Name(s): ___________________________________

CSCI 2200 Foundations of Computer Science Spring 2025

Problem set 3 – More proofs; induction

Instructions: Standard course policies about typesetting, file size, and submission apply. You must show your work to receive credit. Your work must be your own, though you are permitted to get assistance from classmates or instructional staff. Your responses to the submission problems must be uploaded to Submitty by **8:59pm on Thursday, January 30**. *(Please note that is the day after Exam 1. It is highly recommended that you complete these problems before the exam.)*

Practice Problems *(Note: There will be no recitation associated with this homework because of the exam. Solutions to these problems are already posted so that you can check your work.)*

I. Prove using a direct proof: $\forall n \in \mathbb{N}$, if n is not divisible by 3, then $n^2 \div 3$ has a remainder of 1. *(Hint: Two cases.)*

II. Prove using contraposition: If x and y are positive integers such that $xy < 10000$, then $x < 100 \wedge y < 100$.

III. Prove using an indirect proof: There are no positive integers a and b such that $a^2 - b^2 = 6$.

IV. Disprove the following statements:
(a) If $\sqrt{r}$ is irrational, then r is irrational.
(b) $\forall n \in \mathbb{N}, 3^n + 2$ is prime.

Prove the following by induction:

V. $\forall n \in \mathbb{N}, 9^n + 3$ is divisible by 12.
VI. $\forall n \in \mathbb{N}, 1 + 4 + 7 + \cdots + (3n - 2) = \frac{n(3n-1)}{2}$
VII. $\forall n \in \mathbb{N}, 2^{n+2} + 3^{2n+1}$ is divisible by 7.
VIII. $\forall n \in \mathbb{N}, \sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}$

Submission problems

Prove via induction:

1. $\forall n \in \mathbb{N}, \sum_{i=1}^{n} i(i+1) = n(n+1)(n+2) / 3$
2. $\forall n \in \mathbb{N}, n^2 \leq 2^{n+1}$
3. $\forall n \in \mathbb{N}, n$ can be written as a sum of distinct powers of 2. (e.g. $43 = 2^5 + 2^3 + 2^0$)

Prove using any desired method:

4. $\forall m, n, d \in \mathbb{N}$, if m is divisible by d and $(m + n)$ is divisible by d, then n is divisible by d.
5. Every odd positive integer is the difference of two perfect squares.