Name: _____

CSCI 2200 Foundations of Computer Science Spring 2025

Homework 1 – Warm-up problems

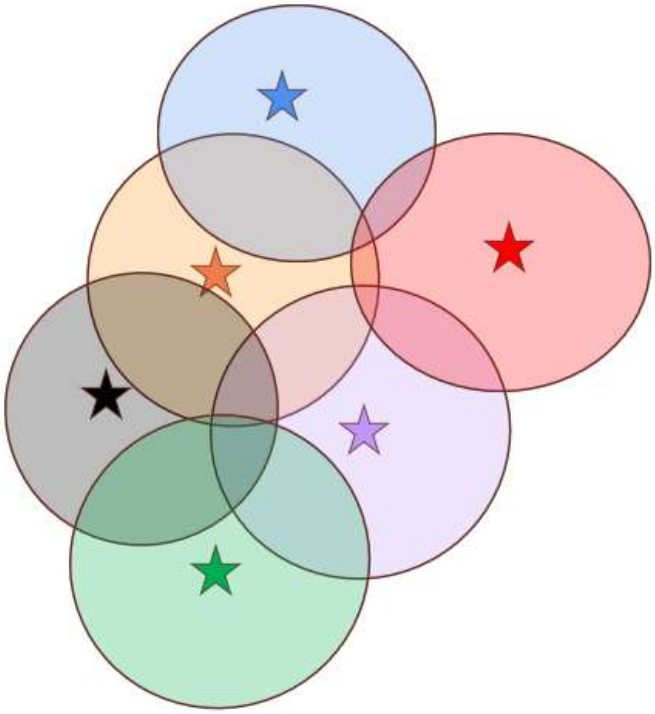
Instructions: Every assignment will have two groups of problems. The recitation problems are ungraded practice problems; solutions to some or all of these problems will be presented during recitation each week. The submission problems will be graded – you must show your work to receive credit. Other than diagrams, you **must** typeset your work. Each assignment **must** be a single PDF file smaller than 10MB in total and uploaded to Submitty **before** 9pm on the due date. Failure to follow these directions will result in zero credit for the assignment.

The HW1 submission problems are due by **8:59 pm on Thursday, January 16.**

Recitation Problems

I. Six cell towers have overlapping coverage areas, as shown in the diagram at right. Towers whose areas overlap must use different frequency bands to communicate with phones in their area.

What is the minimum number of frequency bands needed for this set of cell towers? Explain how you arrived at your answer, and how you know it is the minimum possible.



II. Four people need to cross a rickety rope bridge late at night – the bridge can only hold two of them at one time. The four people move at different speeds: The fastest one can cross the bridge in 2 minutes, the next fastest in 4 minutes, then 8 minutes, then 10 minutes for the slowest one.

Also, they have only one headlamp among them, which is needed to cross the bridge (they need their hands to hold onto the ropes). This means that when two people cross together, the pair moves at the speed of the slower one.

a) The obvious sequence of crossings completes in 26 minutes. What is it?
b) Find the sequence of crossings that takes the least amount of time. (It is less than 26 minutes.)
c) Which person's crossing speed (fastest, second fastest, third fastest, or slowest) has the least impact on the minimum total time needed to get all four people across the bridge? Why?

III. Give formal definitions of the following sets using set builder notation.

A = {0, 1, 8, 27, 64, 125, ...} B = {1, 2, 4, 7, 11, 16, 22, ...}

IV. Create Venn diagrams to illustrate the following combinations of sets. Be sure to include the universe of discourse:

a) $X \cup \bar{Y}$ b) $X \cap Y \cap \bar{Z}$ c) $(X \cap Y) \cup (\bar{X} \cap \bar{Z})$

V. Give the power set of this set: {c, d, {d,e}, e}

VI. List as a set all of the possible 4-bit binary sequences in which 00 does not occur. How many are there?

Submission Problems

[15] 1. There are 25 horses that run at various speeds. (You may find it helpful to label them with letters A through Y, in similar fashion to the lab groups problem from our first lecture.) For simplicity, you may assume that none of them run at precisely identical speeds. You have no timepiece or other way of measuring their absolute speed, but you do have a small racetrack on which you can race five of the horses at one time to determine their *relative* speeds. (In each race, the first-place horse is faster than the second-place horse, which is faster than the third-place horse, etc.)

What is the minimum number of races needed to determine the three fastest horses? (You don't care about the rest.) Explain how you know your answer is the minimum.

[15] 2. A parent is going to purchase gumballs for their young twins. To prevent arguments, they decide to buy enough gumballs to ensure that both kids get the same color of gumball. (Not necessarily the best parenting technique, but let me assure you that every parent of multiple kids has been there at some point.)

Unlike the machine shown at right, the one our young parent sees is mostly empty – it only has remaining 3 blue gumballs, 4 red ones, and 1 yellow one.
(a) How many gumballs must they buy to guarantee two of the same color? Why?
(b) What if they had four kids instead of two? Now how many purchases are needed to guarantee four gumballs of the same color?
(c) Would either answer be different if the machine were full of these three colors?

[15] 3. A group of 10 prisoners is offered a chance at being released. They will be lined up single-file facing forward; each will then have either a green hat or a purple hat placed on their head. Then, starting from the back of the line, each prisoner will in turn try to call out the color of their own hat. If every prisoner guesses correctly, they all go free; if anyone guesses wrong, they all go back to their cell. (They also lose if at any point any of them turn around or say anything other their guess.)

The prisoners are allowed to strategize ahead of time. Devise a strategy that allows them a 50% chance of going free.

[15] 4. True or False? Briefly explain. (a) $\mathbb{R} \subseteq \mathbb{Z}$ (b) $\mathbb{N} \subset \mathbb{Q}$ (c) $(\mathbb{N} \cap \mathbb{Q}) \subseteq \mathbb{Z}$

[10] 5. Give formal definitions of the following sets using set-builder notation.

A = {..., 1/8, 1/4, 1/2, 1, 2, 4, 8, ...} B = {0, 4, 16, 36, 64, 100, ...}

[10] 6. X is a set with 14 elements. Y is a set with 20 elements.

(a) What are the minimum and maximum values for $|X \cup Y|$?

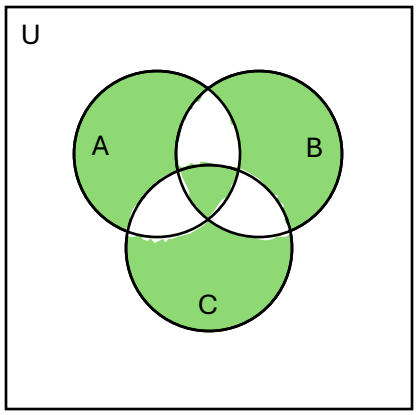
(b) What are the minimum and maximum values for $|X \cap Y|$?

[10] 7. Give a "simple" expression in terms of n for the nth term in each of these sequences.

(a) 2, 5, 10, 17, 26, 37, ... (b) -1, 2, -3, 4, -5, 6, ...

[10] 8. Write expressions using union, intersection, and/or complement for the shaded regions in the following Venn diagrams:

(a)



(b)