Homework 12

Instructions: Standard course policies about typesetting, file size, and submission apply. You must show your work to receive credit. Your work must be your own, though you are permitted to get assistance from classmates or instructional staff. Your responses to the submission problems must be uploaded to Submitty by **8:59pm on Thursday, April 10**.

Attribution: these problems were chosen from the DMC text.

Recitation Problems

I. Find regular expressions for $\mathcal{L}$ and its reversal $\mathcal{L}^R := \{\omega^R \mid \omega \in \mathcal{L}\}$, when $\mathcal{L}$ consists of strings: (a) which contains 100 as a substring at least once, (b) in which every 0 is followed by exactly two 1s.

II. Arvind decides he is going to list out all the languages $\{\mathcal{L}_1, \mathcal{L}_2, \ldots\}$. Will he succeed? That is, will every language eventually appear on his list?

III. Determine which strings can be generated by the corresponding regular expression. Justify your answers.

(a) Regular expression: $\overline{\{1\}^*} \bullet \{0\}^*$. Strings: $\varepsilon, 000; 11; 000111$.
(b) Regular expression: $\{0, 01\}^* \cap \{1, 10\}^*$. Strings: $101110; 00111; 00100; 01100$.

IV. Prove or disprove: there is a language $\mathcal{L}$ for which $\overline{\mathcal{L}^*} = \left(\overline{\mathcal{L}}\right)^*$.

V. Give DFAs for the languages $\mathcal{L}_1 = \{1^{\bullet 5n} \mid n \geq 0\}$ and $\mathcal{L}_2 = \overline{\{10, 100\}}$.

VI. Give DFAs for these languages:

(a) Strings with an odd number of 0s.

(b) Strings that end in 1.

(c) Strings with an odd number of 0s that end in 1. Use product states and the two DFAs from above.

VII. Give a DFA for the language of all strings of length at most ℓ, using a minimal number of states. Explain why the number of states you used is minimal.

VIII. Suppose $\mathcal{L}_1$ and $\mathcal{L}_2$ are both not regular. Prove or disprove that these languages can't be regular:

(a) $\mathcal{L}_1 \cap \mathcal{L}_2$ (b) $\mathcal{L}_1 \cup \mathcal{L}_2$ (c) $\mathcal{L}_1 \bullet \mathcal{L}_2$ (d) $\overline{\mathcal{L}_1}$.

Submission Problems

(1) Give a regular expression for each language when (i) you may and (ii) you may not use complements.

(a) Strings with at most one 1 and at most one 0
(b) Strings whose number of 0s is not divisible by 4.

(2) Find regular expressions for $\mathcal{L}$ and its reversal $\mathcal{L}^R := \{\omega^R \mid \omega \in \mathcal{L}\}$, when $\mathcal{L}$ consists of strings: (a) where the first and last bit are the same (b) in which every 0 is followed by at least one 1.

(3) It is unsatisfactory to define a language as $\{\varepsilon, 0, 11, 101, 1111, 11011, \ldots\}$, since we don't know how to continue the list. A precise definition must be finite and unambiguous. For example, $\{(01)^{\bullet n} \mid n \geq 0\}$ is a precise finite description of the language $\{\varepsilon, 01, 0101, 010101, \ldots\}$.

Assume any finite description can only use the 255 characters of the ASCII code, but it can be arbitrarily long. Prove that some languages cannot be precisely specified by a finite description.

(4) Determine which strings can be generated by the corresponding regular expression. Justify your answers.

(a) Regular expression: $\{0, 01\}^* \bullet \{1, 10\}^*$. Strings: $101110; 00111; 00100; 01100.$
(b) Regular expression: $\overline{\{0, 01\}^*} \bullet \{1, 10\}^*$. Strings: $101110; 00111; 00100; 01100.$

(5) Give DFAs for the following computing problems.

(a) Non-empty strings with an even number of 1s.
(b) Strings which begin with 10 and end with 01.
(c) Strings whose length is not divisible by 2 or 3.
(d) Strings whose even digits alternate between 0 and 1.

(6) Find a DFA for the language $\{0\}^* \bullet \overline{(\{10\} \cup \{11\})^*}$.

(7) Give DFAs for the following computing problems, or prove that the language is non-regular.

(a) Strings with the same number of 01 and 10 substrings.
(b) $\mathcal{L} = \{(01)^{\bullet n}(10)^{\bullet n} \mid n \geq 0\}$.

(8) Let $\mathcal{L} = \{0^{\bullet 2^n} \mid n \geq 0\}$. Is $\mathcal{L}$ regular? What about $\mathcal{L}^*$?