CSCI 2200 Foundations of Computer Science – Spring 2025 – Exam 1

Name: ____

RIN: _________________________________

General instructions: *Legibly write your name and RIN above **and on your crib sheet**. Write as neatly as possible – we must be able to read your responses. Complete all problems to the best of your ability. Be sure to clearly explain your answers, especially in your proofs. You have until 10am to submit your exam.*

No electronic devices of any type (including calculators) are permitted. The back of the exam is blank and may be used for scratch paper.

When you are done, bring your exam, crib sheet, and ID card to the front of the room. Wait until the proctor has confirmed that you are all set before leaving the room.

Part 1 – Multiple choice (5 points each)

*Circle the letter for the **single** most appropriate answer. When indicated with a ♦, you must **show work** in the space to the right of the choices or you will not receive credit even if your answer is correct.*

1. Which is the *easiest* proof technique to use to prove the claim: "If n^5 is odd, then n is odd"? ♦You must give a brief explanation of your choice.

A. Direct

B. Contraposition

C. Indirect (i.e. contradiction)

D. Standard induction

E. Leaping induction

Far easier to work with a fifth power of something than a fifth root.

2. Which of the following is **not** logically equivalent to $p \leftrightarrow q$? ♦You must demonstrate (by giving truth values for p and q) why your choice is not equivalent to $p \leftrightarrow q$.

A. $(p \wedge q) \vee (\neg p \wedge \neg q)$

B. $(p \rightarrow q) \wedge (q \rightarrow p)$

C. $\neg p \leftrightarrow \neg q$

D. $(\neg p \rightarrow \neg q) \vee (\neg q \rightarrow \neg p)$

E. None of the above – all of A-D are logically equivalent to $p \leftrightarrow q$

The problem is the OR in D. When p is true and q is false, the left half of that OR will still be true (and if p is false and q is true, then the righ half of the OR is true, but students only need to specify one or the other).

3. What is the first step in a proof by contradiction of this claim: "$\forall m, n \in \mathbb{N}: 3m + 6n \neq 10$"?

A. Define the predicate $P(m,n)$ as $3m + 6n \neq 10$ and prove the base case $P(1,1)$.

B. Assume that $3m + 6n \neq 10$ for some $m, n \in \mathbb{N}$.

C. Assume that $3m + 6n \neq 10$ for all $m, n \in \mathbb{N}$.

D. Assume that $3m + 6n = 10$ for some $m, n \in \mathbb{N}$.

E. None of the above

4. Let the universe of discourse be "all people on the RPI campus." Let $S(x)$ mean "x is a student", $V(x)$ mean "x owns a vehicle", and $C(x,y)$ mean "x and y are taking a class together". Which of the following is an accurate translation into predicate logic of the sentence: "Every student is taking a class with someone* who owns a vehicle." *(* not necessarily the same someone)*

A. $\forall x \exists y \left(S(x) \wedge C(x,y)\right) \rightarrow V(y)$

B. $\exists y \forall x\, S(x) \rightarrow \left(C(x,y) \wedge V(y)\right)$

C. $\forall x \exists y\, S(x) \wedge \left(C(x,y) \rightarrow V(y)\right)$

D. $\forall x \exists y\, S(x) \rightarrow \left(C(x,y) \wedge V(y)\right)$

E. $\exists y \forall x \left(S(x) \wedge C(x,y)\right) \rightarrow V(y)$

5. The set $S = \{\, n \mid n = (k-1)(-1)^k, \forall k \in \mathbb{N} \,\}$. Which of the following accurately represents part of S?

A. {0, -1, 2, -3, 4, -5, 6, ...}

B. {-1, -2, -3, -4, -5, -6, -7, ...}

C. {0, 1, -2, 3, -4, 5, -6, ...}

D. {-1, 2, -3, 4, -5, 6, -7, ...}

E. {0, -1, -2, -3, -4, -5, -6, ...}

Part 2 – Multiple selection (7 points each)

*Circle the letter of each answer that is correct; **if there are two or more, you must select all that apply**.*

*When indicated with a ♦, you must **show work** or you will not receive credit even if your answer is correct.*

6. Let the universe of discourse be "people who work at RPI." Let $B(x)$ be the predicate "x rides a bicycle to work" and $S(x, y)$ be the predicate "x works in the same building as y." Which of these are logically correct translations of "No one who works in the same building as Dan rides a bicycle to work."?

A. $\neg\exists x\,(S(x, \mathrm{Dan}) \land B(x))$ These are just progressive applications of DeMorgan.

B. $\forall x\,(\neg S(x, \mathrm{Dan}) \lor \neg B(x))$

C. $\exists x\,(\neg S(x, \mathrm{Dan}) \land \neg B(x))$

D. $\forall x\,\neg(S(x, \mathrm{Dan}) \land B(x))$

7. ♦For this question, you must show your work by making truth tables (one for the statement in the question and one for each possible answer).

Consider the logical statement $p \to \neg q$. Which of the following are logically equivalent?

A. $\neg q \to p$

B. $\neg p \land \neg q$

C. $\neg p \lor \neg q$

D. $q \to \neg p$

A

p	q	A
T	T	T
T	F	T
F	T	T
F	F	F

B

p	q	B
T	T	F
T	F	F
F	T	F
F	F	T

C & D (and the original statement)

p	q	C
T	T	F
T	F	T
F	T	T
F	F	T

8. You know, for some predicate P, that $P(1)$ is true and that $P(n) \to P(n+3)$ is true for any n. Which of these can you conclude is definitely true? ♦Show work / give an explanation of why you chose as you did.

A. $P(2)$ You have P(1), P(4), P(7), ... - in other words, P(3k + 1) for any integer k ≥ 0.

B. $P(3)$ Only 25 fits that pattern.

C. $P(25)$

D. $P(90)$

9. You are informed that each card in a set has an integer on one side and an English word on the other. You are then presented with six cards from this set. The visible faces of these cards read: cat, 12, bird, G, 2, 17

A rule is suggested: "If a card has a three-letter word on it, then it has either a single-digit number or an even number on it." Which cards **must** you turn over to verify this rule? ♦ Explain your reasoning.

A. cat

B. 12

C. 9

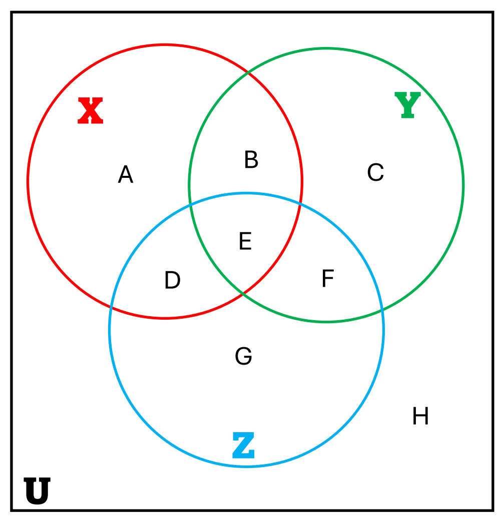
D. bird

E. 2

F. 17

The only way to falsify an implication is when the IF part is true and the THEN part is false. In A, we have the IF part true, and so must check the other side. In F, we have the THEN part false, and so much check the other side. None of the other cards have any chance of falsifying the rule.

10. The Venn diagram at right has three sets (**X**, **Y**, **Z**) represented with circles, as well as the universe of discourse (**U**) represented by a box. The three circles divide the area of the box into eight regions (areas bounded by lines/curves) – these are labeled A through H.



Which of these regions are included in the set $(X \cap \overline{Y}) \cup (\overline{X} \cap \overline{Z})$? (Circle your responses below.)

A B C D

E F G H

Scoring: -1 for each one wrong, to a minimum of 0.

Part 3 – Free response

Respond to each prompt, showing all required work. ***Write legibly!***

11. [15 points] Prove via induction that $\sum_{i=1}^{2n}(3i+1)=6n^2+5n$. *(Hint: How many terms will you add to the sum when you go from* n *to* n+1*?)*

Claim: $\forall n \in \mathbb{N}, \sum_{i=1}^{2n}(3i+1)=6n^2+5n$

Proof by induction.

Base case: n=1. $(3\cdot 1+1)+(3\cdot 2+1)=4+7=11=6\cdot 1^2+5\cdot 1$

Induction step: IF $\sum_{i=1}^{2n}(3i+1)=6n^2+5n$, THEN $\sum_{i=1}^{2(n+1)}(3i+1)=6(n+1)^2+5(n+1)$

12. [15 points] Prove the following claim using any desired method(s): For any positive integer n, n is odd **if and only if** $n^2 - 1$ is divisible by 8.

Required: TWO claims and TWO proofs!

Claim 1: If n is odd, then $n^2 - 1$ is divisible by 8.

Direct proof of Claim 1: Assume n is odd; then it can be written as $n = 2k + 1$, for some $k \in \mathbb{N}_0$. Therefore, $n^2 - 1 = (2k+1)^2 - 1 = 4k^2 + 4k = 4(k^2 + k)$. We have previously proven in class that k and k^2 much match in parity (that is, they are either both odd or both even). Odd + odd = even, and even + even = even, and so $k^2 + k$ is always even, so we can write $k^2 + k = 2m$, for some $m \in \mathbb{N}_0$. Thus, $4(k^2 + k) = 4(2m) = 8m$, which is clearly divisible by 8.

Claim 2: If $n^2 - 1$ is divisible by 8, then n is odd.

(Easiest proof is by contraposition; others possible.)
Proof by contraposition of Claim 2: Assume that n is even. Then (as proven previously in class) n^2 is also even, and thus $n^2 - 1$ must be odd. No odd number is divisible by 8, so $n^2 - 1$ is not divisible by 8. QED

13. [10 points] You have 8 trucks (with drivers), each of which can hold 50 gallons of fuel, and no other fuel containers. The trucks can travel 10 miles per gallon of fuel. Fuel may be freely transferred between trucks at any point.

You have a package that you wish to transport as far as possible. If you do not need the trucks or drivers to return to the starting point, what is the farthest you can possibly deliver the package?

The key idea is to drive all of the trucks until you have used up precisely one tank of fuel collectively across all of the trucks. Then stop, empty one truck to top off the tanks of all the others, and leave the empty truck behind. Repeat until the last truck is empty.

In the first step, drive until each truck has used 1/8 tank (i.e. 6.25 gallons, which gets you 62.5 miles). Use the remaining 7/8 tank in truck #8 to add 1/8 tank of fuel to each of trucks #1-7, bringing them back to full. Then continue with seven trucks, driving until each of them has used up 1/7 tank, etc.

The total distance will be $(1/8 + 1/7 + 1/6 + 1/5 + 1/4 + 1/3 + 1/2 + 1) * 50 * 10 \cong 2.72 * 500 \cong 1358$ miles

(Scoring note: We are not expecting students to get the final numerical answer. If they set up the correct expression, full credit. The "use ½ tank, ditch half the trucks, repeat" strategy (or any other reasonable approach that is not actually optimal) is worth 8 / 10.)