Quiz 2

60 Minutes

First Name: _____

Last Name: _____

RIN: _____

NO COLLABORATION or electronic devices.
Any violations will result in an F.
No questions allowed during the test unless you think there is a mistake.

GOOD LUCK!

Circle at most one answer per question.
10 points for each correct answer.

You **MUST** show **CORRECT** work to get credit.
Correct answers with no explanation will get a 0.

Final Score: _____ / 200

1. Suppose a goody-bag contains 3 candies. Candies come in three colors: red, green and blue. How many types of goody-bags are there in total?

   A $\binom{3}{2}$

   B $3!$

   C $\binom{5}{2}$

   D $5!$

   E None of the above.

2. Suppose a goody-bag contains 3 candies. Candies come in three colors: red, green and blue. If all goody-bags are equally likely, how many goody-bags would I need to buy in order to guarantee I have a candy of each color?

   A $\binom{3}{2}$

   B $3!$

   C $\binom{5}{2}$

   D $5!$

   E None of the above.

3. Suppose FOCS has 10 students and every student tries to shake hands with as many other students as possible. How many handshakes need to occur in total to guarantee a repeat?

   A 10

   B 20

   C 35

   D 46

   E 91

4. Let X be a random variable and let the set of all outcomes be Ω. What is $\sum_{x \in X(\Omega)} \mathbb{P}[X = x]$?

   A 0.5

   B 1

   C 1.5

   D 2

   E None of the above.

5. Suppose $X(\Omega) = \{1, 2, 3\}$ and suppose $\mathbb{P}[X = 1 \vee X = 2] = 0.5$. What is $\mathbb{P}[X = 3]$?

   A 0.2

   B 0.3

   C 0.4

   D 0.5

   E 1

6. Suppose I toss three coins independently. What do we know?

   A   At least two coins must match.

   B   The probability that all coins match is 1/8.

   C   The probability of at least one H is 1.

   D   The probability of at least one T is 1.

   E   None of the above.

7. Suppose X_1 and X_2 are independent and uniform on $\{1, 2, 3, 4, 5\}$. What is $\mathbb{P}[X_1 + X_2 \leq 3]$?

   A $\frac{1}{25}$

   B $\frac{2}{25}$

   C $\frac{3}{25}$

   D $\frac{4}{25}$

   E $\frac{5}{25}$

8. Suppose X_1 and X_2 are independent and uniform on $\{1, 2, 3, 4, 5\}$. What is $\mathbb{E}[X_1 + X_2]$?

   A   3

   B   4

   C   5

   D   6

   E   7

9. Suppose X_1 is uniform on $\{1, 2, 3, 4, 5\}$. If $X_1 \geq 4$, then X_2 is uniform on $\{4, 5\}$; otherwise $X_2 = 5$. What is $\mathbb{E}[X_1 + X_2]$?

   A $\frac{30}{5}$

   B $\frac{33}{5}$

   C $\frac{36}{5}$

   D $\frac{39}{5}$

   E $\frac{42}{5}$

10. Suppose Submitty had a bug and randomly shuffled Quiz 2 grades. Assuming there are 200 students and all grades are different, what is the expected number of students who get their correct grade in Submitty?

   A 1   B 10   C 20   D 50   E None of the above.

11. Suppose the correct answer is not E on any of the 20 questions and you guess randomly among A-D. How many of the 20 questions do you expect to get right?

    A 1

    B 2

    C 3

    D 4

    E 5

12. Suppose you answer A on all 20 questions. How many questions do you expect to get right?

    A 2

    B 3

    C 4

    D 5

    E It cannot be determined from the given information.

13. Suppose it is sunny 1/10 of days in Troy. How much do you expect to wait until a sunny day?

    A 5 days

    B 10 days

    C 15 days

    D 20 days

    E None of the above.

14. Suppose it is sunny 1/10 of days in Troy. Suppose it is always sunny in Philadelphia, except for the days when it is sunny in Troy. How many days is a Philadelphian expected to wait until a sunny day?

    A 10/9

    B 9/10

    C 2

    D 3

    E None of the above.

15. You are in Troy now. If the weather is not sunny, you travel to Philadelphia tomorrow; if it's not sunny in Philadelphia tomorrow, you go back to Troy the day after (and will go back and forth on non-sunny days). How many days do you expect to wait until a sunny day (assuming same probabilities of sunny days as in Question 14)?

    A 91/91   B 100/91   C 190/91   D 290/91   E 182/91

16. Suppose a covid test is correct 90% of the time and 10% of all people have covid. What is the probability that you have covid if you tested positive?

   A 1/10
   B 9/10
   C 1/3
   D 1/2
   E None of the above.

17. Suppose a covid test is correct 90% of the time and 10% of all people have covid. What is the probability that you have covid if you tested positive two times independently?

   A 50/100
   B 50/90
   C 81/100
   D 81/90
   E None of the above.

18. Suppose the correct answer is uniform on $\{A, B, C, D, E\}$. What is the probability that at least 2 of the 20 questions have the same letter for the correct answer?

   A 0
   B $\binom{20}{2}$
   C 0.5
   D 1
   E None of the above.

19. If each question had 26 choices, and the correct answer is uniform on $\{A, \ldots, Z\}$, what is the probability that at least 2 of the 20 questions have the same letter for the correct answer?

   A $\left(\frac{25}{26}\right)^{19} \times \left(\frac{24}{25}\right)^{18} \times \cdots \times \left(\frac{6}{7}\right)$
   B $1 - \left(\frac{25}{26}\right)^{19} \times \left(\frac{24}{25}\right)^{18} \times \cdots \times \left(\frac{6}{7}\right)$
   C $\left(\frac{25}{26}\right)^{19}$
   D $1 - \left(\frac{25}{26}\right)^{19}$
   E None of the above.

20. $X \sim B(p_1)$ and $Y \sim B(p_2)$ are independent Bernoulli random variables. What is $\mathbb{E}[XY]$?

   A 1/4
   B $p_1 p_2$
   C p_1/p_2
   D $p_1 + p_2$
   E None of the above.

Scratch