CSCI 2200 Foundations of Computer Science – Spring 2025 – Exam 2

Name: _____

RCS userid: _____@rpi.edu

RIN: _____

General instructions: *Legibly write your name and RIN above* ***and on your crib sheet****. Write as neatly as possible – we must be able to read your responses. Complete all problems to the best of your ability. Be sure to clearly explain your answers, especially in your proofs. You have until 10am to submit your exam.*

No electronic devices of any type (including calculators) are permitted. The back of the exam is blank and may be used for scratch paper.

When you are done, bring your exam, crib sheet, and ID card to the front of the room. Wait until the proctor has confirmed that you are all set before leaving the room.

Part 1 – Multiple choice (5 points each)

Circle the letter for the ***single*** *most appropriate answer. When indicated with a ♦, you must* ***show work*** *in the space to the right of the choices or you will not receive credit even if your answer is correct.*

1. How many natural numbers less than 55 are relatively prime to 55? ♦Show your work.

A. 14

B. 15

C. 40

D. 54

E. None of the above

2. How many ways are there to select a president, vice-president, secretary, and treasurer from a club with 20 members?

A. ${}_{20}P_4$

B. ${}_{23}P_3$

C. $\binom{20}{4}$

D. $\binom{23}{3}$

E. None of the above

3. Compute $\sum_{i=1}^{2n}(1+3i)$. (*Please note the sum goes to 2n.)* ♦Show your work.

A. $3n^2+5n$

B. $6n^2+5n$

C. $6n^2+8n$

D. $12n^2+8n$

E. None of the above

4. Let $T(1)=1, T(2)=2$, and $T(n)=2T(n-1)-5T(n-2), \forall n \geq 3$. What is $T(6)$? ♦Show your work.

A. -1

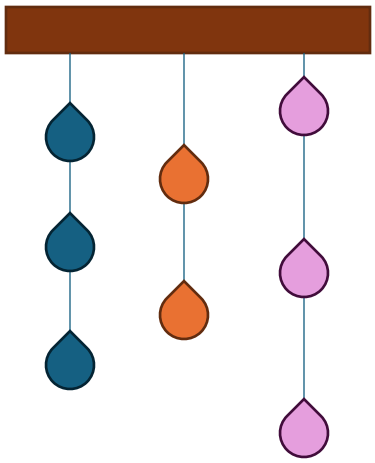
B. 1

C. 12

D. 22

E. None of the above

5. A carnival game involves a set of eight hanging water balloons: three in the first column, two in the second, and three in the third. (See diagram at right.) Players throw darts, and they win if they break all of the balloons, but there is a catch: they must always break the *lowest* remaining balloon in each column. In how many different orders can a player break the balloon while following the rules and thus win the game?



A. $8!$

B. $8! \div 3!$

C. $8! \div (3! \cdot 2! \cdot 3!)$

D. $10! \div (8! \cdot 2!)$

E. None of the above

Part 2 – Multiple selection (7 points each)

*Circle the letter of each answer that is correct; **if there are two or more, you must select all that apply**.*
*When indicated with a ♦, you must **show work** or you will not receive credit even if your answer is correct.*

6. Which of the following situations would be correctly counted using the expression $\binom{15}{3}$?

A. The number of 15-bit strings that have exactly 3 zeros

B. The number of ways to select a dozen donuts at a store that sells four varieties (glazed, frosted, apple cider, and Boston cream)

C. At a county fair pie-baking contest with 15 entrants, the number of ways to award a blue, a red, and a white ribbon

D. The number of different three-person teams that can be selected from a class of 15 students

E. None of the above

7. Which of the following are possible chromatic numbers (minimum # of colors needed) for W_n, the wheel graph on n nodes ($n \geq 4$)? ♦Explain your answer (which could involve drawing graph(s)).

A. 2

B. 3

C. 4

D. 5

8. ♦For this question, give a brief explanation for each choice.
Which of the following functions are $O(n^2)$?

A. $100\ n \log n$

B. $12n^2 + 98n + 16$

C. $6n \cdot 1.5^n$

D. $3n^2\sqrt{n}$

9. Which of the following are equivalent to 1 (mod 11)? ♦Show work / explain why for each choice.

A. 25

B. -10

C. 3^{10}

D. 11^{10}

10. Consider the degree sequence [3,3,3,3,1,1]. Which kind of simple graph could it describe? ♦Explain your answer (which could involve drawing graph(s)).

A. None – it's an impossible sequence

B. A disconnected graph

C. A connected tree

D. A connected planar graph

E. A connected non-planar graph

F. A complete graph

Part 3 – Free response

Respond to each prompt, showing all required work. ***Write legibly****!*

11. A *regular* graph is one in which every vertex has the same degree. Consider a regular bipartite graph, where the two sets of vertices are X and Y.

[n points] (a) Prove that $|X| = |Y|$.

[n points] (b) Prove that the graph must contain a perfect matching.

12. [15 points] Let P be a set of points in the Cartesian plane, defined recursively as follows:

$$(1,0) \in P \quad \text{and} \quad (x,y) \in P \rightarrow (x+1, y+2) \in P$$

Prove using structural induction that $\forall (x,y) \in P, y = 2x - 2$.

13. [10 points] How many integers less than 1,000,000 contain the digit 3? *(There are several ways to approach this problem. It is acceptable to leave your answer in the form of an arithmetic expression without performing the final calculations, as long as you show your work.)*