Midterm

110 Minutes

First Name: _____

Last Name: _____

RIN: _____

NO COLLABORATION or electronic devices.
Any violations will result in an F.
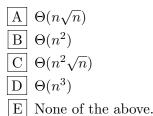**No questions** allowed during the test unless you think there is a mistake.

GOOD LUCK!

10 points per correct multiple-choice answer. Circle exactly one answer.
20 points per correct answer to Problems 2-6.
You **MUST** show **CORRECT** work to get credit.
Correct answers with no explanation will get a 0.

1	**2**	**3**	**4**	**5**	**6**	**Total**
150	**20**	**20**	**20**	**20**	**20**	**250**

1. What is the asymptotic behavior of the sum $S(n) = \sum_{i=1}^{n^2} i^2$?

   A $\Theta(n^3)$
   B $O(n^4)$
   C $\Theta(n^5)$
   D $O(n^6)$
   E None of the above.

2. What is the asymptotic behavior of the function $S(n) = n^{1.5}$?

   A $\Theta(n\sqrt{n})$
   B $\Theta(n^2)$
   C $\Theta(n^2\sqrt{n})$
   D $\Theta(n^3)$
   E None of the above.

3. What is the value of the sum $S = \sum_{i=1}^{10}\sum_{j=1}^{5} i$?

   A 200
   B 225
   C 250
   D 275
   E 300

4. What is the correct asymptotic behavior of the sum $S(n) = \sum_{i=1}^{n} i^{10}$?

   A $\Theta(n^{10})$
   B $\Theta(n^{11})$
   C $\Theta(n^{12})$
   D $\Theta(n^{13})$
   E None of the above.

5. We know that $gcd(290, 310) = 290x + 310y$. Which of the following values are possible for x and y?

   A $x = 4, y = -10$
   B $x = 8, y = -15$
   C $x = 10, y = -20$
   D $x = 14, y = -15$
   E None of the above.

6. What is the remainder when 10^{100} is divided by 11?

   A -2

   B -1

   C 0

   D 1

   E 2

7. Consider a graph G with degree sequence $[2, 2, 2, 2, 2]$. How many edges does G have?

   A 4

   B 6

   C 8

   D 10

   E None of the above.

8. Consider a graph G with degree sequence $[3, 2, 2, 2, 2]$. How many edges does G have?

   A 8

   B 9

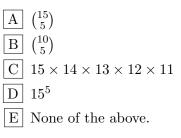   C 10

   D 11

   E None of the above.

9. There are 10 types of people in the world: those who know binary and those who don't. What is 2^{11}?

   A 128

   B 256

   C 512

   D 1024

   E None of the above.

10. Suppose FOCS has 6 sections (with 33 students per section). Each student shakes hands only with students who are in different sections. What do we know?

   A The number of students who shake hands with an odd number of students is even.

   B The number of students who shake hands with an odd number of students is odd.

   C There are 198×165 handshakes in total.

   D The number of handshakes cannot be determined.

   E None of the above.

11. How many subsets of $\{1, 2, 3, 4, 5, 6\}$ contain at least one even number?

A 2^3

B 2^4

C $2^6 - 2^3$

D $2^5 - 8$

E None of the above.

12. Consider all 7-bit binary strings with a 1 in the first position and a 0 in the last position? How many such strings are there?

A 16

B 32

C 64

D 128

E None of the above.

13. Suppose you guess randomly on all 15 multiple-choice questions and you answer 5 correctly. What is the number of all 5-question sets (e.g., a 5-question set is $\{q_1, q_4, q_7, q_{11}, q_{12}\}$)?

A $\binom{15}{5}$

B $\binom{10}{5}$

C $15 \times 14 \times 13 \times 12 \times 11$

D 15^5

E None of the above.

14. In how many ways can you misspell WINTER, assuming you use the same letters?

A $6!$

B 2^6

C $2^6 - 1$

D $6! - 1$

E None of the above.

15. What is the last digit of 11^5?

A 0

B 1

C 2

D 3

E None of the above.

Problem 2. Prove that $\sqrt{12}$ is irrational.

Problem 3. What is the last digit of 102^{1211}? [**Hint:** $11^2 = 121$]

Problem 4. Prove using induction that $\dfrac{1}{1 \cdot 2} + \dfrac{1}{2 \cdot 3} + \cdots + \dfrac{1}{n \cdot (n+1)} = \dfrac{n}{n+1}, \forall n \geq 1.$

Problem 5. Consider the recurrence $A_0 = 0, A_1 = 1, A_n = 2A_{n-1} - A_{n-2}$. **Guess a formula for A_n and prove it using induction. Tinker, tinker, tinker.**

Problem 6. For a graph G, the complement $\overline{G}$ has the same vertices, but the edges in $\overline{G}$ are the complement of the edges in G: distinct vertices u and v are adjacent in $\overline{G}$ if and only if they are not adjacent in G.

Prove that if G is regular, then $\overline{G}$ is regular. [In a regular graph, all vertices have the same degree.]

Scratch

Scratch