QUIZ 1: 60 Minutes

Last Name: Solutions

First Name: ______

RIN: ______

Section: ______

Answer **ALL** questions.
NO COLLABORATION or electronic devices. Any violations result in an F.
NO questions allowed during the test. Interpret and do the best you can.

GOOD LUCK!

Circle at most one answer per question.
10 points for each correct answer.

You **MUST** show **CORRECT** work to get credit.

When in doubt, TINKER.

Total
200

1. Which set below is the set $S = \{2k \mid k \in \mathbb{N}\}$? $= \{2, 4, 6, \ldots\}$

E

- $\boxed{\text{A}}$ All even numbers. ×
- $\boxed{\text{B}}$ All odd numbers. ×
- $\boxed{\text{C}}$ All non-negative even numbers. $= \{0, 2, 4, 6, \ldots\}$
- $\boxed{\text{D}}$ All non-negative odd numbers. ×
- $\boxed{\text{E}}$ None of the above.

2. Define sets $A = \{2k \mid k \in \mathbb{Z}\}$, $B = \{9k \mid k \in \mathbb{Z}\}$ and $C = \{6k \mid k \in \mathbb{Z}\}$. Which is true?

B

- $\boxed{\text{A}}$ $A \cap B = C$.
- $\boxed{\text{B}}$ $A \cap B \subseteq C$.
- $\boxed{\text{C}}$ $A \cap B = \overline{C}$.
- $\boxed{\text{D}}$ $A \cap B \subseteq \overline{C}$.
- $\boxed{\text{E}}$ None of the above.

Popquiz 4.9

$A = \{0, \pm 2, \pm 4, \ldots\}$

$B = \{0, \pm 9, \pm 18, \ldots\}$

$A \cap B = \{0, \pm 18, \pm 36, \ldots\} \subseteq C.$

3. How many rows are in the truth table of $p \to (p \vee q)$?

B

- $\boxed{\text{A}}$ 2.
- $\boxed{\text{B}}$ 4.
- $\boxed{\text{C}}$ 6.
- $\boxed{\text{D}}$ 8.
- $\boxed{\text{E}}$ None of the above.

2 variables $\to$ 2^2 rows

F	F
F	T
T	F
T	T

4. True or false, $p \to (p \vee q)$?

C

- $\boxed{\text{A}}$ Can be true or false, depending on p.
- $\boxed{\text{B}}$ Can be true or false, depending on q.
- $\boxed{\text{C}}$ Always true.
- $\boxed{\text{D}}$ Always false.
- $\boxed{\text{E}}$ None of the above.

p is F $\longrightarrow$ true (LHS is F)

p is T $\longrightarrow$ true (RHS is T)

$\downarrow$

always true.

5. If you majored CS then you took FOCS. Joe took FOCS and Barb majored CS. What else do we know?

B.

- $\boxed{\text{A}}$ Joe majored CS. We don't know anything more about Barb.
- $\boxed{\text{B}}$ We don't know anything more about Joe. Barb took FOCS.
- $\boxed{\text{C}}$ Joe majored CS. And, Barb took FOCS.
- $\boxed{\text{D}}$ Joe did not major CS. And, Barb took FOCS.
- $\boxed{\text{E}}$ None of the above.

CS $\to$ FOCS.

Joe $\to$ RHS is true
Dont know about LHS

Barb $\to$ LHS is true
So know RHS is true
$\therefore$ Barb took FOCS.

6. What is the negation of the claim $\forall m, n \in \mathbb{N} : 3m + 6n \neq 10$?

$\boxed{A}$ $\forall m, n \in \mathbb{N} : 3m + 6n = 10$.

$\boxed{B}$ $\forall m, n \in \mathbb{N} : 3m + 6n \neq 10$.

$\boxed{C}$ $\exists m, n \in \mathbb{N} : 3m + 6n = 10$.

$\boxed{D}$ $\exists m, n \in \mathbb{N} : 3m + 6n \neq 10$.

$\boxed{E}$ None of the above.

C

$\neg \forall m \forall n : 3m + 6n \neq 10$

$\rightarrow \exists m \neg \forall n : 3m + 6n \neq 10$

$\rightarrow \exists m \exists n : \neg(3m + 6n \neq 10)$

$\rightarrow \exists m \exists n : 3m + 6n = 10$.

7. Which proof-method is acceptable to prove the claim p?

$\boxed{A}$ Assume p is true and derive something known to be true, for example $0 = 0$. × Can't assume p.

$\boxed{B}$ Assume $\neg p$ is true and derive something known to be true, for example $0 = 0$. → doesn't prove anything

$\boxed{C}$ Assume p is true and derive something known to be false, for example $1 > 2$. ×

$\boxed{D}$ Assume $\neg p$ is true and derive something known to be false, for example $1 > 2$. ← proof by contradiction

$\boxed{E}$ None of the above.

D

8. Consider the claim $\exists m, n \in \mathbb{Z} : 9m + 21n = 7$. Is the claim true or false?

$\boxed{A}$ True.

$\boxed{B}$ False.

$\boxed{C}$ It depends on m.

$\boxed{D}$ It depends on n.

$\boxed{E}$ None of the above.

B

False: Suppose $\exists m, n$: $9m + 21n = 7$

LHS is div 3, RHS is not → contradiction.

∴ Claim is not true.

[See also Exercise 4.7(a)]

9. How do you *disprove* the claim $\forall n \in \mathbb{N} : \neg P(n) \rightarrow Q(n)$.

$\boxed{A}$ Show that for all $n \in \mathbb{N}$, $P(n)$ is true and $Q(n)$ is false.

$\boxed{B}$ Show that for all $n \in \mathbb{N}$, $P(n)$ is false and $Q(n)$ is false.

$\boxed{C}$ Show that for some $n \in \mathbb{N}$, $P(n)$ is true and $Q(n)$ is false.

$\boxed{D}$ Show that for some $n \in \mathbb{N}$, $P(n)$ is false and $Q(n)$ is false.

$\boxed{E}$ None of the above.

D

Find an n for which the implication is false, that is $\neg P$ is true → P is F, Q is F

10. What is the first step in a proof by contradiction of the claim $\forall m, n \in \mathbb{N} : 3m + 6n \neq 10$.

$\boxed{A}$ Define the predicate $P(m, n) : 3m + 6n \neq 10$ and prove the base case $P(1, 1)$.

$\boxed{B}$ Assume $3m + 6n = 10$ for all $m, n \in \mathbb{N}$.

$\boxed{C}$ Assume $3m + 6n \neq 10$ for some $m, n \in \mathbb{N}$.

$\boxed{D}$ Assume $3m + 6n = 10$ for some $m, n \in \mathbb{N}$.

$\boxed{E}$ None of the above.

D

Assume the negation is true that is.

$\exists m \exists n : 3m + 6n = 10$.

(see prob 6).

11. You decided to *prove* the claim $n^2 \leq 2^n$ for all $n \geq 4$. Which method of proof would you use?

[A] Find a single value $n_* \in \mathbb{N}$ for which $n_*^2 > 2^{n_*}$. → dis proof

[B] Show that the formula $n^2 \leq 2^n$ is true for $n = 1$ up to $n = 1000$. → tinkering not proof

[C] Proof by induction. ✓

[D] Contraposition proof. → for implications

[E] Direct proof. → for implications

C

12. You decided to *disprove* the claim $n^2 \leq 2^n$ for all $n \geq 1$. Which method of proof would you use?

[A] Find a single value $n_* \in \mathbb{N}$ for which $n_*^2 > 2^{n_*}$. ← Find a counterexample.

[B] Show that the formula $n^2 \leq 2^n$ is true for $n = 1$ up to $n = 1000$.

[C] Proof by induction.

[D] Contraposition proof.

[E] Direct proof.

A

13. How do you prove, by induction, the claim "5 divides $11^n - 6$" for all $n \geq 5$?

Show Base Case $P(5)$
Induction Step: $P(n) \to P(n+1)$ for $n \geq 5$

[A] Show 5 divides $11^5 - 6$.

[B] Show 5 divides $11^5 - 6, 11^6 - 6, 11^7 - 6$ all the way up to $11^{1,000,000} - 6$.

[C] Show, for $n \geq 5$, if 5 divides $11^n - 6$ then 5 divides $11^{n+1} - 6$.

[D] $\underbrace{\text{Show 5 divides } 11^5 - 6.}_{\text{Base}}$ And, $\underbrace{\text{show, for } n \geq 5, \text{ if 5 divides } 11^n - 6 \text{ then 5 divides } 11^{n+1} - 6.}_{\text{Induction step.}}$

[E] None of the above.

D

14. You wish to prove $n^4 \leq 2^n$ for $n \geq 16$. You showed that $\underbrace{n^4 \leq 2^n \to (n+3)^4 \leq 2^{n+3} \text{ for } n \geq 16}$. What base cases do you need to prove to complete the proof?

3-leaping induction
Need 3 Base Cases 16, 17, 18

[A] $n = 1$.

[B] $n = 16$.

[C] $n = 1$ and $n = 2$.

[D] $n = 16$ and $n = 17$.

[E] None of the above.

E

15. Define the predicate $P(n) : (2n-1)^2 + 4$ is prime. For which n is $P(n)$ true?

$n = 5$ gives $9^2 + 4 = 85$ ← not prime.

[A] $n \geq 1$. ✗

[B] $n \geq 2$. ✗

[C] $n \geq 3$. ✗

[D] $n \geq 4$. ✗

[E] None of the above.

E

16. Define the sum $S(n) = \frac{1}{1\times 2} + \frac{1}{2\times 3} + \frac{1}{3\times 4} + \cdots + \frac{1}{(n-1)\times n}$ for $n \geq 2$. What is $S(100)$?

This is an induction problem from chapter 5.

[A] 0.1.

[B] 0.01.

[C] 0.9.

[D] 0.99.

[E] None of the above.

n	2	3	4	5	
$S(n)$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{4}{5}$	...

Guess: $S(n) = \frac{n-1}{n}$

$S(2) = \frac{1}{2}$ ✓

Assume $S(n) = \frac{n-1}{n}$

Consider $S(n+1) = S(n) + \frac{1}{n(n+1)} = \frac{n-1}{n} + \frac{1}{n(n+1)}$

$= \frac{n^2 - 1 + 1}{n(n+1)} = \frac{n}{n+1}$ ✓

$\boxed{S(100) = \frac{99}{100} = 0.99}$

D

No work No Credit

17. $f(1) = 1$, $f(2) = 2$, and $f(n) = f(n-2) + 2$ for $n > 2$. What is $f(100)$?

[A] It cannot be computed because the recursion does not have enough base cases.

[B] 50.

[C] 100.

[D] 200.

[E] None of the above.

n	1	2	3	4	5 ...
f	1	2	3	4	5

Guess $f(n) = n$.

$f(1) = 1$, $f(2) = 2$

Assume $f(n) = n$ for $1, 2, \ldots n$.

$f(n+1) = f(n-1) + 2 = n - 1 + 2 = n+1$ ✓

$\therefore f(100) = 100$

C

No work No Credit

18. Define $\mathcal{A}$ recursively: (i) $1 \in \mathcal{A}$ (ii) $x \in \mathcal{A} \to x + 4 \in \mathcal{A}$ (iii) Nothing else is in $\mathcal{A}$. Which is true?

[A] Every number in $\mathcal{A}$ is even. ✗

[B] Every even number is in $\mathcal{A}$. ✗

[C] Every number in $\mathcal{A}$ is odd. ✓

[D] Every odd number is in $\mathcal{A}$. ✗ (what about 3)

[E] None of the above.

$A = \{1, 5, 9, \ldots\}$.

C

19. A rooted binary tree (RBT) has 8 vertices. How many links does it have?

[A] 6.

[B] 7.

[C] 8.

[D] 9.

[E] None of the above.

Links = # vertices − 1

$\therefore$ 7 links

B

20. There are 5 distinct rooted binary trees (RBT) with 3 vertices. How many have 4 vertices?

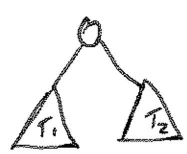
[A] 12.

[B] 13.

[C] 14.

[D] 15.

[E] None of the above.



0 vert — 3 vert → 5
1 vert — 2 vert → 2
2 vert — 1 vert → 2
3 vert — 0 vert → 5

14

← one 1 vert T_1 two 2 vert T_2

C

No work No Credit