Name(s): ________________________________

CSCI 2200 Foundations of Computer Science Spring 2025

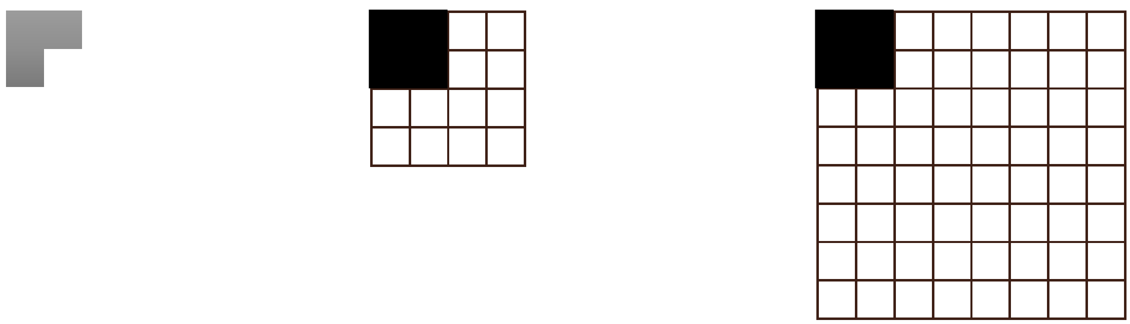
Problem set 4 – Variations on induction; recursion

Instructions: Standard course policies about typesetting, file size, and submission apply. You must show your work to receive credit. Your work must be your own, though you are permitted to get assistance from classmates or instructional staff. Your responses to the submission problems must be uploaded to Submitty by **8:59pm on Thursday, February 6**.

Recitation Problems

I. Prove the following using basic induction:

> $\forall n \geq 2\ (n \in \mathbb{N})$, a 2^n x 2^n square grid with a 2x2 corner removed can be precisely tiled using L-shaped tiles of size 3. ("Precisely tiled" means covering the full area with no tiles overlapping or hanging off the edge of the board. Tiles may be rotated or flipped. A sample L-tile, board with n=2, and board with n=3 are show.)



II. Prove using leaping induction: $\forall n \in \mathbb{N}_0, n^2 + n + 1$ is not divisible by 5.

III. Claim: In any directed graph in which every pair of vertices has a single *one-way* link between them, there is *some* route that touches all vertices. Prove this claim using strong induction.

IV. Unroll the following recursions to find a closed-form formula for T_n.

(a) $T_0 = 2; T_n = T_{n-1} + 3n$ for $n \geq 1$ (b) $T_0 = 3; T_n = 2T_{n-1} + n$ for $n \geq 1$

V. Give a recursive definition of the following set: {1,2,3,4,6,7,8,9,11,12,...}

VI. What is contained in the following recursively defined set S?

$3 \in S; 4 \in S; x, y \in S \rightarrow x + y \in S$

Submission problems

1. Prove that $n^7 < 2^n$ for $n \geq 37$. (a) Use weak induction. (b) Use leaping induction with a step size of 3.

2. Prove via induction that the sum of the first n positive odd integers is a perfect square. (*Hint:* Upgrade the claim.)

3. Prove via strong induction that $n \leq 3^{n/3}$ for all $n \geq 0$.

4. Consider a $5^n \times 5^n$ square grid with the top-left corner removed.
(a) Prove that the number of spaces remaining in the grid is divisible by 3.
(b) Prove that $\forall n \geq 1$, the grid can be L-tiled. *(See Recitation Problem I for definition of L-tiles.)*

5. Describe, in English, the contents of this recursively-defined set. (Assume minimality – nothing else is in the set except those elements permitted by these rules.)

$$3 \in W; x, y \in W \Rightarrow (x + y \in W) \wedge (x - y \in W)$$

6. Give a recursive definition for the set of all binary strings containing equal numbers of 0s and 1s.