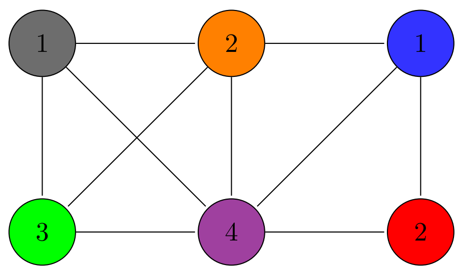
Problem I and Problem VI - by Eric Scheer

(I) Six cell towers have overlapping coverage areas, as shown in the diagram at right.
Towers whose areas overlap must use different frequency bands to communicate with phones in their area.

What is the minimum number of frequency bands needed for this set of cell towers?
Explain how you arrived at your answer, and how you know it is the minimum possible.

We can express this problem as a graph where the nodes of the graph are the cell towers and the edges represent overlapping coverage areas.



Notice that the subgraph consisting of the black, green, orange, and purple cell towers is complete meaning that all four of these cell towers require different frequency bands, meaning that four is a lower bound on the number of frequency bands we need.

Since there exists a solution with four frequency bands (shown above), we know that four is the minimum number of frequency bands needed.

(VI) List as a set all of the possible 4-bit binary sequences in which 00 does not occur.
How many are there?

To find all possible 4-bit binary sequences that do not contain 00, we can find all possible enumerate through different cases. Notice that if our string has less than two zeros, then we are guaranteed to not have 00 as a substring. Therefore the following five sequences fit our pattern: (1111, 1110, 1101, 1011, 0111).

We can also notice that if a string has three or more zeros, we are guaranteed to have two zeros next to each other. Therefore, we must consider strings with two zeros. The following three sequences fit this pattern: (1010, 0101, 0110).

Therefore, we have a total of eight 4-bit binary sequences that do not contain 00.

Problem II and Problem V - by Xingjian Zhao

II. Four people need to cross a rickety rope bridge late at night – the bridge can only hold two of them at one time. The four people move at different speeds: The fastest one can cross the bridge in 2 minutes, the next fastest in 4 minutes, then 8 minutes, then 10 minutes for the slowest one.

Also, they have only one headlamp among them, which is needed to cross the bridge (they need their hands to hold onto the ropes). This means that when two people cross together, the pair moves at the speed of the slower one.

a) The obvious sequence of crossings completes in 26 minutes. What is it?
b) Find the sequence of crossings that takes the least amount of time. (It is less than 26 minutes.)
c) Which person's crossing speed (fastest, second fastest, third fastest, or slowest) has the least impact on the minimum total time needed to get all four people across the bridge? Why?

Let their name be A, B, C, D, with A being the fastest and D being the slowest.

a) A would have the headlamp since they're the fastest, and take D over (10 mins), come back (2 mins), take C over (8 mins), come back (2 mins), and finally take B over (4 mins).
   This takes 10+2+8+2+4 = 26 mins
b) A and B cross first (4 mins), A comes back (2 mins), give the headlamp to C so that C and D can cross together (10 mins), then B brings the headlamp back (4 mins) and bring A over (4 mins).
   This takes 4+2+10+4+4 = 24 mins
c) As we can see from part (b), for the optimal crossing, the third fastest and the slowest go together, which is limited at the speed of the slowest, and the third fastest would not ever need to cross again, so their speed has no effect on the overall time of the optimal crossing.

V. Give the power set of this set: {c, d, {d,e}, e}

P({c, d, {d,e}, e}) ={

∅,
{c},{d},{{d,e}},{e},
{c,d},{c,{d,e}},{c,e},{d,{d,e}},{d,e},{{d,e},e},
{c,d,{d,e}},{c,d,e},{c,{d,e},e},{d,{d,e},e},
{c,d,{d,e},e}

}.

Problem III - by Mei Huang

III. Give formal definitions of the following sets using set builder notation.

A = {0, 1, 8, 27, 64, 125, . . . } B = {1, 2, 4, 7, 11, 16, 22, . . . }

A

These are cubes of natural numbers. We just have

$$A = \{n \mid n = k^3, \text{ where } k \in \mathbb{N}_0\}$$

B

Notice each term is 1 + sum of 1 to n. We start with 1. For index 3, we add 1 + 2 + 3. For index 4, we add 1 + 2 + 3 + 4. So we have

$$\text{Term} = 1 + (\text{sum of numbers from 1 to n})$$

The sum of first n numbers is $\frac{n(n+1)}{2}$ So each term is:

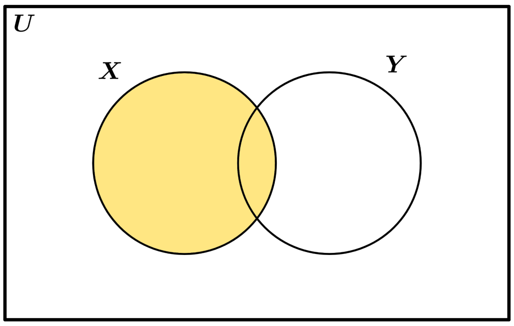
$$1 + \frac{n(n+1)}{2}$$
$$= \frac{2}{2} + \frac{n^2 + n}{2}$$
$$= \frac{n^2 + n + 2}{2}$$

$$B = \{n \mid n = \frac{n^2 + n + 2}{2}; n \in \mathbb{N}_0\}$$

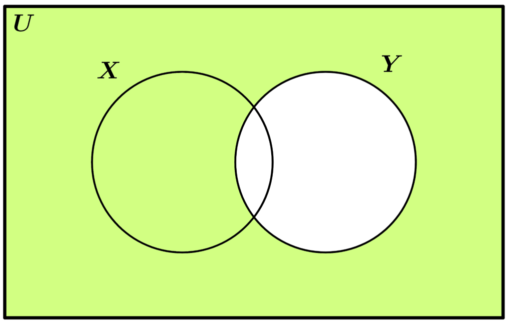
Problem IV - by Junseob Kim

a)

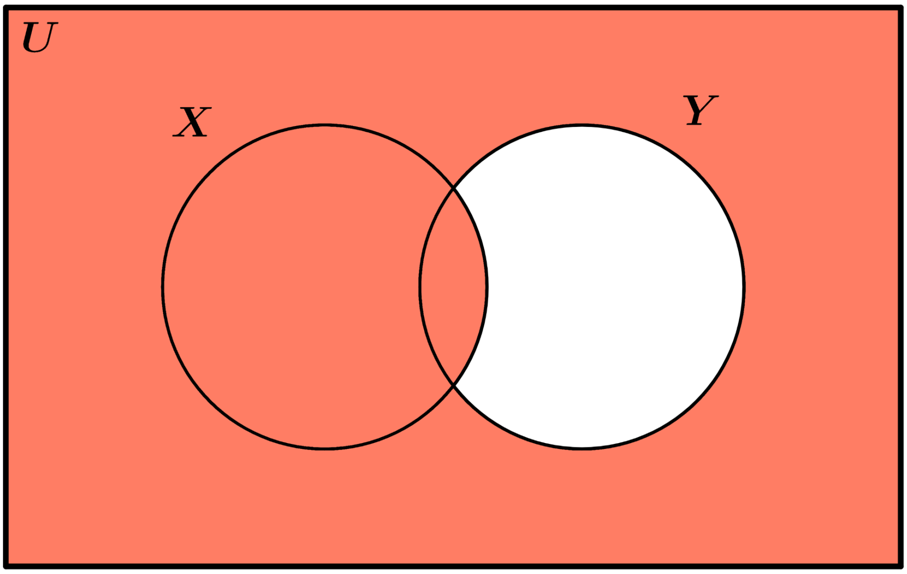
X =



Y' =

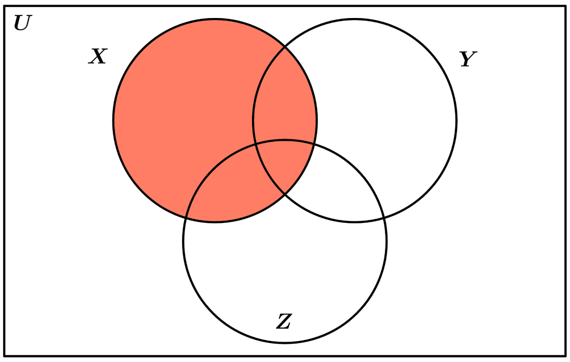


X ∪ Y' =

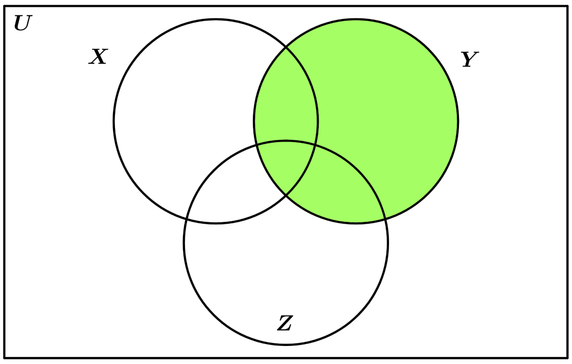


b)

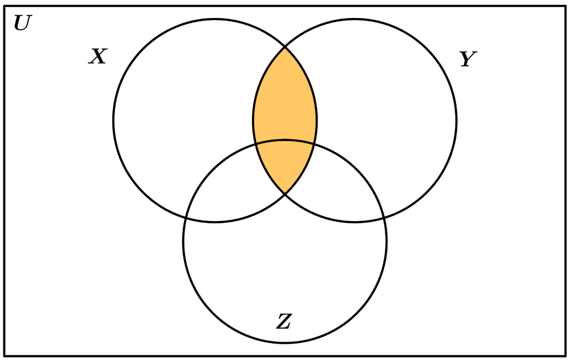
X =



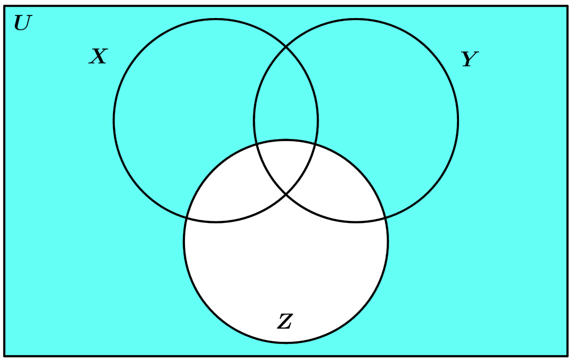
Y =



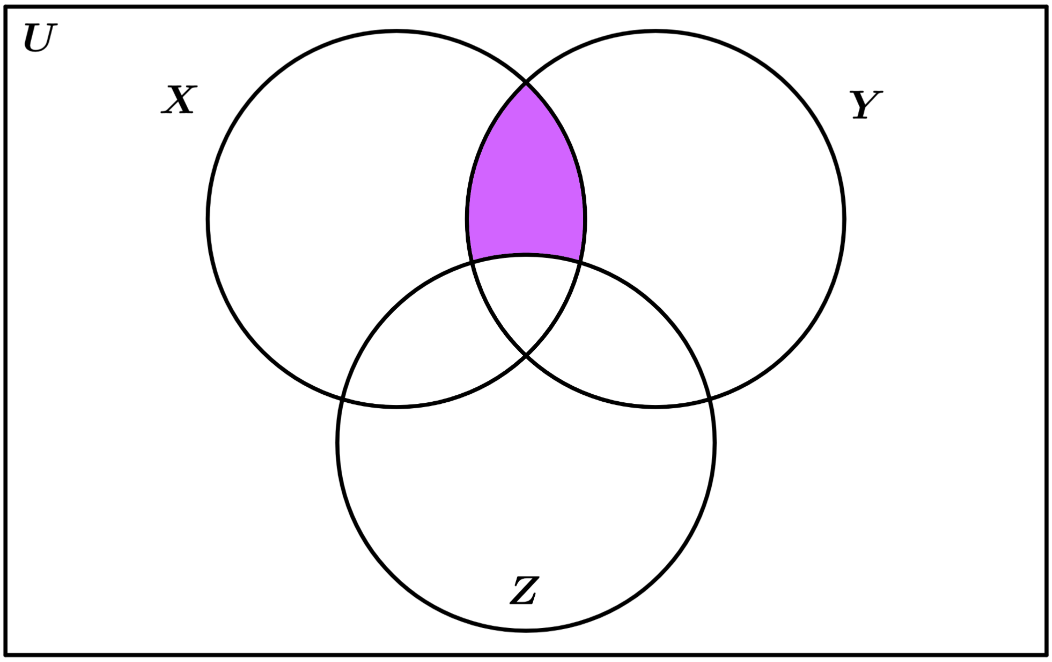
X ∩ Y =



Z' =

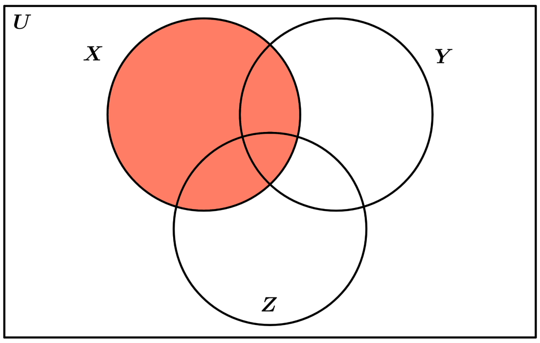


X ∩ Y ∩ Z' =

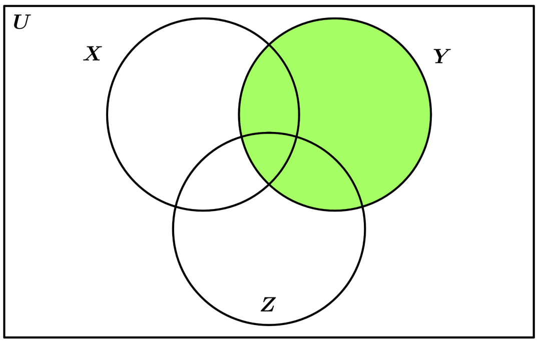


c)

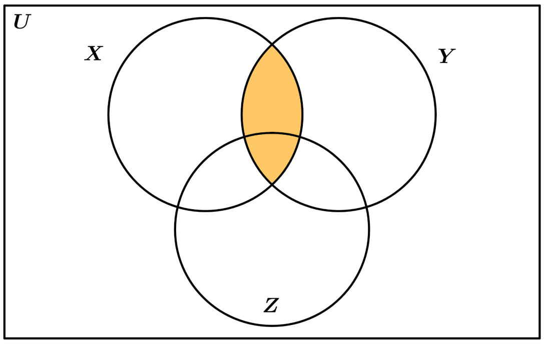
X =



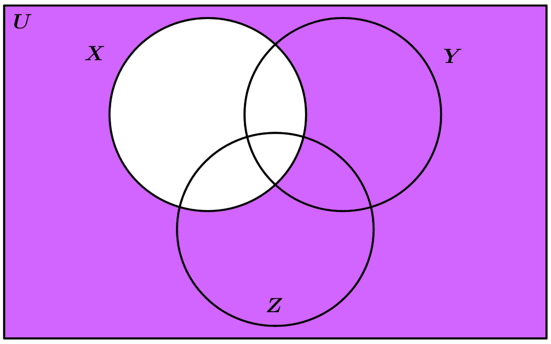
Y =



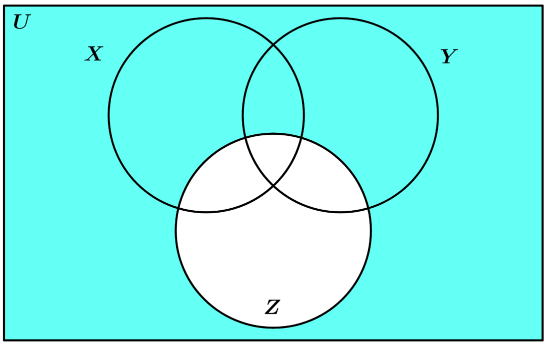
X ∩ Y =



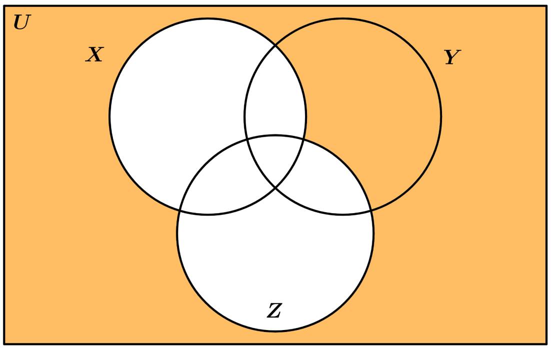
X' =



Z' =



X' ∩ Z' =



(X ∩ Y) ∪ (X' ∩ Z') =