Homework 13

Instructions: Standard course policies about typesetting, file size, and submission apply. You must show your work to receive credit. Your work must be your own, though you are permitted to get assistance from classmates or instructional staff. Your responses to the submission problems must be uploaded to Submitty by **8:59pm on Thursday, April 17**.

Attribution: these problems were chosen from the DMC text.

Recitation Problems

I. Prove that the CFG $S \to \# \mid 0S0 \mid 1S1$ generates all strings of the form $w\#w^R$.

II. For each language, (i) give a CFG that generates the strings in the language, (ii) prove by induction that your CFG generates only strings in that language, and (iii) prove by induction that every string in the language can be generated by your CFG:

(a) $\mathcal{L} = \{\text{strings with an odd number of 1s}\}$
(b) $\mathcal{L} = \{\text{strings with more 1s than 0s}\}$.

III. Find CFGs for these languages:

(a) Strings with more 0s than 1s.
(b) Strings *not* of the form $0^{\bullet n}1^{\bullet n}$.

IV. Give a CFG to generate reversals of the strings in the CFG $S \to 00S1 \mid 1S0 \mid \varepsilon$.

V. In each case: (i) give pseudocode of a Turing Machine for the problem. (ii) Give machine-code for each module in your pseudocode. (iii) Combine your modules to get machine code of a Turing Machine for the problem.

(a) $\mathcal{L} = \{\text{strings with as many 0s as 1s}\}$
(b) $\mathcal{L} = \{\text{strings with twice as many 0s as 1s}\}$

VI. Give high-level pseudocode for a Turing Machine for the problem of adding two: $\mathcal{L} = \{0^{\bullet n}1^{\bullet n+2} \mid n \geq 0\}$.

Submission Problems

(1) For CFG $S \to 0S \mid S1 \mid 0 \mid 1$, prove no string has 10 as a substring.

(2) For each language, (i) give a CFG that generates the strings in the language, (ii) prove by induction that your CFG generates only strings in that language, and (iii) prove by induction that every string in the language can be generated by your CFG:

(a) $\mathcal{L} = \{\text{strings with equal numbers of 0s and 1s}\}$
(b) $\mathcal{L} = \{\text{strings with more 1s than 0s in } \textit{every} \text{ prefix}\}$.

(3) Find CFGs for these languages:

(a) $\{0^{\bullet n} \mid n \text{ is not a multiple of 3}\}$
(b) Strings not of the form ww (non-equality).

(4) Consider the problem $\mathcal{L} = \{0^{\bullet n}1^{\bullet n+m}0^{\bullet m} \mid n, m \geq 0\}$.

(a) Show how a deterministic pushdown automaton (stack memory) can solve $\mathcal{L}$.
(b) Find a CFG for $\mathcal{L}$.

(5) In each case: (i) give pseudocode of a Turing Machine for the problem. (ii) Give machine-code for each module in your pseudocode, in the form of a diagram. (iii) Combine your modules to get machine code of a Turing Machine for the problem, in the form of a diagram.

(a) $\mathcal{L} = \{\text{palindromes } w = w^R\}$
(b) $\mathcal{L} = \{0^{\bullet 2n} \mid n \geq 0\}$ (parity-check)

(6) Give high-level pseudocode for a Turing Machine for the problem of squaring: $\mathcal{L} = \{0^{\bullet n}\#1^{\bullet n^2} \mid n \geq 0\}$.