Name(s): _____

CSCI 2200 Foundations of Computer Science   Spring 2025

Problem set 7 – More graphs; counting

Instructions: Standard course policies about typesetting, file size, and submission apply. You must show your work to receive credit. Your work must be your own, though you are permitted to get assistance from classmates or instructional staff. Your responses to the submission problems must be uploaded to Submitty by **8:59pm on Thursday, February 27**. *(Please note that is the day after Exam 2. It is highly recommended that you complete these problems before the exam.)*
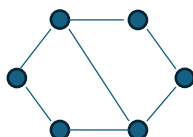
Practice Problems (Note: There will be no recitation associated with this homework because of the exam. Solutions to these problems are already posted so that you can check your work.)

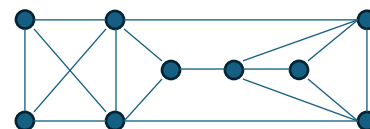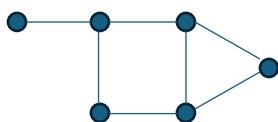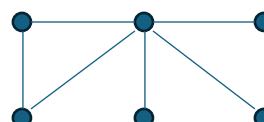I. Find the chromatic number for each of the following graphs:
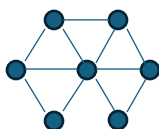
a)
b)
c)

II. A matching is *maximal* if another edge cannot be added to it. A matching is *maximum* (note the difference!) if a larger matching is impossible. For the graphs below, find a maximal matching of the size indicated. Then find a larger maximum matching.

(a) Maximal matching of size 2

(b) Maximal matching of size 1

III. A set of n kids would like to go to summer camp. There are m different summer camps. Each kid has a subset of the camps they are willing to go to (e.g. k_1 might only be interested in camps $\{C_1, C_4, C_5\}$). Additionally, each camp has a limit $(l_1, l_2, \dots, l_m)$ on the number of kids they can handle. Give necessary and sufficient conditions for ensuring that all of the kids can attend a camp they want to go to.

IV. A quiz has 6 true/false questions and 4 multiple choice questions with options A-E. How many different answer sheets are possible for this quiz?

V. Four students (Alex, Bailey, Chris, Dani) each get to pick a T-shirt. There are five different colors of T-shirt available. (a) How many ways can they make their selections if they are allowed to pick the same color as someone else? (b) What if they must each pick a different color?

VI. 300056400 has prime factorization $2^4 \cdot 3^7 \cdot 5^2 \cdot 7^3$. How many divisors does it have?

VII. The most common Tarot deck has 78 cards. (a) A simple type of Tarot reading involves drawing three cards: one for "past", one for "present", and one for "future". How many different drawings are possible? (b) Tarot cards are also used as playing cards. One such game ("French Tarot") involves dealing six cards to a pool called *le chien* ("the dog"). How many sets of six cards are possible?

VIII. To test if a graph with 50 vertices is 3-colorable, you decide to brute-force it and check all possible assignments of {red,green,blue} to vertices to see if any connected vertices share a color. Your computer can check 1,000,000 of these colorings every second. How long will it take to complete this process?

Submission problems

1. Find the chromatic number for C_3, C_4, C_5, C_6. Come up with a rule for C_n, and give a proof of your rule.

2. Prove that every tree has at most one perfect matching.

3. For what values of m, n does $K_{m,n}$ have (a) a Hamiltonian cycle? (b) an Euler cycle? Explain why.

4. A wedding venue offers a reception package where the clients can choose 3 hors d'oeuvres out of 7 offerings, one of three main courses, two side dishes out of six possibilities, and one dessert from five options. How many different menus can clients select?

5. How many strings of 5 lowercase English letters: (a) are there in total? (b) have no repeated letters? (c) begin with "abc"? (d) begin with "abc" and end with "cde"? (e) begin with "abc" and end with "xyz"?

6. A pizzeria offers 11 different toppings. (a) How many different 3-topping pizzas can be ordered if all three toppings must be different? (b) What if you can repeat toppings? (i.e. "triple pepperoni" or "sausage and double mushroom" are valid orders)

7. From a reading list of 50 books, a student is required to read 10 of them. (a) How many sets can the student select? (b) How many sequences of 10 books (with no repeats!) can the student select?

8. How many strings of 10 bits: (a) have more 1s than 0s? (*Hint:* symmetry) (b) have eight or more consecutive zeros?