MIDTERM: <u>90 Minutes</u>

Last Name: ____________________

First Name: ____________________

RIN: ____________________

Section: ____________________

Answer **ALL** questions. You may use one double sided $8\frac{1}{2} \times 11$ crib sheet.
NO COLLABORATION or electronic devices. Any violations result in an F.
NO questions allowed during the test. Interpret and do the best you can.
You **MUST** show **CORRECT** work, <u>**even on multiple choice questions**</u>, to get credit.

GOOD LUCK!

1	2	3	4	5	6	Total
150	20	20	20	20	20	250

1 Circle one answer per question. 10 points for each correct answer.

(1) Compute $S = \sum_{i=1}^{2} \sum_{j=1}^{4} (i + j)$.

$\boxed{\text{A}}$ $S = 28$.

$\boxed{\text{B}}$ $S = 30$.

$\boxed{\text{C}}$ $S = 32$.

$\boxed{\text{D}}$ $S = 34$.

$\boxed{\text{E}}$ None of the above.

(2) Estimate. Which approximation is closest to $S = \sum_{i=1}^{2^{50}} i$.

$\boxed{\text{A}}$ $S \approx 1275$.

$\boxed{\text{B}}$ $S \approx 10^{10}$.

$\boxed{\text{C}}$ $S \approx 10^{20}$.

$\boxed{\text{D}}$ $S \approx 10^{30}$.

$\boxed{\text{E}}$ $S \approx 10^{40}$.

(3) What is the correct asymptotic behavior (order analysis) for the function $S(n) = n\sqrt{n}$.

$\boxed{\text{A}}$ $S(n) \in \Theta(n)$.

$\boxed{\text{B}}$ $S(n) \in \Theta(n^2)$.

$\boxed{\text{C}}$ $S(n) \in \Theta(n^3)$.

$\boxed{\text{D}}$ $S(n) \in \Theta(n^4)$.

$\boxed{\text{E}}$ None of the above.

(4) What is the correct asymptotic behavior (order analysis) for the sum $S(n) = \sum_{i=0}^{n^2} 2^i$.

$\boxed{\text{A}}$ $S(n) \in \Theta(2^n)$.

$\boxed{\text{B}}$ $S(n) \in \Theta((2^n)^2)$.

$\boxed{\text{C}}$ $S(n) \in \Theta(2^{n^2})$.

$\boxed{\text{D}}$ $S(n) \in \Theta(2^{2n^2})$.

$\boxed{\text{E}}$ None of the above.

(5) Compute $\gcd(1045, 2310)$. That is, compute the greatest common divisor of 1045 and 2310.

$\boxed{\text{A}}$ 5.

$\boxed{\text{B}}$ 10.

$\boxed{\text{C}}$ 55.

$\boxed{\text{D}}$ 95.

$\boxed{\text{E}}$ None of the above.

(6) Which degree sequence could be the degree sequence of a friendship network (simple graph)?

A $[5, 3, 3, 2, 1]$.

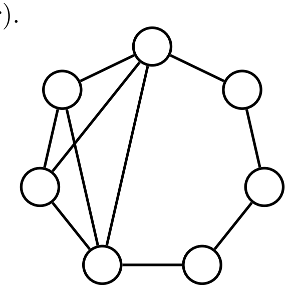
B $[3, 3, 2, 1]$.

C $[3, 3, 3, 3, 3]$.

D $[4, 4, 3, 2, 1]$.

E None of the above.

(7) You wish to color the graph on the right so that linked vertices do not get the same color. What is the minimum number of colors needed (the chromatic number).



A 2

B 3

C 4

D 5

E 6

(8) Boys X, Y, Z and girls A, B, C have the preference lists shown. Which of these is a stable matching?

A A–X, B–Y, C–Z.

B A–X, B–Z, C–Y.

C A–Y, B–X, C–Z.

D A–Z, B–Y, C–X.

E None of the above or there is no stable matching.

	X	**Y**	**Z**
1.	A	A	A
2.	B	B	B
3.	C	C	C

	A	**B**	**C**
1.	X	X	X
2.	Y	Y	Y
3.	Z	Z	Z

(9) What is the minimum number of children that guarantees at least two have the same first and last initial?

A 26

B $26 + 1$

C 26^2

D $26^2 + 1$

E None of the above.

(10) A race has 6 runners. In how many ways can the gold, silver and bronze medal be given?

A 6^3

B $\binom{6}{3}$

C $6 \times 5 \times 4$

D $6!$

E None of the above.

(11) A shirt matches 2 pants. My blue tie matches 3 shirts. My red tie matches 4 shirts. How many matching outfits can I wear? (In a matching outfit, the shirt must match the tie and pants.)

$\boxed{\text{A}}$ 9.

$\boxed{\text{B}}$ 14.

$\boxed{\text{C}}$ 18.

$\boxed{\text{D}}$ 24.

$\boxed{\text{E}}$ None of the above.

(12) A shelf has some books. Alice picks 3 to read and Bob picks 4 to read. Which must be true?

$\boxed{\text{A}}$ Alice has more ways to pick her books than Bob has for picking his books.

$\boxed{\text{B}}$ Bob has more ways to pick his books than Alice has for picking her books.

$\boxed{\text{C}}$ Alice and Bob have the same number of ways for picking their books.

$\boxed{\text{D}}$ Alice and Bob cannot have the same number of ways for picking their books.

$\boxed{\text{E}}$ None of the above.

(13) How many subsets of $\{1, 2, 3, 4, 5\}$ contain 1 and 2 or contain 3 and 4?

$\boxed{\text{A}}$ 10.

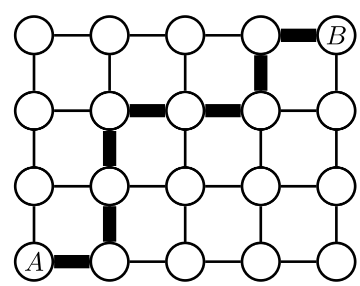
$\boxed{\text{B}}$ 12.

$\boxed{\text{C}}$ 14.

$\boxed{\text{D}}$ 16.

$\boxed{\text{E}}$ None of the above.

(14) How many different shortest paths from A to B are there?



$\boxed{\text{A}}$ 28

$\boxed{\text{B}}$ 32

$\boxed{\text{C}}$ 34

$\boxed{\text{D}}$ 36

$\boxed{\text{E}}$ None of the above.

(15) You write all 1,000 numbers in $\{0, 1, \ldots, 999\}$. How many times did you write the digit 1?

$\boxed{\text{A}}$ 275.

$\boxed{\text{B}}$ 300.

$\boxed{\text{C}}$ 325.

$\boxed{\text{D}}$ 350.

$\boxed{\text{E}}$ None of the above.

2 Prove or disprove: the integer x is odd if and only if $x^2 - 1$ is divisible by 8.

3 For $n \in \mathbb{N}$, prove that $\textsc{remainder}(n, 9) = \textsc{remainder}(\text{sum of } n\text{'s digits}, 9)$.

E.g. 725 has remainder 5 modulo 9. The sum of the digits is 14, which also has remainder 5 modulo 9.

4 Prove by induction for all $n \geq 1$:

$$\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \cdots + \frac{(n-1)}{n!} + \frac{n}{(n+1)!} = 1 - \frac{1}{(n+1)!}.$$

5 A_n **satisfies the recurrence below. Find a formula for** A_n **and prove your answer.**

$$A_0 = 1, A_1 = 2 \textbf{ and } A_n = A_{n-1} + 2A_{n-2} \textbf{ for } n \geq 2$$

6 How many subsets of $\{1, 2, 3, 4, 5, 6\}$ contain some consecutive numbers?.

For example the subset $\{1, 2, 4, 5\}$ has consecutive numbers but the subset $\{1, 3\}$ does not.

SCRATCH

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