Quiz 1

60 Minutes

First Name: ____________________

Last Name: ____________________

RIN: ____________________

Section: ____________________

NO COLLABORATION or electronic devices.
Any violations will result in an F.
No questions allowed during the test unless you think there is a mistake.

GOOD LUCK!

Circle at most one answer per question.
10 points for each correct answer.

You **MUST** show **CORRECT** work to get credit.
Correct answers with no explanation will get a 0.

Final Score: ______ / 200

1. What is a simpler expression for the set $A = \{n \mid n = 12k + 3m, k \in \mathbb{N}, m \in \mathbb{N}\}$?

   $\boxed{\text{A}}$ $A = \{n \mid n = 3k, k \in \mathbb{N}\}$

   $\boxed{\text{B}}$ $A = \{n \mid n = 12 + 3k, k \in \mathbb{N}\}$

   $\boxed{\text{C}}$ $A = \{n \mid n = 12 + 3k, k \in \mathbb{Z}\}$

   $\boxed{\text{D}}$ $A = \{n \mid n = 15 + 3k, k \in \mathbb{N}\}$

   $\boxed{\text{E}}$ None of the above.

2. How did we use the principle of well-ordering to prove that $\sqrt{2}$ is irrational?

   $\boxed{\text{A}}$ We used it to assume $\sqrt{2}$ is rational.

   $\boxed{\text{B}}$ We used it to conclude $\sqrt{2}$ has a factor of 3.

   $\boxed{\text{C}}$ We used it to list the set of all pairs (p, q) such that $\sqrt{2} = p/q$.

   $\boxed{\text{D}}$ We did not use the principle of well-ordering.

   $\boxed{\text{E}}$ None of the above.

3. What is the value of the expression $(p \wedge \neg p) \rightarrow (\neg p \wedge q)$?

   $\boxed{\text{A}}$ Depends on the value of p.

   $\boxed{\text{B}}$ Depends on the value of q.

   $\boxed{\text{C}}$ Always true.

   $\boxed{\text{D}}$ Always false.

   $\boxed{\text{E}}$ None of the above.

4. IF you are in FOCS, THEN you must have taken CS1 AND you must have taken DS. Suppose you have not taken CS1. What do we know?

   $\boxed{\text{A}}$ You are not in FOCS.

   $\boxed{\text{B}}$ You are in DS.

   $\boxed{\text{C}}$ The value of “you are in FOCS” depends on the value of “you have taken DS”.

   $\boxed{\text{D}}$ The value of “you have taken DS” depends on the value of “you are in FOCS”.

   $\boxed{\text{E}}$ None of the above.

5. How many rows are there in the truth table of $(p \wedge q) \vee (\neg p \wedge q) \vee q$?

   $\boxed{\text{A}}$ 2

   $\boxed{\text{B}}$ 3

   $\boxed{\text{C}}$ 4

   $\boxed{\text{D}}$ 8

   $\boxed{\text{E}}$ 16

6. Suppose I want to prove $p \to q$. Which of the following proof techniques will work?

   $\boxed{\text{A}}$ Assume p and show that it leads to a contradiction.

   $\boxed{\text{B}}$ Assume q is false and show that p must be false.

   $\boxed{\text{C}}$ Assume q is false and show it leads to a contradiction.

   $\boxed{\text{D}}$ Use derivations to show that if q is false, then p must be false.

   $\boxed{\text{E}}$ All of the above.

7. What is the negation of the claim: $\forall x \in \mathbb{Z} : \exists y \in \mathbb{N} : x > y$?

   $\boxed{\text{A}}$ $\forall x \in \mathbb{Z} : \exists y \in \mathbb{N} : x \leq y$

   $\boxed{\text{B}}$ $\forall x \in \mathbb{Z} : \forall y \in \mathbb{N} : x > y$

   $\boxed{\text{C}}$ $\forall x \in \mathbb{N} : \exists y \in \mathbb{Z} : x > y$

   $\boxed{\text{D}}$ $\exists x \in \mathbb{Z} : \forall y \in \mathbb{N} : x \leq y$

   $\boxed{\text{E}}$ None of the above.

8. Suppose $p, q \in \mathcal{P}$ are prime ($\mathcal{P}$ is the set of all prime numbers). Which of the following is true?

   $\boxed{\text{A}}$ $\forall p, q \in \mathcal{P} : pq - 1$ is prime.

   $\boxed{\text{B}}$ $\forall p, q \in \mathcal{P} : pq - 1$ is not prime.

   $\boxed{\text{C}}$ $\exists p, q \in \mathcal{P} : pq - 1$ is prime.

   $\boxed{\text{D}}$ $\forall p, q \in \mathcal{P} : p + q$ is prime.

   $\boxed{\text{E}}$ None of the above.

9. Suppose I try to prove $n^2 \leq 2^n, \forall n \geq 1$ using induction. What goes wrong?

   $\boxed{\text{A}}$ The base case is false.

   $\boxed{\text{B}}$ I need more base cases.

   $\boxed{\text{C}}$ I cannot prove $P(n) \to P(n+1), \forall n \geq 1$.

   $\boxed{\text{D}}$ I cannot prove $P(n+1) \to P(n), \forall n \geq 1$.

   $\boxed{\text{E}}$ Nothing goes wrong because the claim is true.

10. How would you *disprove* the claim: $\forall m \in \mathbb{N} : \exists n \in \mathbb{N} : m^2 = n$?

   $\boxed{\text{A}}$ Show that $m^2 \neq n$ for all natural numbers m and n.

   $\boxed{\text{B}}$ Show that $m^2 \neq n$ for all integers m and n.

   $\boxed{\text{C}}$ Find some $m, n \in \mathbb{N}$ for which $m^2 \neq n$.

   $\boxed{\text{D}}$ Find some $m \in \mathbb{N}$ for which $m^2 \neq n$ for all $n \in \mathbb{N}$.

   $\boxed{\text{E}}$ None of the above.

11. Consider the recurrence $T_0 = 1, T_n = T_{n-2} + 2$. What is T_{179}?

   $\boxed{\text{A}}$ 178

   $\boxed{\text{B}}$ 179

   $\boxed{\text{C}}$ 180

   $\boxed{\text{D}}$ It is not defined.

   $\boxed{\text{E}}$ None of the above.

12. Consider the set $\mathcal{S}$ defined as follows: (1) Base case: $1 \in \mathcal{S}$; (2) Constructor: $x \in \mathcal{S} \to x + 2 \in \mathcal{S}$. Which of the following <u>cannot</u> be the set $\mathcal{S}$?

   $\boxed{\text{A}}$ All odd natural numbers.

   $\boxed{\text{B}}$ All odd integers.

   $\boxed{\text{C}}$ All natural numbers.

   $\boxed{\text{D}}$ $\mathbb{N} \cap \{n \mid n = 2k - 3, k \in \mathbb{N}\}$.

   $\boxed{\text{E}}$ $\mathbb{N} \cap \{n \mid n = 2k, k \in \mathbb{N}\}$.

13. Define the predicate $P(n) : 3n^2 \leq n^3$. For which n is $P(n)$ true?

   $\boxed{\text{A}}$ $n \geq 1$

   $\boxed{\text{B}}$ $n \geq 2$

   $\boxed{\text{C}}$ $n \geq 3$

   $\boxed{\text{D}}$ Only for $n = 4$ and $n = 5$

   $\boxed{\text{E}}$ None of the above

14. Which of the following proof techniques can be used to prove $n \leq 3^{n/3}, \forall n \geq 0$? Suppose $P(n) : n \leq 3^{n/3}$.

   $\boxed{\text{A}}$ Show $P(0)$ and $P(1)$ are true and show that $P(1) \wedge \cdots \wedge P(n) \to P(n+1), \forall n \geq 0$.

   $\boxed{\text{B}}$ Show that $P(n) \to P(n+1), \forall n \geq 0$.

   $\boxed{\text{C}}$ Show that $P(0) \wedge P(1) \wedge \cdots \wedge P(n) \to P(n+1), \forall n \geq 0$.

   $\boxed{\text{D}}$ Define $Q(n) = P(0) \wedge P(1) \wedge \cdots \wedge P(n)$ and show that $Q(n) \to Q(n+1), \forall n \geq 0$.

   $\boxed{\text{E}}$ None of the above.

15. What can we say about this claim: $\exists x \in \mathbb{R} : \forall y \in \mathbb{R} : xy = y$?

   $\boxed{\text{A}}$ True

   $\boxed{\text{B}}$ False

   $\boxed{\text{C}}$ Depends on the value of x

   $\boxed{\text{D}}$ Depends on the value of y

   $\boxed{\text{E}}$ None of the above.

16. What is another expression for the set $A \cap \overline{(A \cap B)}$?

$\boxed{\text{A}}$ $A \cap \overline{B}$

$\boxed{\text{B}}$ $A \cup \overline{B}$

$\boxed{\text{C}}$ $A \cap \overline{A} \cap B$

$\boxed{\text{D}}$ $A \cup \overline{A} \cup A \cap B$

$\boxed{\text{E}}$ None of the above.

17. What is a formal way to say "Every positive real distance is realized by some points on the plane"?

$\boxed{\text{A}}$ $\exists x \in \mathbb{R} : \forall x_1, x_2, y_1, y_2 \in \mathbb{R} : x = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$\boxed{\text{B}}$ $\exists x \in \mathbb{R} : \exists x_1, x_2, y_1, y_2 \in \mathbb{R} : x = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$\boxed{\text{C}}$ $\forall x \in \mathbb{R} \cap \{z \mid z > 0, z \in \mathbb{R}\} : \forall x_1, x_2, y_1, y_2 \in \mathbb{R} : x = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$\boxed{\text{D}}$ $\forall x \in \mathbb{R} \cap \{z \mid z > 0, z \in \mathbb{R}\} : \exists x_1, x_2, y_1, y_2 \in \mathbb{R} : x = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

$\boxed{\text{E}}$ None of the above.

18. Let $A_n = 2A_{n-1} - 1$ and $A_0 = 2$. What is a general formula for A_n for $n \geq 1$?

$\boxed{\text{A}}$ $A_n = 2n - 1$

$\boxed{\text{B}}$ $A_n = 2n + 1$

$\boxed{\text{C}}$ $A_n = 2^n$

$\boxed{\text{D}}$ $A_n = 2^n + 1$

$\boxed{\text{E}}$ None of the above.

19. Suppose I create a new type of rooted tree, called perfect binary tree (PBT), as follows:

① The tree with one vertex is a PBT. **[base case]**

② If PBTs T_1 and T_2 with roots r_1 and r_2 have the same structure, then linking r_1 and r_2 to a new root r gives a new PBT with root r. **[constructor]**

③ No other tree is a PBT. **[minimality]**

What do we know about PBTs (recall RBT stands for a rooted binary tree)?

$\boxed{\text{A}}$ The number of vertices of any PBT is some number n such that $n = 2^k - 1, k \in \mathbb{N}$.

$\boxed{\text{B}}$ The sets of all PBTs and all RBTs are the same.

$\boxed{\text{C}}$ All RBTs are PBTs.

$\boxed{\text{D}}$ All PBTs have an even number of vertices.

$\boxed{\text{E}}$ None of the above.

20. Suppose a rooted binary tree has 8 vertices in its left subtree (ignoring the root). How many vertices are in the right subtree (ignoring the root) if the tree has 17 links in total?

$\boxed{\text{A}}$ 9 $\boxed{\text{B}}$ 10 $\boxed{\text{C}}$ 11 $\boxed{\text{D}}$ 12 $\boxed{\text{E}}$ It cannot be determined.

Scratch