Name(s): _____

CSCI 2200 Foundations of Computer Science Spring 2025

Problem set 2 – Logic & proof

Instructions: Standard course policies about typesetting, file size, and submission apply. You must show your work to receive credit. Your work must be your own, though you are permitted to get assistance from classmates or instructional staff. Your responses to the submission problems must be uploaded to Submitty by **8:59pm on Thursday, January 23**.

Recitation Problems

I. The symbol ≡ is used to indicate that two statements in formal logic are *logically equivalent* – that is, that they always have the same truth value regardless of the propositions involved.

Use truth tables to verify the following:
(a) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$
(b) $\neg((p \wedge q) \vee r) \equiv (\neg p \wedge \neg r) \vee (\neg q \wedge \neg r)$

II. You are given the following predicates defined on the domain of "everyone on the RPI campus": $S(x)$ = "x is a student"; $W(x)$ = "x is wise"; $F(x, y)$ = "x is a friend of y". Use these predicates to formalize the following English sentences in predicate logic.
(a) Puckman is a student. *# Note: Just like Python, you may use a literal as function input.*
(b) No students are wise.
(c) All wise students are friends with Puckman.
(d) There is exactly one student who is a friend of Dan's. *# Note: $x \neq y$ is a proposition you may find useful here.*

III. Given the information, answer the question *yes*, *no*, or *uncertain*.
(a) If you ace the midterm and the final, you get an A. You aced the final. Did you get an A?
(b) If you ace the midterm or the final, you get an A. You aced the final. Did you get an A?
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IV. Give direct proofs of the following statements.

(a) $(x \in \mathbb{Q} \wedge y \in \mathbb{Q}) \Rightarrow xy \in \mathbb{Q}$ (b) $n \in \mathbb{N} \Rightarrow n^2 + n$ is even

V. Prove the following by contraposition:

(a) If x is irrational, then $\sqrt{x}$ is irrational.
(b) $\forall m, n, d \in \mathbb{N}$, if mn is not divisible by d, then neither m nor n is divisible by d.

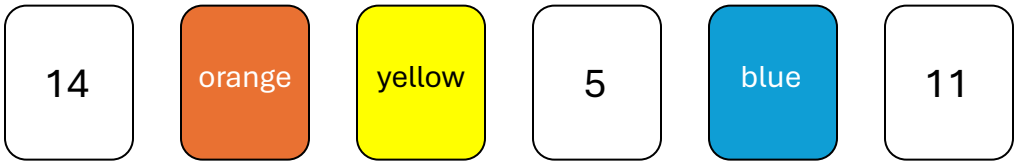
VI. Prove the following by contradiction:

(a) There is no smallest positive rational number.
(b) $\log_2 9$ is irrational.

Submission problems

1. A famous psychology experiment (the Wason selection task – look it up **after** you submit your assignment!) shows participants a set of cards that have integers on one side and colors on the other.

You participate in this experiment and are dealt six cards, showing the following faces:



You are told: "If a card is yellow on one side, then it has an odd number on the other side." Which cards do you need to turn over to verify that the rule is true? Explain your answer.

2. Using the same predicates as in problem II above, write the following in predicate logic:

(a) Marty is a friend of every student.
(b) Every student is a friend of some other student.
(c) All wise students have a friend.
(d) No student is a friend of every other student.

3. Under which of the domains $\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}$ are the following statements true? (If there are two variables, they must use the same domain.) Be sure to justify your answer.

(a) $\exists x : x^2 = 4$
(b) $\exists x : x^2 = 2$
(c) $\forall x : (\exists y : x^2 = y)$
(d) $\forall y : (\exists x : x^2 = y)$

4. Prove each of the following statements using the indicated technique.

(a) **Direct:** For all integers a, b, c, if a is divisible by b and b is divisible by c, then a is divisible by c.

(b) **Contraposition:** If $n \in \mathbb{N}$, then $\frac{n}{n+1} \notin \mathbb{N}$. *(Hint: You may use the fact that there are no integers between 0 and 1 as an axiom.)*

(c) **Contradiction:** $\sqrt[3]{2}$ is irrational.

5. Prove each of the following statements using any desired technique. (Be sure to state which type of proof you are doing at the start of the proof.)

(a) The product of any two odd integers is also odd.

(b) There is no fixed constant c such that, $\forall n \in \mathbb{N}, n^3 \leq cn^2$.

(c) $\forall x \in \mathbb{R}, x > 0 \Rightarrow x^2 + x^{-2} \geq 2$