

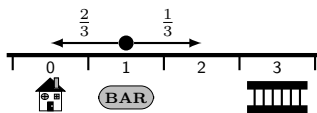
Instructions: Standard course policies about typesetting, file size, and submission apply. You must show your work to receive credit. Your work must be your own, though you are permitted to get assistance from classmates or instructional staff. Your responses to the submission problems must be uploaded to Submitty by **8:59pm on Thursday, March 13**.

Attribution: these problems were chosen from the DMC text.

Note: you can employ Monte Carlo simulations to check some of your answers. But of course your arguments must be sound and not reference any simulations.

Recitation Problems

- I. Eight pawns are placed randomly on different squares of a chessboard.
 - (a) Compute the probability they are in a straight line (including diagonals).
 - (b) Compute the probability no two are in the same row or column.
- II. A bag contains 10 identical envelopes each with some amount of money; the amounts are all different. You and a friend randomly draw an envelope. Do you want to go first or second, or does it not matter?
- III. An urn has m blue balls and n red balls. You randomly pick the balls one by one and lay them in a line. What is the probability that the last ball is red?
- IV. A drunk leaves the bar at position 1, and takes random steps: left (L) with probability $\frac{2}{3}$ or right (R) with probability $\frac{1}{3}$. What is the probability the drunk reaches home (at position 0) before reaching the lockup (at position 3)?



- V. A parent picks a boy's name as Sue with probability $0 < \beta < 1$. Monica has two children with different names. What is the probability Monica has two boys if:
 - (a) Monica has a boy.
 - (b) Monica has a boy named Sue.
 - (c) Monica does not have a boy named Sue.
- VI. One out of n coins is 2-headed. A random coin is picked and flipped k times. All flips were H.
 - (a) What is the probability the coin flipped is 2-headed?
 - (b) For $n = 10^6$, how high should one pick k to be 99.9% sure the 2-headed coin was flipped?
- VII. Cards with distinct values v_1, \dots, v_m are dealt in random order. The k th card is largest among the cards already dealt. What is the probability it is the largest in the pack?

Submission Problems

- (1) In a bag are 12 balls, 2 balls in each of 6 colors. You randomly pick 5 balls without replacement. Compute $\mathbb{P}[i]$, the probability that you get i colors in your sample, for $i = 1, \dots, 6$.
- (2) Each pair from $\{\text{Dan, Lilian, Eric, Mei}\}$ randomly decides whether or not to be friends. If someone hears a rumor, they tell it to their friends. Mei got a juicy piece of gossip. What are the chances that Eric hears it?
- (3) Three monkeys A, B, C have a 6-shooter pistol loaded with 2 bullets. Starting with A , each spins the bullet-wheel and shoots their foot. Compute probabilities p_A, p_B, p_C for each monkey to be the first shot.
- (4) A random cut on a circular pizza picks two random points on the circumference and cuts along the chord joining the two points. You make two random cuts. What is the probability to get 4 pieces of pizza?
- (5) There are two beavers, brown and black. What are the chances both are male? What if you know:
 - (a) one is male
 - (b) one is male and one is born on Tuesday
 - (c) one is a male born on a Tuesday?
- (6) A class has 10 boys and 5 girls. Three children are selected one after another. Compute the probability that the first two are boys if both sexes are represented.
- (7) Dan, Lilian, Eric, and Mei each choose a random number in $\{1, 2, 3, 4, 5\}$. What are the chances that some pair chooses the same number? What if there are k people and n numbers?
- (8) A biased coin is tossed repeatedly, with probability p of getting heads on each toss. How likely is a run of m heads before a run of n tails?