $\mathcal{L} = \{1\}$

D $\mathcal{L} = \{1, 00\}$

E None of the above.

12. Consider the DFA in Question 11. Which state does the DFA terminate in for the input string 0010?

A q_1

B q_2

C q_3

D q_4

E None of the above

13. Consider the language $\mathcal{L} = \{0\}^*\{1\}^*\{10\}^*$. Which word is not in $\mathcal{L}$?

A 0110

B 1110

C 0011

D 0100

E They are all in $\mathcal{L}$.

14. Which of the following languages cannot be solved by a DFA?

A $\mathcal{L} = \{$strings with at least five 1s$\}$

B $\mathcal{L} = \{$strings with at least five million 1s$\}$

C $\mathcal{L} = \{$strings with no 1s$\}$

D $\mathcal{L} = \{$strings with same number of "10" and "01" substrings$\}$

E They are all regular languages.

15. Which of the following strings *cannot* be generated by the CFG: $S \to \varepsilon \mid 1S0 \mid 0S1$?

A 1001

B 0011

C 1100

D 1010

E None of the above.

16. Which CFG generates all strings of even length?

- [A] $S \to \varepsilon \mid 1S0 \mid 0S1$
- [B] $S \to \varepsilon \mid 1S0S \mid 0S1S$
- [C] $S \to \varepsilon \mid 1S0 \mid 0S1 \mid 0S0 \mid 1S1$
- [D] $S \to 1S0S \mid 0S1S \mid 0S0S \mid 1S1S$
- [E] None of the above.

17. Compare the two CFGs G_1 and G_2 below. What do we know about them?

$G_1 : A \to \varepsilon \mid 1A0 \mid 0A1 \mid 0A0 \mid 1A1$

$G_2 : B \to \varepsilon \mid 1B0B \mid 0B1B \mid 0B0B \mid 1B1B$

- [A] They generate the same languages.
- [B] They can both generate the word 1010.
- [C] They may produce different parse trees for the same word.
- [D] They generate regular languages.
- [E] All of the above.

18. What is the relationship between DFAs and pushdown automata (PDAs)?

- [A] They can decide the same set of languages.
- [B] DFAs can decide some languages that no PDA can decide.
- [C] PDAs can decide some languages that no DFA can decide.
- [D] They are the same model.
- [E] None of the above.

19. Why are Turing Machines (TMs) more expressive than pushdown automata (PDAs)?

- [A] TMs can have more discrete states.
- [B] There are uncountably many TMs.
- [C] TMs model the human brain.
- [D] TMs have random memory access, which allows them to decide more computing problems.
- [E] They are equally expressive.

20. Which statement is true about the language $\mathcal{L} = \{ww \mid w \in \{0,1\}^*\}$?

- [A] A DFA can solve this language. A TM can solve this language.
- [B] A DFA cannot solve this language. A TM can solve this language.
- [C] A DFA can solve this language. A TM cannot solve this language.
- [D] A DFA cannot solve this language. A TM cannot solve this language.
- [E] None of the above.

Scratch